

Review of the doctoral dissertation by
mgr Moisés Bermejo Morán:
*Characterizing Quantum Behaviours:
Polynomial Optimization in Quantum
Information.*

Introduction to the Review

The dissertation titled *Characterizing Quantum Behaviours: Polynomial Optimization in Quantum Information* by mgr Moisés Bermejo Morán presents a thorough exploration of the intersection between quantum physics and advanced mathematical techniques, specifically polynomial optimization. This work comprises **91 pages** structured into **five main chapters**, along with supplementary sections including an abstract and bibliography.

The structure of the dissertation is logical and well-organized. It begins with an introduction to foundational concepts in quantum mechanics, algebra, and convex optimization, setting the stage for subsequent technical discussions. The core chapters delve into specialized topics:

- **Chapter 2:** Focuses on special constraints, such as Gröbner bases and commutation relations, vital for optimizing quantum behaviors.
- **Chapter 3:** Explores extensions of Bell scenarios and other correlation settings, linking theoretical frameworks to practical quantum information problems.
- **Chapter 4:** Addresses uncertainty relations, providing both theoretical insights and applications in quantum technologies.

The title of the dissertation aptly reflects its content, as it emphasizes the characterization of quantum behaviors through polynomial optimization methods. The title's focus is realized in the text, which marries mathematical rigor with the physical principles underlying quantum information.

The narration is dense yet methodical, catering to an expert academic audience. The author systematically develops the discussion, introducing concepts progressively while anchoring them in established theories and innovative contributions. This style ensures clarity in presenting complex ideas while maintaining a high level of precision and depth.

Subject of the Dissertation

The dissertation explores the intricate mathematical and physical principles underlying quantum information theory. The central theme of the work is the characterization of quantum behaviors, employing advanced algebraic techniques from the domain of polynomial optimization. These techniques include sum-of-squares decompositions and their translation into semidefinite programming frameworks.

The primary objectives of the dissertation are twofold:

- To distinguish quantum behaviors from classical and post-quantum theories through algebraic characterizations.
- To develop computational tools that enhance the understanding and practical application of quantum behaviors in areas such as cryptography, information processing, and quantum computation.

The research specifically focuses on:

- *Generalized Bell Scenarios*: Studying measurement processes on subsystems with overlapping components, thereby extending classical notions of locality in quantum mechanics.
- *Uncertainty Relations*: Introducing algebraic frameworks to analyze and bound quantum behaviors, particularly in contexts involving uncertainty in measurement outcomes.
- *Partially Commutative Variables*: Developing computational methods to efficiently handle physical systems with local structures and constraints, significantly improving computational accuracy and performance.

This dissertation uniquely integrates mathematical rigor with physical insight. The algebraic methods not only address fundamental theoretical questions but also propose practical solutions for characterizing and optimizing quantum behaviors

in various scenarios. The work's contributions have implications for the foundational understanding of quantum mechanics and for applied fields such as quantum cryptography and secure communication.

By bridging the gap between mathematical abstraction and physical application, this dissertation provides a valuable framework for advancing both theoretical and practical aspects of quantum information science.

Final Assessment and Conclusions

The dissertation *Characterizing Quantum Behaviours: Polynomial Optimization in Quantum Information* by mgr Moisés Bermejo Morán represents a significant contribution to the fields of quantum physics and mathematical optimization. Through a meticulous exploration of quantum behaviors and their characterization using polynomial optimization, the author bridges theoretical physics and computational mathematics in an innovative manner.

One of the most significant scientific achievements of mgr Moisés Bermejo Morán is the development of a novel framework for characterizing quantum behaviors using polynomial optimization techniques. By extending traditional methods to incorporate partially commutative variables, the author provides an innovative approach to dealing with physical systems that exhibit local structures, thereby boosting computational efficiency and accuracy. This work not only advances the theoretical understanding of generalized Bell scenarios—where measurements act on overlapping subsystems—but also introduces powerful algebraic tools that have practical applications in quantum information processing. These contributions, grounded in rigorous mathematical foundations, open new pathways for analyzing uncertainty relations, cryptographic protocols, and the fundamental limits of quantum mechanics. The author's ability to bridge abstract algebraic methods with concrete physical phenomena marks a valuable and original contribution to the field.

Strengths of the Dissertation

- **Innovative Approach:** The integration of polynomial optimization techniques, such as sum-of-squares decompositions and semidefinite programming, provides novel tools for addressing challenges in quantum information science.
- **Breadth and Depth:** The work covers a wide array of topics, including Bell scenarios, uncertainty relations, and the handling of partially commutative variables. Each topic is treated with both theoretical rigor and practical relevance.

- **Impactful Contributions:** The dissertation's findings have implications for quantum cryptography, computation, and information processing, making it valuable for both foundational and applied research.
- **Clarity and Structure:** The document is well-organized, with a logical progression from basic principles to advanced topics, making it accessible to readers with varying levels of expertise.

Areas for Potential Improvement While the dissertation achieves its objectives admirably, there are areas that could be further developed:

- **Practical Applications:** While the theoretical results are strong, additional emphasis on real-world implementations and experimental validation could enhance the practical impact.
- **Accessibility for Broader Audiences:** The dense mathematical exposition may limit accessibility for readers outside the immediate field. Including more intuitive explanations or graphical representations could broaden the reach of the work.

Conclusions In conclusion, this dissertation demonstrates outstanding academic quality and originality. It successfully addresses complex theoretical questions while laying a foundation for future research in quantum information science. The author's ability to connect algebraic techniques with physical phenomena showcases a high level of expertise and creativity.

I believe that the presented dissertation meets the statutory requirements doctoral theses in the field of exact and natural sciences in discipline of physical sciences and I request that it be accepted by Disciplinary Council of Physical Sciences of the Jagiellonian University in Kraków, and o allowing for public defense.