
A thesis presented for the degree of
Doctor of Philosophy (Ph.D.)

Study of e^+e^- production in π^-p collisions in HADES at GSI

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*This thesis is dedicated to my parents.
For their endless love, support and encouragement*

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Jestem świadomy, że niezgodność niniejszego oświadczenia z prawdą ujawniona w dowolnym czasie, niezależnie od skutków prawnych wynikających z ww. ustawy, może spowodować unieważnienie stopnia nabytego na podstawie tej rozprawy.

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But if you don't have the right guidance, you can never bite the right target".*

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"The whole purpose of education is to turn mirrors into windows".

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Abstract

The e^+e^- pairs (dileptons) produced in collisions of atomic nuclei are excellent probes that carry the information from hot and dense nuclear matter created during collision. This is possible because, unlike hadron probes, they do not interact strongly and leave the nuclear matter almost undisturbed. Experimental studies of the dilepton production in heavy-ion collisions at energies of a few GeV per nucleon are conducted by the HADES collaboration at the SIS-18 synchrotron in GSI research center in Darmstadt, Germany. For the interpretation of dilepton spectra measured in heavy ion collisions, HADES also studies the production of dileptons in the elementary proton-proton, neutron-proton and pion-proton collisions. The latter are especially important for understanding the production of dileptons in the Dalitz decays of baryon resonances.

This work presents the analysis of data collected with the HADES spectrometer for the dilepton production in π^-p collisions at a pion beam momentum of 0.685 GeV/c. The measurements were performed with a polyethylene target and, additionally, with a carbon target to determine the contribution to the dilepton production from the beam scattering on carbon in polyethylene. The main aim of the work was to determine the size of the dilepton production signal due to emission of the virtual bremsstrahlung photons in the elementary reaction $\pi^-p \rightarrow \pi^-p \gamma^*(\gamma^* \rightarrow e^+e^-)$. We searched for the signal from this process through the analysis of three exclusive channels: $\pi^-p e^+e^-$, $\pi^-e^+e^-X$, $p e^+e^-X$, and one inclusive channel: e^+e^-X . To estimate the production of dileptons from various possible processes, we performed Monte Carlo simulations based on theoretical models of these processes. The virtual photon bremsstrahlung was simulated using theoretical description based on the soft-photon approximation. In turns, the dilepton production in π^0 and $\Delta(1232)$ Dalitz decays were simulated based on the partial wave analysis of one- and two-pion production data from pion-nucleon scattering and photo-production experiments.

The experimental e^+e^- invariant mass spectra are reasonably well described by the simulations. In the range below the π^0 mass, these spectra are dominated by the π^0 Dalitz decay. Various experimental distributions of the differential cross sections determined for this process are in a good agreement with the simulations based on the partial wave analysis. We find that in the exclusive $\pi^-e^+e^-X$ and $p e^+e^-X$ channels, the distributions of the e^+e^- invariant mass in the range above the π^0 mass are dominated by the contribution from the bremsstrahlung emission. This contribution is roughly two times larger for the π^-C collisions than for the π^-p collisions. According to the performed simulations, the contribution from the Δ resonance Dalitz decay is about two orders of magnitude smaller than that from the bremsstrahlung.

We used a clean signal for the bremsstrahlung emission process, observed in the exclusive $\pi^-e^+e^-X$ analysis, to determine the cross-section for this process in the collisions of the π^- beam with a polyethylene target. The value of the cross-section determined within the HADES acceptance and for the e^+e^- invariant mass larger than 0.14 GeV/c² is $(5.87 \pm 0.94_{stat} \pm 0.70_{sys}) \times 10^{-6}$ mb. This value, extrapolated to the full solid angle and the entire range of the e^+e^- invariant mass using simulations based on the soft-photon approximation model, gives the total cross-section of $(2.58 \pm 0.44_{stat} \pm 0.31_{sys}) \times 10^{-2}$ mb. We also determined the π^-p bremsstrahlung cross section by subtracting the carbon contribution in the missing mass spectra for the three exclusive channels measured with the polyethylene target. The mean value of the obtained cross-sections, extrapolated to the full solid angle and the entire range of the e^+e^- invariant mass is $(4.68 \pm 1.44_{stat} \pm 0.56_{sys}) \times 10^{-3}$ mb. To our knowledge, this is the first cross section data

for the virtual bremsstrahlung emission in the collision of charged pions with protons.

The thesis is organized in the following way:

Chapter 1 presents the motivation of studies of the dilepton production in heavy-ion collisions and the state of research of the dilepton production in the elementary nucleon-nucleon and pion-nucleon collisions. It also provides the basics of the theoretical description of the bremsstrahlung emission process based on the soft-photon approximation.

The accelerator complex in GSI research center, the pion beam-line and details of the HADES detection system are described in Chapter 2.

Algorithms developed by the HADES collaboration for track and momentum reconstruction as well as for identification of charged particles are presented in Chapter 3. Particular attention is paid to the methods of electron identification using the HADES RICH detector.

Chapter 4 presents Monte Carlo simulations of various processes contributing to the production of dileptons in the studied reaction channels.

The analysis of the collected experimental data, carried out to extract the signal from the production of dileptons and, in particular, from the pion-proton bremsstrahlung, is presented in Chapter 5.

Interpretation of the obtained data by comparing them with results of simulations based on theoretical models of investigated processes is given in Chapter 6. The estimation of the total cross section for the bremsstrahlung process is also presented.

The summary of the work and the main conclusions drawn from the conducted research are given in Chapter 7.

Streszczenie

Pary e^+e^- (dileptony) produkowane w zderzeniach jąder atomowych są doskonałymi próbnikami niosącymi informację z gorącej i gęstej materii jądrowej wytwarzanej w zderzeniach. Jest to możliwe dzięki temu, że w przeciwieństwie do hadronów, nie oddziałują silnie i mogą opuścić materię jądrową prawie niezakłócone. Badania eksperymentalne produkcji dileptonów w zderzeniach ciężkich jonów przy energiach rzędu kilku GeV na nukleon są prowadzone przez zespół HADES na synchrotronie SIS-18 w ośrodku badawczym GSI w Darmstadt w Niemczech. Dla interpretacji widm dileptonów otrzymywanych dla zderzeń ciężkich jonów, HADES bada również produkcję dileptonów w elementarnych zderzeniach proton-proton, neutron-proton i pion-proton. Te ostatnie są szczególnie ważne dla poznania produkcji dileptonów w rozpadach Dalitza rezonanów barionowych.

Niniejsza rozprawa przedstawia analizę danych zebranych za pomocą spektrometru HADES dla produkcji dileptonów w zderzeniach π^-p przy pędzie wiązki pionów 0.685 GeV/c. Pomiary przeprowadzono z tarczą polietylenową i dodatkowo z tarczą węglową dla określenia przyczynku do produkcji dileptonów od rozpraszania na węglu w tarczy polietylenowej. Głównym celem pracy było określenie wielkości sygnału od produkcji dileptonów poprzez emisję wirtualnych fotonów promieniowania hamowania w elementarnej reakcji $\pi^-p \rightarrow \pi^-p \gamma^*(\gamma^* \rightarrow e^+e^-)$. Sygnał od tego procesu był poszukiwany poprzez analizę trzech kanałów ekskluzywnych: $\pi^-p e^+e^-$, $\pi^-e^+e^-X$, $p e^+e^-X$, oraz jednego kanału inkluzywnego: e^+e^-X . Aby oszacować wkład w produkcję dileptonów od różnych możliwych procesów, przeprowadzono symulacje oparte na modelach teoretycznych tych procesów. Emisja wirtualnych fotonów promieniowania hamowania była symulowana przy użyciu modelu teoretycznego opartego na tzw. przybliżeniu miękkich fotonów (ang. soft-photon approximation (SPA)). Z kolei produkcję dileptonów w rozpadach Dalitza mezonów π^0 i bari-onów $\Delta(1232)$ symulowano wykorzystując wyniki analizy fal cząstkowych danych dla produkcji jednego i dwóch pionów w rozpraszaniu pion-nukleon i w procesie fotoprodukcji.

