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Report on the PhD Thesis of Albert Rico

Convex Structures in Quantum Information Symmetric and Geometric Methods

The PhD thesis is devoted to the analysis of several interesting and important problems in quantum information theory. Convex structures play a key role in various aspects of this theory. Typical objects, such as quantum states, separable states of multipartite systems, quantum channels, and quantum superchannels (to name a few), define convex sets. Therefore, the analysis of new geometric methods is of primary importance. **I find the thesis to be excellent. It is very well written, with a coherent presentation and clearly stated results.**

The thesis is based on the following original papers of Albert Rico:

1. **Albert Rico** and Felix Huber, Entanglement detection with trace polynomials, Phys. Rev. Lett., **132**, 070202 (2024).
2. **Albert Rico** and Karol Życzkowski, Discrete dynamics in the set of quantum measurements, J. Phys. A: Math. Theor. **57**, 435302 (2024)
3. **Albert Rico**, Moises Bermejo Moran, Fereshte Shahbeigi, Karol Życzkowski, Certifying nonlocal properties of noisy quantum operations, arXiv:2407.02570
4. **Albert Rico**, Moises Bermejo Moran, Fereshte Shahbeigi, Karol Życzkowski, Channel nonlocality under decoherence, arXiv:2408.10317

The thesis consists of six chapters (all together 150 pages). The extensive bibliography includes 287 entries and contains all the most essential papers related to the subject of the dissertation.



Chapter 1 Basic concepts provides a mathematical background required for the analysis of the main problems of the thesis. Author introduces definitions of states, quantum maps, entanglement witnesses, and generalized measurements. Brief introduction to semidefinite programming, and nonlocality of bipartite states and quantum channels is also included.

Chapter 2 Entanglement detection with trace polynomials provides a new method for nonlinear entanglement detection. Traditional methods are based on the idea of linear entanglement witnesses, i.e. Hermitian operators with at least one negative eigenvalue and nonnegative mean values on all separable states. Interestingly, nonlinear correction can make the entire detection method more effective. In this way one can identify witnesses for which linear detection of an entangled state fails, but for which nonlinear detection succeeds. The author analysis the following problem: given a linear entanglement witness W , can it be employed also in a nonlinear fashion as to detect entanglement in multiple copies of a given quantum state? The analysis of this Chapter proves that it is possible in general multipartite scenario. Moreover, an interesting nonlinear scenario bases on particular matrix inequalities is proposed. In particular, the so-called immanant inequalities are used to construct a family of entanglement witnesses. Now, introducing a tensor product of witnesses constructed this way one obtains an effective nonlinear detection scenario. In particular, it is shown that this approach is suitable to randomized measurements. Presented techniques were used for constructing a new family of positive maps that extends existing results. For example the well known reduction map is generalized to a family of maps able to detect more random entangled states. I find this results presented in this chapter very interesting and well presented.

Question: It is not clear for me how this method works for bound entangled states.

Chapter 3 Unitary invariant semidefinite programs: a user's guide analyses semidefinite programming under the presence of additional symmetries. Such invariant semidefinite problems are quite common in quantum information theory. The author shows how to reduce unitary invariant problems in practice. Presented analysis includes the symmetry-reduction through the standard Schur-Weyl duality, reduction of the number of variables with the good permutations basis, and further simplification by optimizing over expectation values. Presented approach allows to solve several problems in quantum information.

Chapter 4 Discrete dynamics in the set of quantum measurements describes discrete transformations in the set of quantum measurements in terms of blockwise stochastic matrices (composed of semipositive blocks that sum columnwise to identity). A general measurement represented by a POVM is encoded into a column vector and then an appropriate sequential (or a star) product of matrices enables one transform an original POVM to another POVM via a blockwise stochastic matrix. This providea a natural generalization of the well known classical

scenario. The author presents several interesting observations. In particular it is shown that two POVMs $\{P_i\}$ and $\{Q_j\}$ are compatible if and only if P can be transform to Q via some blockwise stochastic matrix (Proposition 7). Another elegant result generalizes a classic theorem stating that a stochastic matrix B is bistochastic if and only if any probability vector p majorizes its image Bp . Introducing a class of blockwise bistochastic matrices (Definition 6) the author shows that a star product of any blockwise bistochastic matrix and sortable POVM is always majorized by an original (sortable) POVM. Results presented in this chapter were already published (with K. Życzkowski).

Questions: Is it possible to introduce thermomajorization in this scenario? Is it possible to generalize Sinkhorn algorithm for blockwise (bi)stochastic matrices?

Chapter 5 Certifying nonlocal properties of noisy quantum operations provides a framework to certify nonlocal properties of quantum channels in the presence of dephasing noise. Proposition 9 establishes interesting relation between nonlocal properties of bipartite quantum channels and the probability distribution obtained by an appropriate measurement protocol (illustrated on Fig. 5.3.1). This includes three important classes of channels: LOSR (local operation assisted by shared randomness), LOSE (local operation assisted by shared entanglement), and qNS (quantum non-signaling). Introducing the concept of classical and quantum superchannels the author found the necessary and sufficient conditions allowing to transform a bipartite channel E to channel E' by a convex combination of product quantum superchannels (Proposition 10). This result provides explicit necessary and sufficient conditions to certify non-LOSR operations. Another interesting observation (Proposition 11) states that a dephasing channel acting on a separable bipartite state can generate an entangled state, and a dephasing quantum superchannel acting on an LOSR operation can generate a nonlocal bipartite quantum channel. Finally, a complete decoherence (corresponding to diagonal Choi matrices) is analyzed. Proposition 12 shows that the set of local/quantum/non-signalling correlations coincides with the set of decoherent actions of LOSR/LOSE/qNS bipartite quantum channels. I find the analysis in this chapter to be excellent!

Chapter 6 Nonlocality of decoherent quantum channels quantifies nonlocality of bipartite quantum channels and identifies its component resisting the effects of dephasing noise. The nonlocal resources of a channel are quantified in terms of a suitable functional which essentially measures the "distance" of a given bipartite channel from a convex set of LOSR channels. Its properties are fully characterized by Proposition 13. Typical examples of such functional include trace distance, diamond distance and relative entropy (here defined as relative entropy between corresponding Choi matrices). The author studies nonlocal properties under complete decoherence superchannel. Proposition 14 characterizes the decoherent nonlocality of a channel and the nonlocal resources present in the classical distribution defining its decoherent

action. Interestingly, the measures of decoherent nonlocality can be seen as the amount of nonlocal resources that remain after complete decoherence occurs.

Question: is it possible to distill nonlocality to obtain maximally nonlocal channel?

The final conclusion: in my opinion the PhD thesis of Albert Rico makes an important contribution to the field of quantum information theory. It presents significant developments of new methods and provides important and very interesting results. It should be noted that several of these results have already been published in two articles in Physical Review Letters and Journal of Physics A. In view of the above, I conclude that the dissertation submitted for my review meets all the statutory and customary requirements for doctoral dissertations, and I recommend it for admission to the next stage of the procedure. **In my opinion, Albert Rico fully deserves to be awarded a PhD in Physics. Furthermore, this PhD thesis should be considered for acceptance with honors.**

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