

Report onThe thesis
HIGHER GAUGE FIELDS AND FERMIONS IN LATTICE MODELS
by **Błażej Ruba**.

The doctoral dissertation thesis of Błażej Ruba is concerned with lattice formulations of gauge theories and fermions. Lattice theories have a long history in physics, where they have been used not only to describe systems with crystalline structure, for example in models of magnetism such as the Ising model, but also as a way to regularize the infinities that appear in perturbation theory formulations of quantum field theory. This leads to lattice field theory, a formulation of quantum field theory where the regularization explicitly breaks Poincaré symmetry but may keep other symmetries, such as gauge invariance, manifest. The lattice formulation, involving only a finite number of degrees of freedom, makes it possible to study non-perturbative phenomena using Monte Carlo methods. By letting the lattice grow while the lattice spacing decreases, one is able to derive results in the continuum theory, which are inaccessible using ordinary path integral methods.

In the thesis, the concept of a lattice is generalized to configurations including vertices and edges, but connected in a way that does not have to exhibit the standard translational invariance of a regular lattice. Following Yetter, the author and his collaborators construct theories where the lattice is, roughly speaking, replaced by a product of Eilenberg-MacLane spaces, giving the structure of the classifying space of a crossed module. It turns out that this beautiful mathematical structure naturally incorporates all the parts that are needed for constructing lattice gauge theory, but also naturally generalizes this to something known in the physics community as higher gauge theory.

Higher symmetries and higher gauge theory have recently attracted attention both in particle physics and in condensed matter physics, where it has been used to gain information about the phase structure and stability of various theories. For instance, the comparison of higher global symmetries can tell if two different descriptions can actually describe the same phase of a system, or 't Hooft anomalies of the higher symmetries can help constrain the running under the renormalization group. In general, higher-form symmetries give many constraints on the infrared dynamics of gauge theories. Higher symmetries have also been used in order to find constraints on effective theories including gravity that have a reasonable UV completion in the so-called Swampland program. The higher symmetries are intimately associated with topology and topological defects.

Given the importance of higher symmetries in the recent progress in particle physics and condensed matter theory, it is clear that the topic of this thesis is at the forefront of modern research in theoretical physics.

Another area where 1-form symmetries are important is in the bosonization of fermionic theories in higher dimensions. The spin structure needed for the formulation of the fermionic theory is inherited as a 1-form symmetry in the bosonic theory. Given this, it is natural that the author goes on to study bosonization using his lattice formulation of gauge theory. Apart from this purely theoretic motivation, this topic is also of utmost importance since lattice gauge theories play an important role in the high energy phenomenology community. Lattice calculations of physical quantities give an alternative and non-perturbative way to calculate many results that are inaccessible using perturbation theory. Even when perturbation theory is available, the lattice theory can provide important input, as can be seen in the recent debate over the muon $g - 2$ discrepancy, where experimental results disagree with perturbative standard model calculations up to 4σ . It has been known for a long time that putting fermions on a lattice is problematic, and many tricks and techniques have been invented to overcome this issue. A very natural way to bypass the problem would be if one could find an equivalent theory with only bosonic degrees of freedom. An original approach to this problem, using the mathematical techniques developed for dealing with the lattices, is discussed in the second part of the thesis.

The thesis itself is, as is usually the case, composed of two parts. In the first part, various introductory material is collected together with a review of the key points of the research on which the thesis is built. It serves as a commentary on the published articles that make up the main content of the thesis. The second part consists of five published papers in which the results were published in some of the most renowned journals in the field. The thesis is well written, in English that is easy to read. Only in a few places have I found less clear passages. For example, I do not think that the phrase "well-define" can be used as a verb. Otherwise, the thesis is remarkably free from misprints for a text of this high technical level.