Eksperymentalne widma masy niezmienniczej par e^+e^- są dobrze opisywane przez symulacje. W zakresie poniżej masy π^0 widma te są zdominowane przez rozpad Dalitza π^0 . Wyznaczone różne rozkłady różniczkowych przekrojów czynnych dla tego procesu są w dobrej zgodności z symulacjami opartymi na analizie fal cząstkowych. Stwierdzamy, że w ekskluzywnych kanałach $\pi^-e^+e^-X$ i $p e^+e^-X$ rozkłady masy niezmienniczej e^+e^- w zakresie powyżej masy π^0 są zdominowane przez przyczynę od procesu emisji promieniowania hamowania. Przyczynę ten jest około dwa razy większy dla zderzeń π^-C niż dla zderzeń π^-p . Zgodnie z przeprowadzonymi symulacjami, wkład od rozpadu Dalitza rezonansu Δ jest o około dwa rzędy wielkości mniejszy niż ten od promieniowania hamowania.

Czysty sygnał od procesu emisji promieniowania hamowania zaobserwowany w ekskluzywnym kanale analizie $\pi^-e^+e^-X$ został wykorzystany do wyznaczenia przekroju czynnego dla tego procesu w zderzeniach wiązki pionów z tarczą polietylenową. Otrzymana wartość odpowiadająca akceptancji spektrometru HADES i wartościom masy niezmienniczej par e^+e^- powyżej 0.14 GeV/c² wynosi $(5.87 \pm 0.94_{stat} \pm 0.70_{sys}) \times 10^{-6}$ mb. Wartość ta, ekstrapolowana do pełnego kąta bryłowego i pełnego zakresu masy niezmienniczej przy użyciu symulacji opartych na modelu SPA, daje całkowity przekrój czynny $(2.58 \pm 0.44_{stat} \pm 0.31_{sys}) \times 10^{-2}$ mb. Wyznaczyliśmy również przekrój czynny na emisję promieniowania hamowania w zderzeniach π^-p , odejmując udział węgla w widmach masy brakującej dla trzech ekskluzywnych kanałów analizie zmierzonych z tarczą polietylenową. Średnia z otrzymanych wartości

przekroju czynnego, ekstrapolowana do pełnego kąta bryłowego i pełnego zakresu masy niezmienniej e^+e^- wynosi $(4.68 \pm 1.44_{stat} \pm 0.56_{sys}) \times 10^{-3}$ mb. Według naszej wiedzy są to pierwsze dane dla całkowitego przekroju czynnego dla emisji wirtualnych fotonów promieniowania hamowania w zderzeniu naładowanych pionów z protonami.

Układ pracy jest następujący:

W Rozdziale 1 przedstawiono motywację badań produkcji dileptonów w zderzeniach jąder atomowych oraz stan badań produkcji dileptonów w zderzeniach nukleon-nukleon i pion-nukleon. Podane są w nim także podstawy opisu teoretycznego procesu emisji promieniowania hamowania oparte na przybliżeniu miękkich fotonów.

Kompleks akceleratorów w ośrodku badawczym GSI, linia wiązki pionów oraz szczegóły układu detekcyjnego HADES zostały opisane w Rozdziale 2.

Algorytmy do rekonstrukcji torów i pędów cząstek oraz identyfikacji cząstek naładowanych, opracowane przez zespół HADES, przedstawiono w Rozdziale 3. Szczególną uwagę poświęcono metodom identyfikacji elektronów z wykorzystaniem detektora HADES RICH.

W Rozdziale 4 przedstawiono symulacje Monte Carlo różnych procesów wnoszących wkład do produkcji dileptonów w badanych kanałach reakcji.

Analizę zebranych danych eksperymentalnych, przeprowadzoną w celu wydobywania sygnału produkcji dileptonów, a w szczególności sygnału od promieniowania hamowania, przedstawiono w Rozdziale 5.

W Rozdziale 6 podano interpretację uzyskanych danych poprzez porównanie ich z wynikami symulacji opartych na modelach teoretycznych badanych procesów. Przedstawiono również oszacowanie całkowitego przekroju czynnego dla procesu emisji promieniowania hamowania.

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Chapter 1

Introduction

*"In the beginning the Universe was created.
This has made a lot of people very angry and
has been widely regarded as a bad move."*

Douglas Adams
The Hitchhiker's Guide to the Galaxy

This chapter presents the motivation behind the performed studies of the dilepton production in π^-p collisions. They are related to the main goal of the HADES experiment, which is to learn about the properties of hot and dense nuclear matter created in heavy-ion collisions by measuring dileptons produced during the collisions. This goal is discussed in the first section of the present chapter. To understand the spectra of dileptons registered in the nucleus-nucleus collisions, it is important to measure the corresponding spectra in the elementary nucleon-nucleon and pion-nucleon collisions. In Section 2, results of measurements of dilepton production in $N - N$ collisions are presented. One of the main sources of dileptons are Dalitz decays of baryon resonances. Measurements of these decays can be used to determine the baryon transition form-factors in the time-like region, as discussed in Section 3. Analysis of the production of dileptons in π^-p collisions performed in the frame of this thesis, has led to the conclusion that a significant contribution to the dilepton production comes from the virtual bremsstrahlung. In Section 4, the current state of the experimental research on the production of bremsstrahlung in collisions of charged pions with protons is presented and in Section 5, the basic theoretical description of the bremsstrahlung emission used in this work is discussed.

1.1 Dilepton production in heavy-ion collisions

Studies of properties of nuclear matter at various densities and temperatures are one of the central issues of nuclear physics. Understanding these properties is important for the theoretical description of the transition from the strong-interaction matter consisting of quarks and gluons to hadrons, that occurred in the early phase of the Big Bang. Studies of the dense nuclear matter are also important for the description of neutron stars and of the binary neutron star mergers observed with gravitational waves and light [1]. Another interesting goal of this research is to test the predictions of the theory of strong interaction - quantum chromodynamics (QCD) concerning the transition between "confined phase" of nuclear matter and quark-gluon plasma. This transition is associated with the phenomenon of the chiral symmetry restoration.

The basic experimental method of studying the properties of nuclear matter is the collision of heavy ions on particle accelerators. By choosing different energies and collision systems, different states of matter, that appear for a short time during the collision, can be investigated. A phase diagram of strong interaction matter is typically presented on a plane defined by temperature (T) and baryochemical potential (μ_B), as shown in Fig. 1.1. In the early universe, the strong-interaction matter consisting of quarks and gluons

occurred at very high temperatures ($T > 150$ MeV) and zero baryochemical potential. The highest energy experiments conducted at RHIC and LHC try to recreate these conditions to observe the transition between hadronic matter and quark-gluon plasma.

