

Report on the PhD thesis

"Convex Structures in Quantum Information: Symmetric Structure in Quantum Information" **by Albert Rico**

General overview. The thesis entitled *"Convex Structures in Quantum Information: Symmetric Structures in Quantum Information"* was written by Albert Rico at the Jagiellonian University in Kraków under the supervision of dr Felix Huber. It is over 150-pages long and is based on four articles co-authored by the Candidate, of which one was published in the very good *Physical Review Letters*, whereas the other three can be found on the open-access preprint repository arXiv.org. The thesis contains also some additional unpublished material. All the publications are multi-author, but the Candidate is the first author in all of them. Overall, this is a substantial piece of work both in results and size that in my opinion goes beyond typical doctoral theses.

The thesis is written in very good English which significantly facilitates reading and understanding the material. Still, I was able to spot a few linguistic errors here and there, which, however, are not worth mentioning here. On the other hand, there is something that I must point out here, namely, the Polish abstract lacks some of the Polish letters like „ę” or „ą”.

The thesis consists of a preface and seven chapters, of which five contain original results (Chapters 2–6), and in fact each of them, except for Chapter 3, corresponds to one of the publications mentioned above. These will be described in a more detailed way in the next part of this report. Here, let me shortly comment on the remaining parts of the thesis. Preface outlines the research context and motivation. Chapter 1 contains a substantial introduction to the basics of quantum theory as well as to the most relevant concepts and techniques originating from quantum information theory, convex optimization or group theory which are exploited later in the thesis. The thesis concludes with Chapter 7 which summarizes the obtained results and also outlines a couple of open questions that are inspired by the work. While all this reads well, I am not sure whether I can agree with the first sentence of Preface „Quantum information theory describes quantum physics in non-relativistic regimes, ...”. I would say non-relativistic quantum physics is a proper theory on its own and doesn't need quantum information to be described. If the author meant that quantum information is based on quantum physics in non-relativistic regimes, then I would be in favor of such a formulation, noting, however, that one can also have quantum information theory in relativistic regimes. In the same spirit, I don't quite agree with the first sentence of the abstract, i.e., “Quantum information theory aims to establish a mathematical ground to describe nature at small scales”, because in my opinion that's exactly the aim of quantum theory.

Results. The main aim of this thesis is to address several problems in quantum information and the foundations of quantum theory, such as entanglement detection and the simulability of quantum channels using various resources. This is achieved through approaches that leverage the symmetries and geometric properties of the systems under consideration. Such approaches have proven highly effective across many areas of quantum information, particularly for studying entanglement in bipartite and multipartite states with symmetries, with the works of Werner and collaborators standing out, or for developing tools to determine whether correlations are Bell nonlocal or belong to the quantum set, leveraging the convexity of the underlying sets, with the

Doherty-Parillo-Spedalieri or Navascués-Pironio-Acín hierarchies being the most known examples. Here, the Author follows this research direction and identifies several areas where these approaches can still offer new insights. In fact, the thesis contains a significant number of new results that not only provide answers to the questions mentioned above but also in my opinion open new interesting lines for further research. Let me now describe the obtained results in a more detailed way and shortly comment on them.

Chapter 2. The first research chapter focuses on entanglement detection using nonlinear entanglement witnesses, specifically those defined on multiple copies of a given quantum state. The study and development of such multicopy witnesses is motivated by their generally greater effectiveness in detecting entanglement compared to the standard linear entanglement witnesses. This advantage comes nevertheless at the cost of increased implementation complexity for multicopy witnesses.

The chapter starts by presenting a general method for constructing multicopy entanglement witnesses from linear ones. A very interesting observation is then made that this construction can lead to nonlinear witnesses that detect entanglement of quantum states for which the underlying linear witnesses fail to do so. Next, the Author applies this prescription to linear witnesses generated by matrix immanant inequalities, which leads to a set of interesting multicopy trace polynomial entanglement witnesses; these are listed in Table 2.1. The detection power of an exemplary witness from this class is tested on the three-qutrit Werner states. At the same time, the Author provides a way to implement the trace polynomial witnesses through randomized measurements on a single copy of a state which allows to avoid measuring a witness on many copies of a state which apart from some simple cases is rather unachievable in experiment.

