

Reviewer's report on doctoral thesis

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Title: Convex structures in quantum information – Symmetric and geometric methods

Thesis is a combination of results from four different research subjects – entanglement detection, convex optimisation, dynamics of observables and channel's nonlocality. In all of these areas student achieved relevant and nontrivial results. The fact he was able to cover and master all these subjects is demonstrating the abilities and research potential of Albert Rico. The achieved results are opening new interesting research directions of practical relevance. For me the most important achievements are development of theory of multipartite nonlocal witnesses and exploitation of decoherence to certify channel's nonlocality.

The introductory part clearly demonstrates the student acquired deep knowledge in mathematics of quantum information theory. Section 2 contains very interesting and relatively nontrivial results regarding polynomial entanglement witnesses, especially for multipartite systems. Section 3 investigates enhancement of SDP for problems with local unitary symmetries. Applications for computational problems is described. Section 4 defines a form of dynamics of observables reflecting stochastic transformations of effects. Section 5 defines nonlocality features of bipartite quantum channels and analyzes their behavior under special decoherence models. Section 6 studies the introduced nonlocality under complete decoherence. The key results of the thesis are nicely summarized in Section 7 and there is no need to repeat its content. Thesis is based on five research papers (one of them in preparation, one published in Phys. Rev. Letters and three submitted).

The thesis is written with erudition, contains negligible number of typos and achieves solid and interesting results. There is no doubt the both the reported research outcomes and the style meets the highest standards and criteria for PhD degree. Therefore, I recommend the thesis to be accepted as PhD thesis and evaluated with the highest possible mark. In particular, I believe the Thesis deserves to be distinguished by the 'Summa cum Laude' honour.

Comments:

- Section 1.1.6, especially the paragraph containing the figure - nothing is missing technically, but I would recommend to motivate more and provide more explanation of the “physics” of the decoherence in Choi matrix formulation.
- I do not completely follow why Section 2.5. Conclusions was not placed at the end of the whole Chapter and joined with Section 2.6.5. It is not something incorrect. I just found it quite puzzling and I am not sure whether there was some deeper reason for that. Similar thing is also in Chapter 4, where Section “Concluding remarks” is preceding Section “Further results”.
- I would expect this paper (Physical Review A 77, 030301(R) (2008)) to be mentioned in the Section 2. For me this was one of the first results on two-qubits entanglement detection using a four-copy witness.
- I found the definition of dynamics in Section IV very restricted. I would expect the perspective of higher-order structures to be discussed here and the considered form of dynamics will be introduced as special case we are going to be focused on. Again, there is nothing incorrect and there is no technical problem with the used concepts, only the approach is a bit restrictive to my taste.
- Defining properties in Equations 5.2.4x would deserve better explanation, or motivation. It would help potential reader, especially to someone not familiar with this concept.
- Especially in Section 5.3.3. I would recommend to use brackets in the mathematical expressions to avoid potential confusion.
- Figure 6.3.1. contains parameter λ , but the caption is using parameter μ .
- My last minor comment is related to the choice of the title. I do not have any alternative suggestion, but it matches with the content only in a very broad sense.

Questions:

- 1) In conclusion 3.10 it is mentioned that performed analysis suggests dominance for one numerical package over another one. Is there any intuitive understanding for this?
- 2) Is there any update on conjecture in 4.6.3.?
- 3) In Fig.5.3.2 there is Bell functional γ achieving values higher than 4 for some bipartite operations, which is a bit suspicious, because 4 should be the algebraic maximum of Bell functional. Might be I missed something in its definition. Could you pls explain?

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Mário Ziman