At lower energies of 1-2 AGeV accessible in experiments with the HADES spectrometer at the GSI- SIS-18 accelerator, a baryon density 3-4 times greater than normal nuclear density and temperatures of the order of order 100 MeV may be reached in a fireball with a mean lifespan of 10 fm/c for heavy-ion collisions. At these temperatures and densities, the conditions for the chiral-symmetry restoration and deconfinement of hadrons are not yet accessible. However, they may be sufficient for a partial restoration of the chiral symmetry which may manifest in a substantial modification of the masses and decay width of hadrons embedded in the hot and dense nuclear medium.

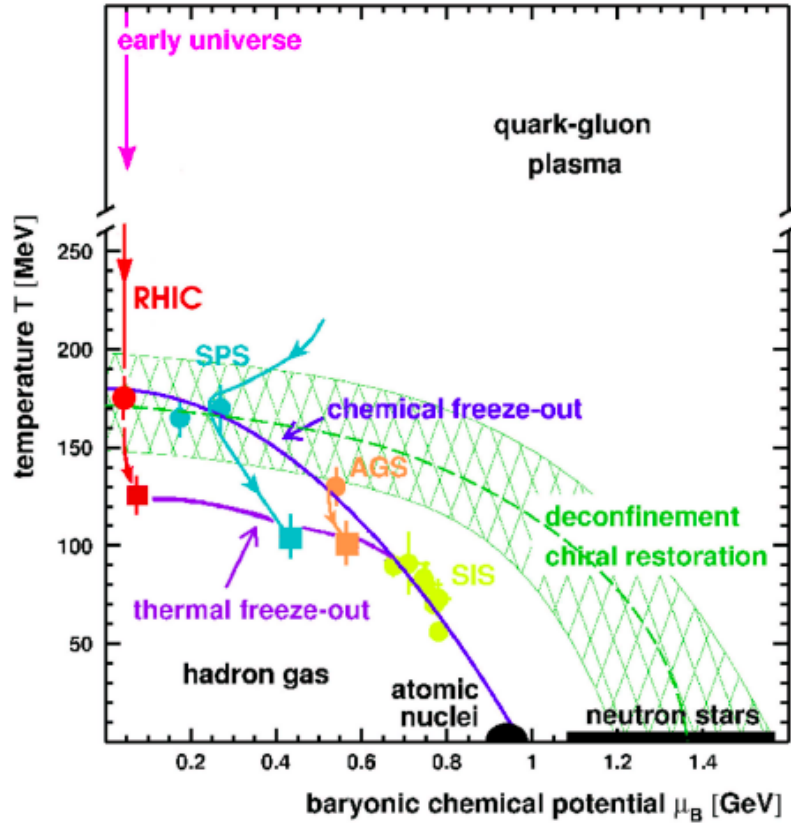


Figure 1.1.: The phase diagram of strongly interacting matter on a plane defined by temperature T and baryochemical potential μ_B . Dark blue and violet line represent, respectively, chemical, and thermal freeze-out calculated within the framework of a statistical model. The chemical freeze-out points are determined from a thermal model analysis of heavy-ion collision data at SIS, AGS, SPS and RHIC energies. The hatched area represents the region of deconfinement.

At energies of a few GeV per nucleon, heavy-ion collision can be separated into three stages. At the beginning of a collision, ions have normal matter properties. When they meet, hard scattering of nucleons with high energy transfer occurs resulting in an increase of the density and temperature of the system, with a formation of a fireball. At SIS-18 energies, fireball exists for 10-20 fm/c [2], while it expands and cools down. Finally, the system reaches chemical and thermal equilibrium, indicating that the heavy-ion collision has come to an end. The electromagnetic radiation, including real and virtual photons, represents an excellent probe for investigations of heavy-ion collisions as it is emitted during the whole time evolution of the collision carrying information about hot and compressed nuclear matter created in the collision. Besides, unlike hadron probes, the electromagnetic radiation does not interact strongly and, therefore,

leaves the nuclear matter almost undisturbed. Virtual photons decaying into a pair of leptons (dilepton) have an extra observable in terms of their invariant-mass, in contrary to real photons, which are massless.

Nuclear matter studies with the use of dileptons have been conducted in experiments at various energy scales. The dilepton spectroscopy was initiated in the mid-1980s at CERN on the SPS accelerator at energies 40-158 GeV/nucleon by experiments Na34-HELIOS and Na38, and was then continued by CERES and other experiments. Dilepton production research have been also conducted at ultra-relativistic energies available at the collider RHIC by the experiment PHENIX and STAR and at LHC by experiments CMS, ATLAS and ALICE.

First measurements of the dilepton production at lower energies, in the order of 1.0 GeV/nucleon, were performed using the Dilepton Spectrometer (DLS) which was built and was operational during the late 1980s and early 1990s at the Bevatron/Bevalac accelerator at Lawrence Berkeley Laboratory (LBL). It was a two arm dipole spectrometer capable of measuring electron-positron pairs [3, 4].

A schematic drawing of the DLS setup is shown in Fig. 1.2. Each of the two identical DLS arms consisted of a large-aperture dipole magnet, three drift chambers, two segmented 1-atm gas Cherenkov counters and two scintillation hodoscope. The detection efficiency for electrons was 95% and the rejection factor of pions was about 10^5 .

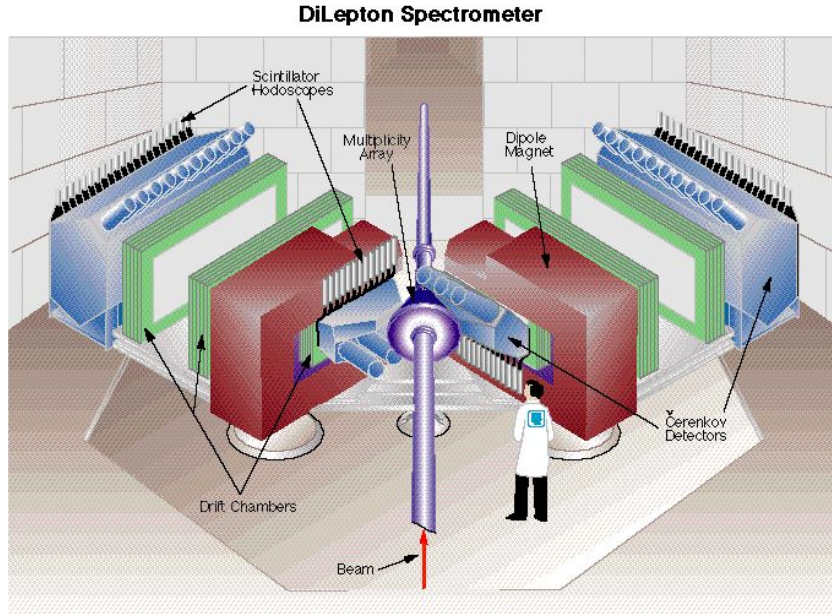


Figure 1.2.: Schematic view of DLS experimental setup.

The DLS collaboration measured the dilepton invariant mass spectra for $p + p$ and $p + d$ collisions [5] with beam energies from 1.04 GeV to 4.88 GeV, as well as in collisions of heavier nuclei such as C+C at 1.04 AGeV and Ca+Ca at 1 and 2 AGeV [5,6]. Fig. 1.3 shows invariant mass spectra of e^+e^- pairs measured in C+C and Ca+Ca collisions at 1.0 AGeV. In the invariant mass range from about 0.2 to 0.6 GeV/ c^2 the experimental data points were strongly underestimated by available theoretical calculations based on transport models for example the Hadron String Dynamics (HSD) model [7]. This discrepancy has been called the DLS puzzle.

