

Report on Jakub Janarek's PhD thesis

General overview

The PhD thesis reports new results about (quantum) transport in disordered systems, more precisely about the boomerang effect (BE), i.e. the fact that the center of mass of a wave packet, when launched with a finite velocity, goes back to its initial position. This phenomenon does not occur for a classical particle and is a direct consequence of Anderson localization; the later resulting from an interplay between disorder and interference effects. More specifically, the thesis studies two possible scenarios that could break the perfect BE: (i) breaking time reversal symmetry (TRS); (ii) the impact of the interactions between the particles.

The manuscript is generally very well written, easy to follow, with, in each chapter, the relevant amount of information for the reader to follow and understand the novelties brought by this work.

Comments

- Chapter 1:

This chapter is a general introduction to the subject of (quantum) transport in disordered systems. It nicely explains the main concepts (weak and strong localization, scaling approach) and tools (diagrammatic tools, numerical computation) involved. It also nicely reviews the main experimental results, in particular based on cold atoms, together with a detailed introduction to the main topic of the thesis, namely the boomerang effect. This introduction is well balanced and clearly emphasizes the broad overview by the student about the transport in disordered systems.

Even though it is not a crucial point, it would have been nice to mention more explicitly the difference between the mean-free path and the localization length, for instance, by explaining the difference between the disorder average of the wavefunction $|\Psi(x, t)|^2$ (or of the Green's function), governed by Dyson's equation, and of the intensity $|\Psi(x, t)|^2$ governed by Bethe-Salpeter's equation. The later does not appear in the manuscript even though the average of the product of the retarded/advanced Green functions is a central object in the thesis.

- Chapter 2:

This chapter presents new surprising results about the boomerang effect (BE): contrary to what people believed, breaking TRS does not prevent a perfect return of the wavepacket to the origin. The results are based on a spinful model with a spin-orbit coupling.

The study in this chapter is extremely thorough: (i) it leaves no loophole for a possible exact hidden TRS-like that would explain the perfect BE; (ii) a rather intricate diagrammatic approach has been developed for the system emphasizing a perfect agreement with the exact numerical data. All in all, it gives very convincing results for a perfect BE even when TRS is broken.

I would like to mention that the work presented in this chapter is very impressive: the diagrammatic approach is a really difficult one, but, at the same time, it is very nicely explained.

Concerning the breaking of TRS, even if it seems that there is no exact TRS-like symmetry for the Hamiltonian, would it be possible to explain the perfect BE with the $\mathcal{PK}$ symmetry, i.e. the product of the parity with the complex conjugation? This is symmetry of the system without disorder and even if it is broken for each disorder configuration, it is restored after averaging over the disorder, like in section 1.3.3 for the usual BE.

- Chapter 3:

This chapter is dedicated to the impact of interactions on the BE in the case of a Bose-Einstein condensate, a situation relevant for experiments performed with cold atoms. Using the Gross-Pitaevskii equation (GPE), i.e. a meanfield description of the condensate, the BE is shown to be only partial, a direct consequence of the nonlinearity of the GPE, known to (partially) break Anderson localization. The final displacement is shown to increase with the interaction strength (quadratically at low strength and linearly at large strength). Furthermore, performing a thorough study of the amplitude of the final displacement as a function of the different parameters (disorder strength, interaction, width of initial wavepacket), it is shown that the BE actually only depends on a universal scaling parameter the nonlinear energy, a measure of the interactions among the particles computed after the break time; the later roughly corresponds to the time at which the interactions “stop” the BE. The nonlinear energy is also shown to emerge as a natural quantity in the self-energy, i.e. in the mean-free time.

There are many new results in this chapter. In addition, the ideas and the reasoning that lead to the finding of this scaling parameter are quite fascinating. This emphasizes again the excellent level of the work and the quality of the manuscript.