I find this part of the thesis highly interesting and at the same time the most complete among the research chapters of the thesis. In fact, it proposes a general construction of entanglement witnesses and studies the construction really thoroughly. What is more, I am impressed by the number of concepts and techniques that are implemented here to provide and analyze the construction. I believe it will have a strong impact on the field. Nevertheless, I have a few comments:

1.1. I think it would be very useful to compare the new nonlinear witnesses with the other multicopy entanglement witnesses already existing in the literature, such as for instance those based on the entropic inequalities introduced already quite some time ago [see, e.g., <https://arxiv.org/pdf/quant-ph/0202058>] and prove superiority of the new construction.

1.2. It would have been interesting to explore the effectiveness of these new witnesses in detecting PPT entangled states, in particular in the bipartite case. In the case of NPT states, I believe that the problem of entanglement detection through multicopy witnesses is sort of solved: in any fixed Hilbert space, all entangled NPT states can be detected through a finite set of multicopy witnesses that are constructed from the moments of the partially transposed density matrix; how to measure such moments was described in [H. Carteret, Phys. Rev. Lett. **94**, 040502 (2005)]. For two-qubit or qubit-qutrit systems this can be further simplified by measuring the symmetric polynomials of eigenvalues of partially transposed density matrix: in the two-qubit systems a single witness on four copies of the state is sufficient and for qubit-qutrit systems three witnesses on six copies are sufficient.

1.3. As for the literature I think that the Author could have mentioned some of the experimental works or proposals devoted to entanglement detection with multicopy witnesses such as [F. Bovino *et al.*, Phys. Rev. Lett. **95**, 240407 (2005)], where two-copy witnesses corresponding to a particular entropic inequality are measured or [K. Bartkiewicz *et al.*, Phys. Rev. A **91**, 032315 (2015)] where a method to experimentally implement a four-copy witness detecting all bipartite entangled state is proposed.

1.4. I have also a small remark of an editorial nature. After conclusions on page 62, the Author added some more material which has its own conclusions. This is quite confusing. I understand that the additional results are simply appendices from the corresponding publication, however, in my opinion the Author could have reorganized all the results in the thesis to avoid this type of division. The same happens in Chapter 4.

Chapter 3 is devoted to semi-definite programming which is an instance of convex optimization methods. Due to its effectiveness and the relative simplicity of formulation, semidefinite programming is currently one of the most widely used exact numerical methods in the field and has led to many important breakthroughs as well as allowed to formulate several relevant tools in the field such the Doherty-Parillo-Spedalieri hierarchy for entanglement detection or the Navascués-Pironio-Acín hierarchy used to bound the set of quantum correlations in Bell scenarios; both hierarchies are in fact used in the thesis. Yet, the applicability of this technique is severely constrained by the dimension of the underlying quantum system which for multipartite systems grows exponentially with the number of individual particles making the SDP problems intractable with standard desktops.

Here the Author provides a guide how to significantly simplify the semidefinite programs if the problem under study is invariant under the multilateral action of the unitary group. The chapter collects the known tools and concepts such as Schur-Weyl duality and good permutation basis and shows how they can be combined to reach such a simplification; concrete numbers are presented in a few tables such as Table 3.1 which shows how the number of free variables drops in case of real problems. The approach is illustrated in the case of the SDP programs for selected problems such as the max cut problem or the marginal problem for the Werner states.

While this part of the thesis will be a valuable contribution to the field, it leaves some dissatisfaction. Namely, the Author shows how to reformulate the semidefinite programs for several relevant problems in quantum information. However, the thesis doesn't really explore the power of these new formulations in comparison to the results in the literature. For instance, it would be very useful to clearly demonstrate the advantages of the present formulation of the semidefinite program in terms of computation time or the system size for the max cut problem as compared to the previous, symmetry-based formulation of Refs. [164,165].

Chapter 4 is concerned with discrete transformations of quantum measurements as a generalization of transformations of classical probability vectors through the action of stochastic matrices. More specifically, probability vectors are generalized to vectors of measurement effects, whereas the stochastic matrices to blockwise stochastic matrices, and the action of the latter on the former is defined through the blockwise product. Such a framework is motivated by the sequential measurements scenario in which measurements are performed in a sequence on a quantum state where similar transformation of measurement operators occur. Then, the properties of those objects as well as the blockwise product are thoroughly studied and these are confronted with the classical case of stochastic matrices and probability vectors. While there is no point in describing all the findings here—they are all nicely summarized in Table 4.1 in the thesis—it is worth mentioning that a well-known majorization criterion for transformability of one probability vector into another via stochastic matrices was generalized to the case of quantum measurement and is formulated through certain majorization for measurement effects; yet, the criterion can only be formulated for the so-called sortable measurement vectors. Moreover, the authors prove that one measurement vector can be transformed into another one iff both are jointly measurable.