Solving the DLS puzzle had become one of the main research goals of the new experiment HADES at SIS-18 accelerator in GSI Darmstadt, Germany, which was designed to study the production of dileptons in a similar energy range as DLS, but with higher data rates and improved mass resolution. Measurements performed by HADES in C+C collisions at 1 AGeV [8] turned out to be consistent with the corresponding data from DLS and thus confirmed the correctness of these data. Further studies of the dilepton production

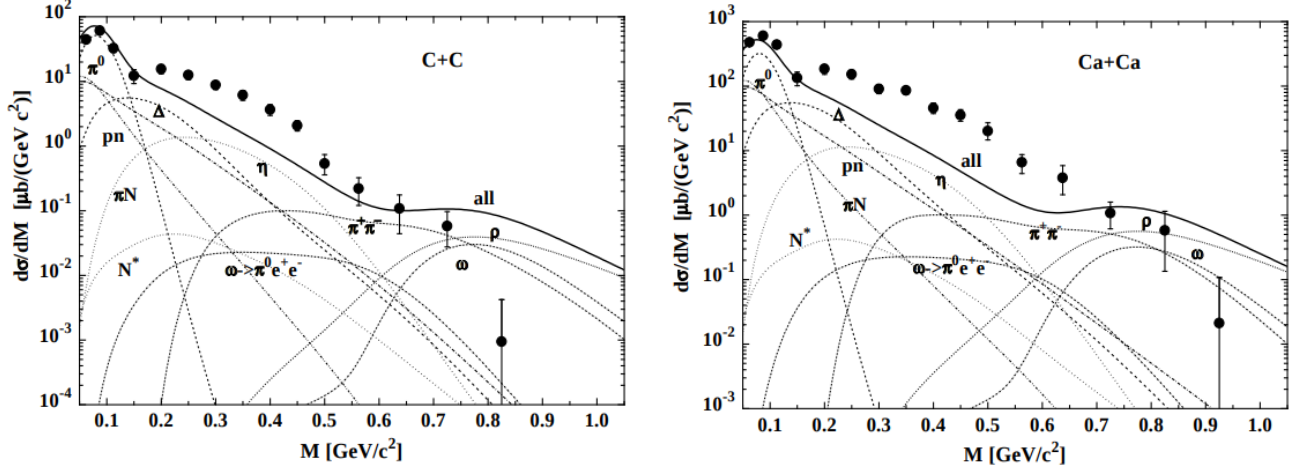


Figure 1.3.: Dilepton invariant mass spectra for Ca+Ca and C+C at 1.04 AGeV compared with the sum of all contributions included in the Hadron String Dynamics (HSD) transport code.

in $p - p$ and $d - p$ collisions at 1.25 GeV/nucleon performed at HADES showed that the dilepton pair spectra measured in C+C collisions can be explained by superposition of elementary $p + p$ and $n + p$ collisions [9]. More details on this topic are given in the next section.

HADES also measured the dilepton production in collisions of the medium-heavy Ar+KCl systems at 1.756 AGeV [9,10] and of the heavy Au+Au systems at 1.23 AGeV [11]. In these collisions a systematic excess of the dilepton emission, compared to the expectations based on the superposition of spectra for elementary $N - N$ collisions, was observed [12,13].

This excess originates mainly from the hot and dense hadronic medium (fireball) created during collision [1]. It shows a near-exponential dependence on the dilepton invariant mass. From this dependence, an average temperature of the radiation from fireball formed in the Au+Au collisions of $T = 71.8 \pm 2.1$ MeV is deduced. According to calculations using the transport models, the estimated nuclear matter densities reached in the collisions are of up to three times the normal nuclear matter density. As pointed out in Ref. [1], these properties are similar to ones expected to occur in the final state of a neutron star merger. This demonstrates the importance of studying the dense and hot nuclear matter produced in heavy ion collisions for the understanding of astrophysical phenomena.

In the energy range explored by HADES, the main dilepton sources can be divided into the following categories:

1. The Dalitz decay of mesons:

- $\pi^0 \rightarrow \gamma e^+e^-$ – with branching ratio 1.2 %,
- $\eta \rightarrow \gamma e^+e^-$ – with branching ratio 7.0×10^{-3} %,
- $\omega \rightarrow \pi^0 e^+e^-$ – with branching ratio 8.0×10^{-4} .

2. The Dalitz decay of baryonic resonances:

- $\Delta \rightarrow N e^+e^-$ – with branching ratio 4.2×10^{-5} ,
- $N(1535), N(1520), \dots \rightarrow N e^+e^-$.

3. The vector meson decays:

- $\rho \rightarrow e^+e^-$ – with branching ratio 4.44×10^{-5} ,
- $\omega \rightarrow e^+e^-$ – with branching ratio 7.07×10^{-5} ,

- $\phi \rightarrow e^+e^-$ – with branching ratio 3.09×10^{-4} .

4. Nucleon-nucleon and pion-nucleon bremsstrahlung: $NN \rightarrow NN e^+e^-$, $\pi^-N \rightarrow \pi^-N e^+e^-$.

To investigate the above dilepton sources, HADES performed measurements of the dilepton production in the elementary proton-proton, neutron-proton and pion-proton collisions. Results of the first two measurements are presented briefly in the next subsection. In turns, the production of dileptons in pion-proton collisions is the subject of the current work with particular emphasis on the virtual photon bremsstrahlung.

1.2 Studies of dilepton production in nucleon-nucleon collision

Production of e^+e^- pairs in $p+p$ and $p+d$ collisions for beam kinetic energies in the range 1.0-4.9 GeV/c was studied by the DLS collaboration [14]. They investigated the ratio of dilepton yield in $p+d$ interaction to $p+p$ interaction. This ratio is sensitive to the contribution to the dilepton production from decays of hadrons (e.g. Dalitz decays of Δ resonances and π^0 mesons) and from the virtual bremsstrahlung. If the dileptons were produced predominately by the decays of hadrons, then the pd/pp ratio should be around 2-4, as the hadron production in $p-n$ collisions is typically factor 3 times higher than in $p-p$ collision. In turns, in the case of bremsstrahlung this ratio would have a value around 10, since the $p-n$ bremsstrahlung is expected to be an order of magnitude higher than the $p-p$ one [15]. For the beam kinetic energy at 1.0 GeV and the e^+e^- invariant mass larger than 0.15 GeV/c², the pd/pp ratio is around 10 (see Fig. 1.4), which the authors of the publication [14] interpret as an effect resulting from the bremsstrahlung emission in the $p-n$ collisions.