The introduction

The introductory part is quite short, providing some material on the mathematical foundations of gauge theory and topological field theory, as well as introducing some less well-known concepts, including the Gamma model and the crossed modules used in several of the articles on which the thesis is built. In my opinion, this part of the thesis could have been much longer. Some mathematical concepts that arguably are not part of the standard knowledge among physicists, such as the CW-complex, are not introduced at all. For someone who is not an expert in the field I think that it would have been useful to provide many simple examples, toy models, where the formalism is applied to known systems so that the reader would be able to build up some intuition in order to be ready for the non-trivial situations that are presented in the papers.

The second part of the thesis consists of five articles, all published in distinguished refereed journals with high impact factors. The first two papers deal with the generalization of lattice gauge theory using crossed modules whereas the last three papers deal with the generalized bosonization method.

Paper I

In the first paper, the model is introduced in full generality. Many new mathematical concepts are needed to define the model and explain the basic ideas. It is clearly explained how the mathematical structures govern the structure of the model and the behavior of the resulting lattice theory. The most complicated issues are discussed in a collection of Appendices. I find this paper extremely interesting. It introduces

several new ideas and contains a wealth of new material that inspires thoughts in many new directions. The paper is technically advanced, utilizing highly abstract mathematical concepts in new and interesting ways. From the thesis, it can be seen that the main driving force behind this paper is the author of this thesis. The paper represents a valuable and significant improvement of our knowledge of higher gauge theory in lattice theories. I highly value this paper.

Paper II

In the second paper, the results of the first paper are applied in a more concrete setting. The lattice is chosen to be cubic with periodic boundary conditions and the crossed module is chosen to be one of the simplest nontrivial possibilities, based on the group $\mathbb{Z}_4$. This has the advantage that explicit calculations can be performed, and in particular numerical simulations using Monte Carlo techniques become feasible. The authors convincingly show that the model is sensitive to topological features of the theory, and the phase structure of the theory is roughly mapped out. Order parameters for higher symmetries are suggested and shown to behave appropriately.

Paper III

In paper three the theoretical underpinnings for the new models of bosonization are laid out. Using the results of the previous papers, it is shown that the so-called Γ model can be interpreted in terms of a lattice gauge theory with gauge group $\mathbb{Z}_2$. This result is very satisfying and nicely ties together the second part of the thesis with the first part. The paper is purely theoretical/mathematical in scope and is more concerned with proving results than with actually performing calculations. It is again clear that the author of this thesis was instrumental in the writing of this paper.

Paper IV and V

Papers four and five turn to more practical applications of the new bosonization techniques. In both cases, the authors turn to computers to perform the calculations. In paper four a symbolic algebra program is used to explicitly show the reduction of Hilbert space when the constraints are imposed in the case of some small lattice sizes. In the fifth paper, there are more examples of models where the bosonization techniques can be applied. In particular, theories with different number of degrees of freedom at different lattice points, such as theories with a boundary, are discussed. Simulations using Monte Carlo methods are presented in some cases with small lattices.

Given these facts I can safely declare that:

I hereby confirm that the submitted thesis meets the requirements for doctoral dissertations and I request that Błażej Ruba be admitted to the further stages of the doctoral procedure

Application for distinction

It is clear to me that this thesis represents a major contribution at the forefront of modern theoretical physics. Błażej Ruba is an exceptionally talented young physicist with a deep knowledge of many areas of physics, as well as a thorough grasp of highly technical ideas from modern mathematics, which he applies to physics in new and interesting ways. I have served several times as a member or even chairman of the committee for the international Václav Votruba Prize for the best thesis in theoretical physics during the past year. We receive contributions from all over the world, including some of the best universities, such as Cambridge

University in England or Perimeter Institute in Canada, and I can safely say that the thesis of Błażej Ruba is comparable to the very best contributions I have seen as part of the awarding process for the Votruba Prize, and I highly recommend that the thesis of Błażej Ruba, Higher gauge fields and fermions in lattice models, should be awarded the distinction from the Jagiellonian University that I have been informed is possible.

A handwritten signature in blue ink, reading "von Unge". The signature is fluid and cursive, with a long horizontal stroke extending from the end.

Rikard von Unge

Professor of Theoretical Physics and
Astrophysics