I found this part of the work very interesting. It not only introduces and explores a general framework for describing transformations of quantum measurements, which, in particular, provides a nice interpretation of measurement incompatibility, but also includes a great deal of nice mathematics. It certainly reflects Albert's high technical skills very well.

I would have here two questions that the Candidate might want to address during the defence.

2.1. The first one is whether it is possible to describe in this language a situation in which more than two measurements are performed in a sequence on a given quantum state.

2.2. The second question is whether the Author has tried to exploit this formalism to design some criteria for checking whether two measurements, not necessarily projective, are incompatible.

Chapter 5. Here the Author provide schemes to certify nonlocality of quantum channels, or, more precisely, to verify whether a given channel belongs to one of the class: LOSR (local operations assisted by shared randomness), LOSE (local operations assisted by shared entanglement) and qNS (quantum nonsignalling), where the first class of channels is included in the second one which is then included in the third one. Certain specific schemes are considered which are summarized on Fig. 5.3.1. Then, a series of conditions (mostly necessary) are derived for a channel to belong to one of the above classes which are formulated in terms of the resulting bipartite correlations. This in turn allows one to employ the known methods from the area of Bell nonlocality to make nontrivial statements on the membership of a channel in a given set. In fact, a necessary and sufficient condition for a channel to belong to the LOSR class is formulated in terms of Bell functionals. Next, it is discussed how decoherence and in particular dephasing can affect the possibility of detecting nonlocality of a channel where a few situations are considered depending on whether dephasing acts jointly or separably on the inputs and outcomes of a quantum channel. An important observation here is that the joint dephasing might result in drawing wrong conclusions about a channel: namely, a channel that initially belonged to the LOSR class might be certified as not belonging to that class and vice versa.

Then, **Chapter 6**, provides continues on characterization of the impact of complete decoherence on nonlocality of bipartite quantum channels. This time, the Author provides means to quantify the channel nonlocality. In particular, the notion of decoherence nonlocality is introduced which quantifies the nonlocality which remains in the channel after the action of decoherence on its inputs and outputs. Next, the Author relates this decoherent nonlocality to the performance of certain quantum protocols where various numerical methods used to achieve this such as for instance a certain adaptation of the Doherty-Parillo-Spedalieri hierarchy presented on page 131.

This part of the thesis is an interesting continuation of the previous works on characterization of nonlocal properties of bipartite quantum channels which also provides a framework for experimental verification of whether a channel belongs to one the above three types within various frameworks, in particular the device-independent one. Finally, it nicely relates the geometry of various sets of bipartite correlations to the geometry of the sets of quantum channels.

While, at the end of Chapter 5 the Author states some open questions, I would have yet another one, namely, whether one can use the NPA or the Lasserre hierarchies used to bound the sets of quantum and classical correlations, respectively, to introduce analogous hierarchies for the sets of quantum channels in the device-independent setting. It seems that the Author uses the NPA hierarchy in this context, which is reflected on Fig. 6.3.1, but there is no deeper discussion in the text. Another hierarchy is presented on page 131 but this one is defined in the device-dependent context.

Conclusion. The introduces several highly nontrivial and original results which in my opinion will significantly impact the fields of quantum information and foundations of quantum physics. Of all the results, the ones that impressed me the most are those presented in Chapter 2 and Chapter 4, that is, the construction and analysis of multicopy entanglement witnesses and the general framework for discrete quantum measurement transformations. Another highly positive aspect of the thesis that should be highlighted here is the number of different question it asks (entanglement detection, nonlocality of quantum channels, transformations of measurements) and also the large variety of concepts, tools and techniques it brings together. All in all, this

proves that the Candidate's knowledge of the field is very broad implying that the candidate has gained the necessary knowledge to obtain the doctoral degree in physics.

Taking all this into consideration, I am sure that the thesis satisfies all the formal requirements established by the respective act of laws and as such I recommend that Albert Rico be admitted to the subsequent stages of the doctoral proceedings, in particular the public defense.

Also, given that: (i) the thesis contains highly nontrivial and valuable results, (ii) it is thematically very broad demonstrating that the Author has mastered a wide range of concepts, notions and techniques from quantum theory and quantum information, and (iii) the Author made major contribution to all the papers on which the thesis is based, I would like to request that the doctoral thesis by Albert Rico be distinguished.

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