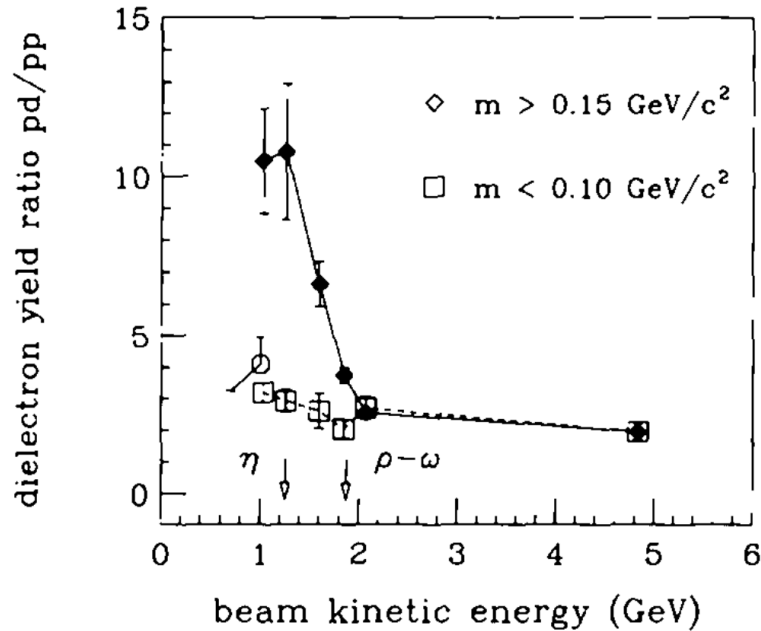


Figure 1.4.: Ratio of dilepton yield in $p-d$ and $p-p$ collisions as a function of the beam kinetic energy, for e^+e^- invariant mass range $0.05 < m < 0.10 \text{ GeV}/c^2$ (squares) and $m > 0.15 \text{ GeV}/c^2$ (diamonds). Kinematic thresholds for the production of η and ρ, ω mesons are indicated with arrows. Figure is taken from Ref. [14].

The first measurement of the $pp \rightarrow pp e^+e^-$ channel at an energy below the pion production threshold was performed at the superconducting-cyclotron AGOR of the Kernfysisch Versneller Instituut (KVI) in

Groningen [16]. All final state particles were measured in coincidence and a total of 600 events were collected. Obtained differential cross sections are compared to predictions of the low energy calculation model (LET) [17] and a fully relativistic microscopic model [18]. The LET calculation provides better description of the experimental data than the microscopic model.

Production of e^+e^- pairs in $p-p$ and $d-p$ collisions was also studied at HADES [19–22]. The pp measurements were performed at a beam kinetic energy of 1.25, 2.2 and 3.5 GeV and the dp data were taken at a deuteron beam energy of 1.25 GeV per nucleon. The $d-p$ experiment was used to investigate quasi-free $n-p$ reaction. In this experiment, a deuteron beam was scattered on liquid hydrogen target and the spectator proton from the deuteron projectile was registered by the Forward Wall consisting of 320 plastic scintillators arranged in a matrix, placed 7 meters downstream of the target and covering the forward region of 0.5° – 7° .

Fig. 1.5 presents distributions of the e^+e^- invariant mass measured in the $p-p$ and $n-p$ reactions. The spectrum for $n-p$ reaction shows much higher dilepton yield than the one for the $p-p$ reaction. At invariant masses below the π^0 mass, the yield can be understood by larger (factor 2) cross section for π^0 production in $n-p$ collisions and the π^0 Dalitz decay. At higher invariant masses the difference between the two spectra is larger and more difficult to understand.

However, as pointed out in Ref. [9], by superposition of the dilepton spectra measured by HADES in elementary the $p-p$ and $n-p$ collisions, the excess in the dilepton spectra measured in C+C collisions can be explained. In this way, the "DLS puzzle" is solved. However, the puzzle has shifted to understanding the increased production of dileptons in neutron-proton collisions.

In 2010, Shyam and Mosel [23] presented results of theoretical description of the dilepton production in $n-p$ and $p-p$ collisions measured at HADES. Their calculation was based on the One-Boson Exchange model [24] which was extended by inclusion of the electromagnetic form factors in the nucleon diagrams using the Vector Dominance Model (VDM) [25]. In the case of $p+p$ reaction, their calculation including the π^0 Dalitz decay, the Δ Dalitz decay and the nucleon-nucleon bremsstrahlung describe the HADES data very well (see Fig. 1.5, upper panel). The introduction of the electromagnetic form-factors at the nucleon vertices does not have a significant impact on the results. A different situation is in the case of the $n p \rightarrow n p e^+e^-$ reaction. Without inclusion of the electromagnetic form-factors at the pion and nucleon vertices, the HADES data points are strongly underestimated at higher e^+e^- invariant masses (see Fig. 1.5, lower panel). As shown in the corresponding figure in Ref. [23], taking into account the electromagnetic form factors allows for a proper description of the invariant mass distribution also at higher invariant mass values.

Theoretical calculations to explain the invariant mass spectra measured at HADES were also carried out by Bashkanov and Clement [26]. In the calculations they took into account various contributions to the dilepton production including Dalitz decays of π^0 and η mesons as well as baryon resonances, predominantly Δ , virtual bremsstrahlung emission and leptonic vector meson decay: $\rho \rightarrow e^+e^-$. The ρ mesons are formed due to the final state interaction between two pions originating from decays of a pair of Δ resonances excited in the $n-p$ collisions:

$$n p \rightarrow \Delta \Delta \rightarrow n p \pi^+ \pi^- \rightarrow n p \rho^0 \rightarrow n p e^+ e^-$$

Taking the ρ meson decay into account allows for the explanation of the dilepton surplus observed at high invariant masses in the $n-p$ collisions. In Fig. 1.6, the HADES data for the invariant mass distributions of e^+e^- pairs measured in $p-p$ scattering and quasi-free $n-p$ scattering are compared to the results of theoretical calculations of Bashkanov and Clement [26]. Both, $p-p$ and $n-p$ data are well described by the calculations.