- Chapter 4:

This chapter is also dedicated to the impact of the interactions on the BE, but now using an exact method, namely TEBD (again nicely summarized). The numerical simulations are extremely demanding such that results here are limited to a few particles. Nevertheless, they confirm the findings of the preceding chapter in the low interaction strength U , but with a final displacement found to be larger than the one predicted by the GPE. The main reason is that the exact quantum state, measured by the numerical one-body density matrix, is far from being the one of a pure condensate and therefore imperfectly described by GPE. On the opposite limit, i.e. extremely strong interactions, the Tonks-Girardeau gas, the mapping to weak interacting fermions shows that the BE is almost perfectly restored, with a behavior mirroring the weak interaction limit after a mapping $U \rightarrow 1/U$. Finally, for both situations, the study of the interaction energy and the break time seems to indicate a similar universal dependence on a single parameter.

This chapter is really the state of the art of exact numerical simulations and of getting the best informations from them. At the same time, there is always a great care about providing (simple) physical interpretations of the results.

Concerning the one-body density matrix: for a $\mathcal{PT}$ symmetric state it fulfills $\rho(x, x') = \rho(-x', -x)$. From that point of view, the BE provides an information about this symmetry, but along the diagonal $x' = x$ only. Even though for long time, $\overline{\rho(x, x')}$ is essentially correlated along the diagonal, it might be interesting, in particular for the exact propagation of the ground state, to study how much the $\mathcal{PT}$ of ρ is broken/restored as a function of time.

Conclusion

All in all, the level of the work presented in this thesis is really excellent and the so is the quality of the manuscript. Therefore there is absolutely no doubt that the thesis should be defended and that the student should be granted a PhD degree.

Minor comments-typos

- Introduction, p. 1: "the particle will stop its movement". The residual motion is diffusive, not really stopped.
- Introduction, p. 7: Eq. 1.17: a minus sign is missing in the exponential. Also, the same notation, P , is used for the distribution of V and for the correlation of the disorder.
- Chapter 2: The notation $|\uparrow\rangle$ or $|\downarrow\rangle$ is a bit confusing: it's not always immediately clear when it means the computational basis and when it means the eigenstate at a given k of H_0 . Maybe defining right after eq. 2.3, the eigenstates of H_0 as follows: $|k\rangle = \begin{pmatrix} |k, \uparrow\rangle \\ |k, \downarrow\rangle \end{pmatrix}$, i.e. keeping k in the ket $|\sigma\rangle$ rather than using $|\sigma\rangle_{\pm}$ would help. This is somehow done in Eq. 2.29, but without using the ket notation.
- Chapter 2, p. 43: I think one needs a bit more information about the spin transfer, i.e. the evolution of the populations in each spin component, during the time evolution. From the analysis done in the thesis, can we guess that it saturates after a few τ ?
- Chapter 2, p. 58: One could have use a different notation for the " L_m " that does not depend on x .
- Chapter 3, p. 81: just after 3.1.1 : neglectig $\rightarrow$ neglecting.
- Chapter 3, p. 91: just before Eq. 3.25 : brak $\rightarrow$ break.
- Chapter 4, p. 129. Eq. 4.54: It's not fully obvious for everyone that V_F is attractive. Either writing it in momentum space or providing, for instance, the computation for a 2 fermion wavefunction would help.
- Chapter 4, sec. 4.7: "Now, with weakly interacting fermions, we do not have such help...". In the presence of attractive interaction for the fermionic system, even though it's not local, would it be possible to perform a BCS-like meanfield self-consistent approach to the time evolution and compare with the exact numerical results?
- Chapter 4, sec. 4.8: For a spinful fermionic system with attractive interaction the typical size of the correlation diverges exponential fast at weak pairing, i.e. in the BCS regime, such that the finite size of the system becomes rapidly an issue. Is this difficulty relevant here?
- Chapter 4, p. 134 Eq. 4.60: how to understand the relation between the two timescales, i.e. comparing a non-disordered model to the disordered one?

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