

On the Doctoral Thesis in Physics by Paweł Duch

“Massless fields and adiabatic limit in quantum field theory”

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(supervised by Prof. dr. hab. Andrzej Herdegen)

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1 The setting

Perturbative quantum field theory (pQFT) underpins the construction of the Large Hadron Collider at Geneva and prior accelerators of particles, leading to the discovery of the fundamental forces of nature, the most successful branch of fundamental physics to this day.

Nevertheless, the mathematical machinery behind the predictions of pQFT has long been obtaining its spectacular results by rather sickly methods of “ultraviolet renormalization”. Namely, by extracting finite values from expressions of the type “ $\infty - \infty$ ”. To date, the most satisfactory attempt to make the subject logically and mathematically respectable stems from a sound physical principle of relativistic causality, which goes back to pioneer work by von Stückelberg in the forties and Bogoliubov in the fifties, and was clearly stated and followed through by Henri Epstein and Vladimir Jurko Glaser in a landmark series of contributions in the seventies, particularly [1].

In the view of Epstein and Glaser, the problem of ultraviolet renormalization in pQFT is reduced to distribution theory, concretely to the problem of organizing and controlling the ambiguities in *extending distributions*, originally defined for non-coincident arguments; thus eschewing infinities. Their method (EGR) was definitely a great intellectual achievement, fundamental, elegant and correct, whose validity extends to curved spacetimes. However, in

spite of its conceptual appeal, it suffers from a reputation – only partially justified – of being less accessible from the computational viewpoint.

It is fair to say that EGR is still unknown to most workers in the field; whereupon methods based on regularization techniques with doubtful manipulations of infinite “counterterms” for the physical parameters still hold sway. Part of the reason for this slow percolation into mainstream physics is that, whereas EGR does not require the introduction-cum-removal of cutoffs in momentum space, it still requires cutoffs in position space, due to the so-called infrared problems of field theory. The price to pay to allay these is to replace the coupling constants of interacting particle physics models by coupling functions that dwindle away at long spacetime separations. To make then the bridge between EGR and the more physical constructs of “scattering matrix” theory, allowing understanding of collision processes at high energy, demands to “undo” the “undoing” of such a replacement. The latter process is the (so called by the founders) *adiabatic limit* of the theory.

As it turns out, the existence of the adiabatic limit is problematic as well. The adiabatic limit was established well enough for models involving exclusively *massive* particles, by Epstein and Glaser themselves – see that work in [2]. Their proof rests on nature of the spectrum of energy-momentum for multi-particle fields. Between the vacuum and the one-particle state space there is a gap, since the latter possesses energy at least equivalent to its mass. (There is again a gap between the one-particle and the two-particle states, after which supervenes the continuum.) This happens for a key “commutator function” for EGR. When going to position space, it means that the adiabatic long-distance limit can be performed. Typically, however, the carriers of interaction, the quantum of light in particular, are *massless* particles.

2 The thesis

This is where Paweł Duch’s thesis comes in. Adiabatic limit theory is, or was until his work, still anchored in the seventies. Whereas the Bogoliubov–Epstein–Glaser scattering matrix in general does not possess a proper limit, the latter is not strictly necessary for confidently making predictions of physical import. It is enough to establish the existence of a limit with good properties for a class of vacuum expected values of quantum fields, the so-called *n*-point or Green’s functions containing the physical information of the theory. This so-called *weak adiabatic limit* in models including massless fields was studied and established in a couple of models by Philippe Blanchard and Roland Sénéor [3] in 1975. And so the matter rested until today.

The work by Paweł Duch under examination amounts to *nothing less than a breakthrough* in this subject of great interest. The author proves existence, for all models of physical relevance, of the weak asymptotic limit. In this evaluator’s opinion, the methods leading to these results in the thesis are as important as the results themselves. The author establishes the

latter by going deeper into distribution theory, in particular bringing to bear the mathematical theory of *distributional values at a point*, due to the Polish mathematician Łojasiewicz [4].

The insight of Łojasiewicz runs against the received wisdom that distributions cannot take definite values at given points. However, under some circumstances such values can be defined and employed – as quantum field theorists do in some instances, without realizing it most of the time. It so happens that vanishing *in the distributional sense* can replace vanishing in the usual sense, while still keeping the expected properties of the resulting EG renormalization.¹

As a piece of scholarly work, Duch's thesis is well organized. Besides the preview of results, the four first chapters rather painstakingly and extensively summarize Epstein–Glaser theory in a form suitable for the sequel, introduce interacting models and the concepts of adiabatic limits. Chapters 4 to 7 show the existence of the asymptotic limit for the n -point functions. Chapter 5 is the heart of the thesis. After a sobering amount of mathematical spadework, the author proves the main theorem as advertised, and then Chapters 6 and 7 go on to show that essentially all the good properties of the renormalized vacuum expected values, and in particular the “central normalization condition”, are preserved under the new dispensation. This is bound to be useful in future applications of that theorem.

This reviewer would like to see concrete applications of the central normalization condition, and how they would relate to the expectations and inroads by Günter Scharf and others on the same matter. He does not think that scalar theory with two interaction vertices and scalar QED with the time-ordered product as given in Eq. (6.3.2) are truly distinct theories. These are not complaints: I just express the hope that the new construction of the n -point functions will soon be put to work in concrete models. The fledgling theory of string-local fields for pQFT [5, 6], which seems absolutely to require EGR, might be great a beneficiary of this thesis' conclusions and techniques.

There is no doubt in my mind that Paweł Duch is a very able mathematical physicist, and his thesis ought to be considered a lasting contribution to the development of pQFT.

Papers mentioned in this report

- [1] H. Epstein and V. J. Glaser, “The role of locality in perturbation theory”, *Ann. Inst. Henri Poincaré A* **19** (1973), 211–295.
- [2] H. Epstein and V. J. Glaser, “Adiabatic limit in perturbation theory”, in *Renormalization theory*, G. Velo and A. S. Wightman, eds., Springer, 1974; pp. 193–254.
- [3] Ph. Blanchard and R. Sénéor, “Green's functions for theories with massless particles (in perturbation theory)”, *Ann. Inst. Henri Poincaré* **23** (1975), 147–209.

¹This reviewer was fortunate in having learned in the past from Ricardo Estrada (the foremost expert nowadays in Łojasiewicz's theory) about distributional values. Otherwise, his task would have been even more difficult.

- [4] S. Łojasiewicz, “Sur la valeur et la limite d’une distribution en un point”, *Studia Math.* **16** (1957), 1–36.
- [5] J. Mund and E. T. de Oliveira, “String-localized free vector and tensor potentials for massive particles with any spin: I. Bosons”, *Commun. Math. Phys.* **355** (2017), 1243–1282.
- [6] J. M. Gracia-Bondía, J. Mund and J. C. Várilly, “The chirality theorem”, arXiv:1702.03383.



Signed by Prof. dr. José M. Gracia-Bondía, Heredia, Costa Rica, 22 August 2017