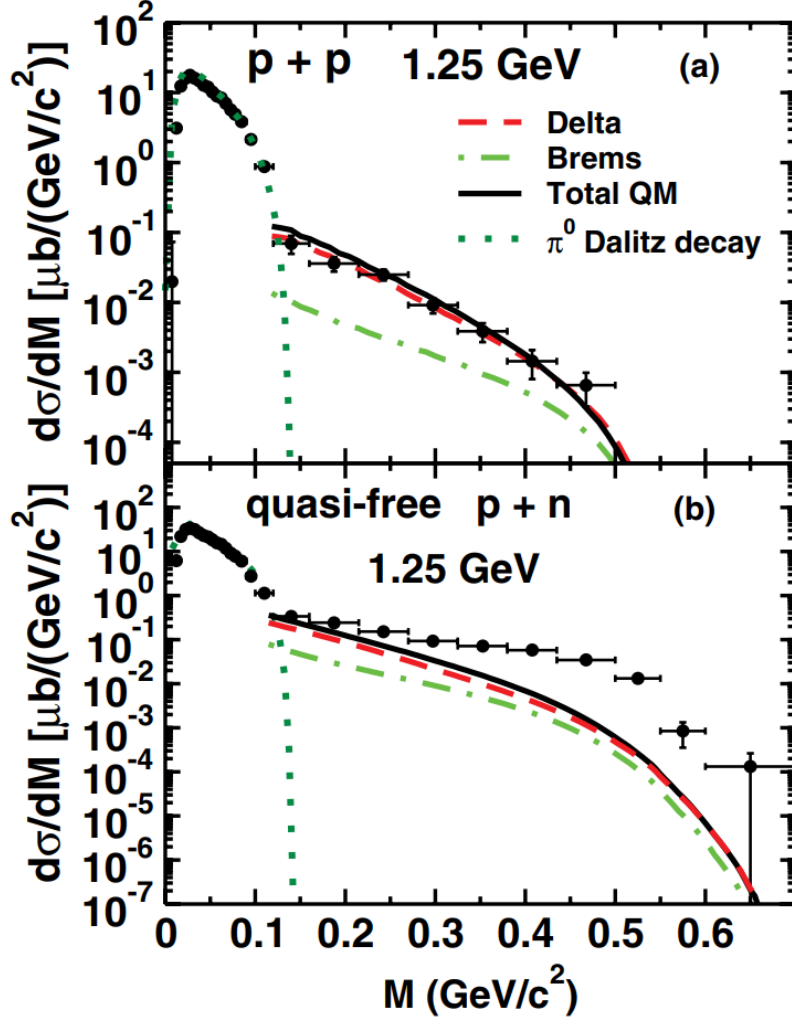


Figure 1.5.: Invariant mass distribution of e^+e^- pairs produced in $pp \rightarrow pp e^+e^-$ reaction (upper panel) and $np \rightarrow np e^+e^-$ reaction (lower panel) at beam kinetic energy of 1.25 GeV. Results of theoretical calculations of Shyam and Mosel [23] are shown with various lines. The π^0 Dalitz decay contributions are shown with dotted lines. The Delta isobar and $N-N$ bremsstrahlung contributions are shown with dashed lines and dashed-dotted lines, respectively, and their coherent sum is represented by solid lines. Figure is taken from Ref. [23].

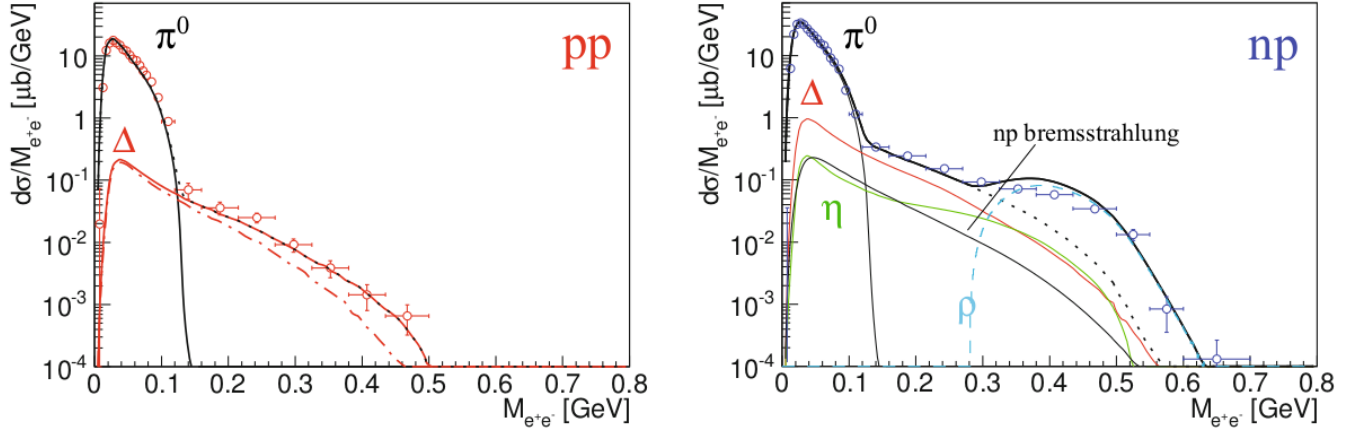


Figure 1.6.: Invariant mass distributions of e^+e^- pairs produced in $N - N$ collisions at 1.25 GeV beam kinetic energy. Left panel: HADES data for $p - p$ collisions (red open circles) with two leading contributions: π^0 Dalitz (black line) and Δ Dalitz decay including form factor (red solid line) and the point-like form factor (red dot-dashed line). Right panel: HADES data for $n - p$ collisions (blue open circles) with π^0 Dalitz decay and $n - p$ bremsstrahlung (black line), Δ Dalitz decay (red line), η Dalitz decay (green line). The dotted curve represents the sum of these processes. The dashed cyan curve represents the contribution from ρ decay ($\rho \rightarrow e^+e^-$) and the thick solid curve the total sum.

1.3 Electromagnetic form factors

A significant contribution to the production of dileptons comes from the Dalitz decay of baryons. The baryon Dalitz decay $B \rightarrow N \gamma^*(\gamma^* \rightarrow e^+e^-)$ is an electromagnetic transition that is sensitive to the electromagnetic structure of the involved baryons.

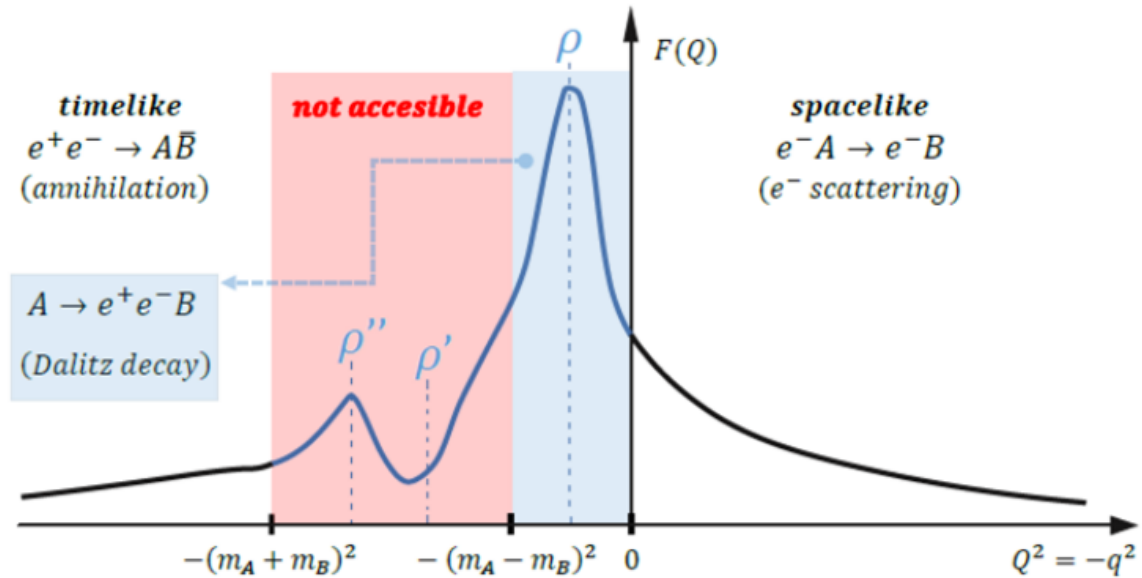


Figure 1.7.: A schematic dependence of the electromagnetic form-factor in function of Q^2 ($= -q^2$) for the space-like and time-like regions.

This structure is encoded in a function called electromagnetic Transition Form-Factor (eFTT) which de-

depends on the four-momentum squared q^2 of the virtual photon γ^* . The electromagnetic form factors are analytical functions of q^2 and for a consistent understanding of the baryon electromagnetic structure, experimental information on them is required both in the space-like region ($q^2 < 0$) and in the time-like region ($q^2 > 0$). The dependence of the form-factor on q^2 is illustrated in Fig. 1.7.

The baryon form-factors have been extensively studied in recent years by the CLAS collaboration at JLAB using electrons scattered on a proton target [27]. The corresponding process of the baryon resonance excitation through the exchange of a virtual photon is shown in Fig. 1.8, left panel. The four-momentum squared of the exchanged virtual photon (q^2) has in this case negative value. Thus, the CLAS measurements provide access to the baryon form-factors in the space-like region, as schematically marked in Fig. 1.7.

