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Marseille, 2024, May 28

Report on

Geometric spectral functionals
Paweł Zalecki
PhD thesis

This thesis is dedicated to understanding how classical notions such as curvature, the Einstein-Cartan tensor, and torsion can be transformed in noncommutative geometry (NCG).

Differential geometry plays a fundamental role in both mathematics and physics. Classical geometry of manifolds has been encompassed within the broad field of NCG thanks to the work of A. Connes. It quickly became essential to generalize fundamental concepts such as the metric, scalar curvature, and torsion to this new framework. However, progress in this direction has been slow and marked by unconvincing direct attempts.

This thesis, which is part of a collaboration with Ludwig Dąbrowski and Andrzej Sitarz, presents a realistic and original approach based on the notion of spectral functionals. Indeed, it is possible to calculate these functionals in several quite generic cases. This long-term work has proven to be rewarding and especially promising for future analyses. It is expected to have consequences both in mathematics and in theoretical physics, where NCG has proven its worth.

After an introduction to the subject, recalling the relevant tools, the thesis is structured into three published articles.

In the first, the authors introduce new notions of spectral functions for application to spectral triples in NCG, whose densities cover the classical cases of metrics and the Einstein tensor for a compact Riemannian manifold. The technique used is based on the calculation of Wodzicki's residue. This important notion is indeed related to the Dixmier trace and Connes noncommutative integral.

In a second article, the authors extend the previous case from vector fields to differential forms and define the notion of spectrally closed spectral triples. This notion, although the terminology may not be the best, will prove to be important in NCG.

In the third article, the spectral torsion is introduced, controlled in the commutative case, and then applied to different cases : almost commutative geometry, noncommutative tori, and a quantum group. This proves the efficiency of the approach.

Furthermore, Paweł Zalecki introduces several personal improvements between each chapter compared to the published articles.

The thesis is well written, but I do have one regret : Paweł Zalecki taped together the three articles he published rather than rewriting the entire work from scratch. This would have avoided repetitions in the introductions of each chapter, unified the references, and better integrated his personal improvements, for instance avoiding the need to generalize a previously typed result.

It should also be noted that the works of Chinese researchers Li, Hong, Wang&Wang, Wu and Yang deserve more citations, especially when their works have titles such as : "A General Dabrowski-Sitarz-Zalecki Type Theorem for Manifolds with Boundary", "The Hodge-Dirac Operator and Dabrowski-Sitarz-Zalecki Type Theorems for Manifolds with Boundary," or "The General Kastler-Kalau-Walze Type Theorem and the Dabrowski-Sitarz-Zalecki Type Theorem for Odd Dimensional Manifolds with Boundary." Even though all these Chinese works concern only the commutative case, it would have been an opportunity to highlight that the results in this thesis are more oriented towards noncommutative geometry.

There are a few misprints and some inaccuracies that can be easily addressed during the defense.

To accomplish this, he had to not only deepen his knowledge of classical geometry but also immerse himself in the extremely sophisticated mathematics of NCG. The results of this thesis show that he has fully assimilated the necessary knowledge and is also capable of manipulating the tools to generate new results that interest the international community.

Despite the technical difficulties, the details of the lengthy calculations are clearly exposed, proving that Paweł Zalecki has a perfect mastery of this original subject matter.

The thesis meets all the criteria for a good thesis in mathematical physics at an international level. There is no doubt that Paweł Zalecki will be able to conduct independent research on a competitive and promising topic in the future.

I therefore recommend without hesitation its acceptance by the Discipline Council for Physical Sciences at Jagiellonian University and its authorization for public defense.

Moreover, given the interest and originality of the results and the overall quality of the writing of this thesis, I propose awarding this thesis with distinction.

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