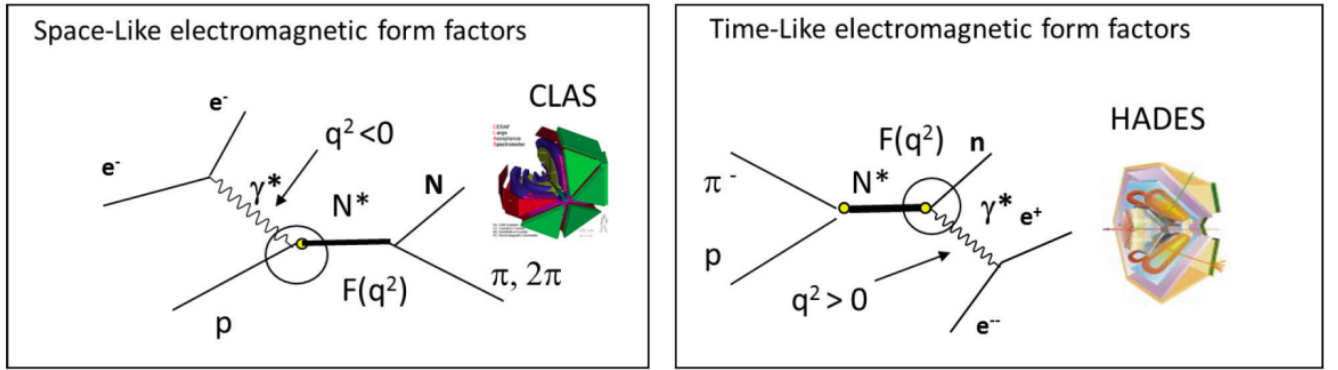


Figure 1.8.: Graph corresponding to excitation of baryon resonance N^* in e^-p collision (left panel) and to formation of N^* resonance in π^-p collision with subsequent Dalitz decay to ne^+e^- final state (right panel).

In the time-like region corresponding to positive value of q^2 , there exist very scarce data from the annihilation reaction: $e^+e^- \rightarrow B N$ [28]. In this reaction, the form-factors can be studied for the four-momentum squared higher than the sum of baryon masses squared: $(m_B + m_N)^2$ (see Fig. 1.7.)

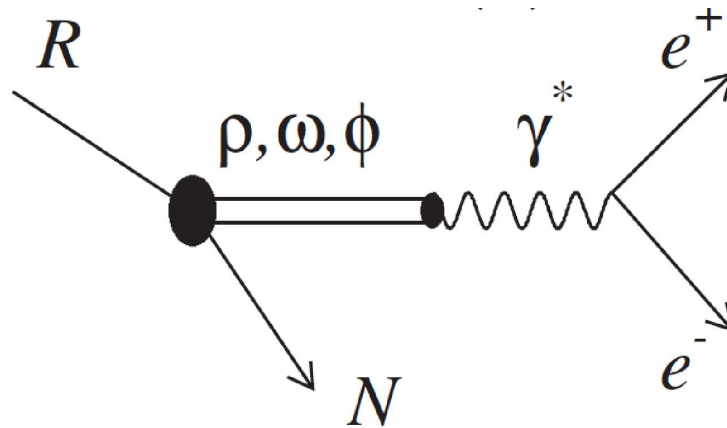


Figure 1.9.: Coupling of virtual photon to baryon via intermediate vector mesons, that have the same quantum numbers as the photon.

Measurements of the Dalitz decay of baryon resonances provide access to baryon electromagnetic form-factors in the time-like region but in the range of small q^2 values below the square of the mass difference $(m_B - m_N)^2$. According to the Vector Dominance Model (VDM) [29], the coupling between photon and

baryon is significantly influenced by vector mesons (ρ , ω , ϕ) as illustrated in Fig. 1.9. VDM was used for description of eFTT in Dalitz decays of neutral mesons [30]. In Dalitz decays of baryon resonances, the influence of the vector mesons on the photon-baryon coupling may be very important due to the proximity of q^2 values to the vector meson masses.

HADES measurements of dilepton production in proton-proton collisions at kinetic energy 1.25 GeV were used to study the Dalitz decay $\Delta(1232) \rightarrow p e^+ e^-$. In Fig. 1.10, ratio of experimental dilepton yield from the Δ Dalitz decay to results of simulations assuming a point-like electromagnetic vertex is presented [31]. A slight increase of the data is observed with increasing invariant mass squared of $e^+ e^-$ pair. This increase agrees with predictions of the covariant quark model of Ramahlo and Pena including the baryon electromagnetic form-factor [32] represented by blue line in Fig. 1.10. Contribution from the coupling of the virtual photon to pion cloud coupling (red dashed line) and to the quark core (black dashed line) is also indicated in the figure.

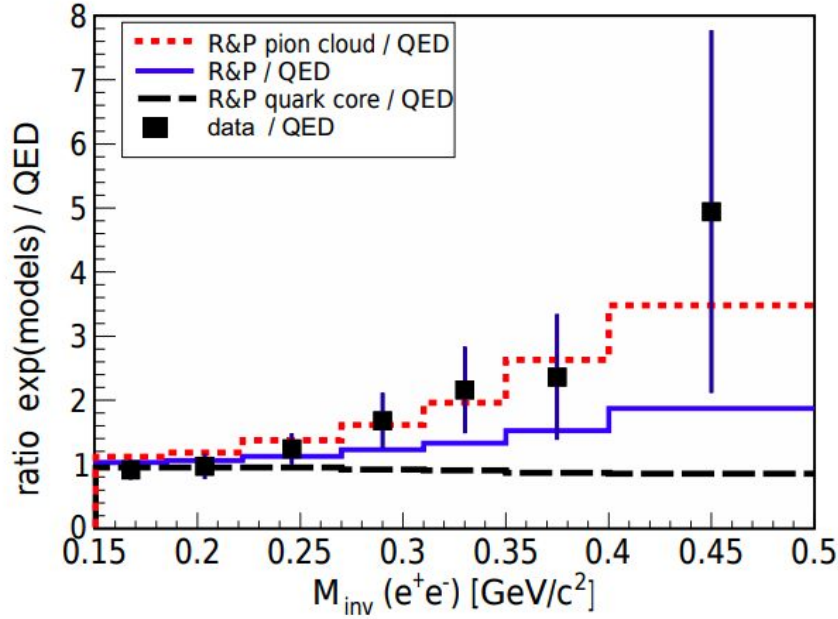


Figure 1.10.: Ratio of the measured dilepton yield from the $\Delta(1232) \rightarrow p e^+ e^-$ decay to the model of point-like particle (QED) as a function of $e^+ e^-$ invariant mass, compared to the Ramahlo and Pena (RP) full model (blue) and assuming dominance of the photon-quark core (black dashed) and photon-pion-cloud contributions (red dashed). Figure is adopted from Ref. [31].

The electromagnetic form-factors of baryons were also studied by HADES in $\pi^- p$ collisions. The formation of baryon resonances with masses in the region of the N(1520) resonance in the $\pi^- p$ collision with subsequent Dalitz decay to $n e^+ e^-$ final state, illustrated in Fig. 1.8, right panel, was analyzed in Ref. [33]. A publication on the electromagnetic structure of these resonances is in preparation.

1.4 Experimental studies of pion-nucleon bremsstrahlung

Dilepton production measurements using a low energy pion beam and a proton target were carried out in several experiments in which the pions were either captured at rest [34–36] or collided with protons at momentum of 300 MeV/c [37]. The measured reactions were $\pi^- p \rightarrow n \pi^0 (\gamma e^+ e^-)$ and $\pi^- p \rightarrow n \gamma^* (e^+ e^-)$. The studies were focused on the energy and angular distributions of the produced dileptons as well as on the electromagnetic form factor of the π^0 . To our knowledge, no measurements of the virtual bremsstrahlung production in pion-nucleon scattering have been performed so far. However, measurements of the production of real bremsstrahlung photons were carried out in several experiments.

The first, high statistics measurements of real bremsstrahlung production in pion-proton collisions, $\pi^+ p \rightarrow \pi^+ p \gamma$ and $\pi^- p \rightarrow \pi^- p \gamma$, were performed at the Lawrence Berkeley Laboratory (LBL) 184 in. cyclotron [Leu,1976, Nef1978]. Pions were produced in the external proton beam of the cyclotron. Data were taken for three different kinetic energies: 269, 298 and 324 MeV of π^- and π^+ beam. The pion beam was scattered on a liquid hydrogen target. The outgoing protons, pions and photons were registered in coincidence in a detection system presented schematically in Fig. 1.11. In the experiment, five-fold

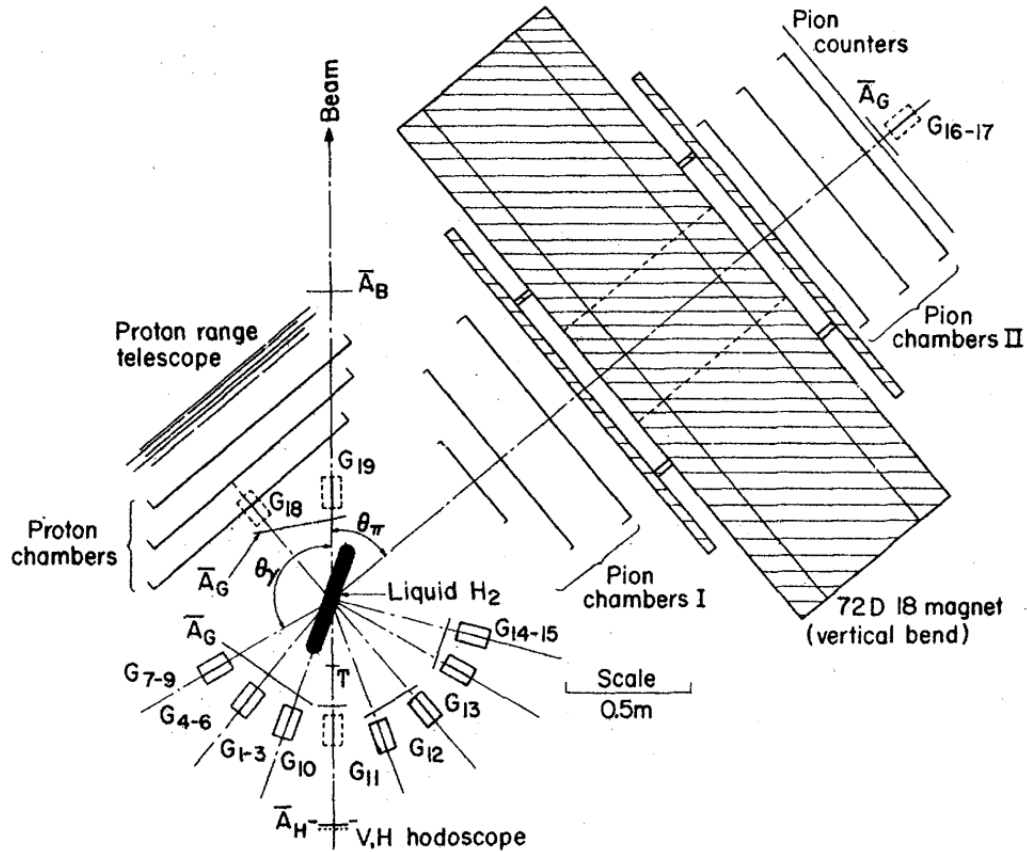


Figure 1.11.: Detection system used for registration of protons, pions and photons originating from pion-proton collisions. G1 through G19 are lead-glass counters applied for the registration of photons. Charged pions are registered in magnetic spectrometer equipped with a set of wire spark chambers (pion chambers) and with scintillation's counters (pion counters). The proton detector consisted of three wire spark chambers and a range telescope comprising scintillation counters and cooper absorbers. Figure is taken from Ref. [38].

differential cross sections $d^5\sigma/d\Omega_\pi d\Omega_\gamma dE_\gamma$ as a function of the photon energy were determined. A characteristic feature of these cross sections is a monotonic decrease with increasing photon energy, which is consistent with the soft-photon approximation (SPA) calculation by Liou and Nutt [39] (see also Chapter 1.5). An example the differential cross-section dependence on the photon energy compared with the SPA calculations is presented in Fig. 1.12. The five-fold differential cross-sections for the $\pi^- p \rightarrow \pi^- p \gamma$

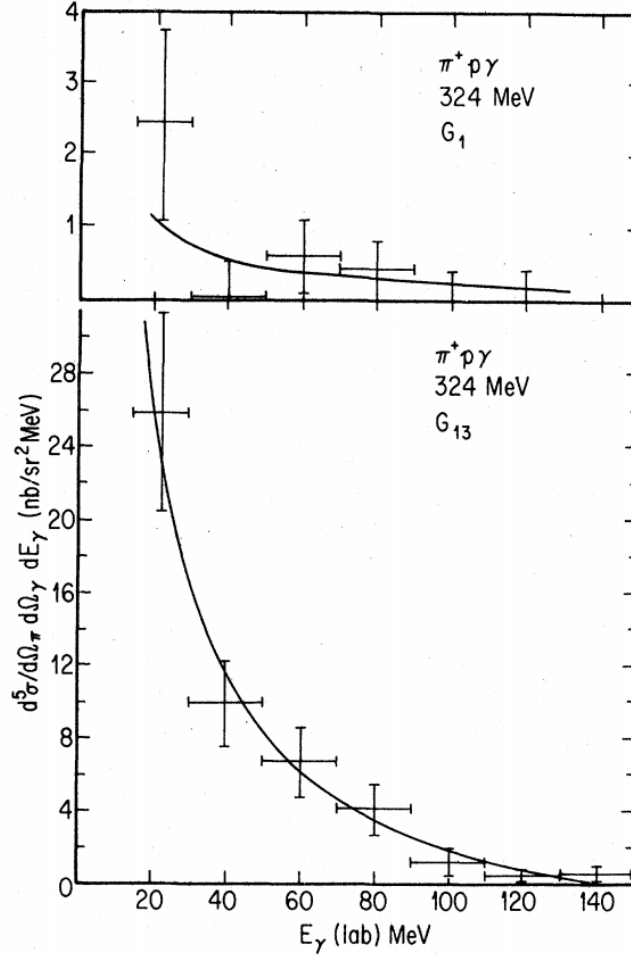


Figure 1.12.: Differential cross-section as a function of photon energy measured at two photon angles with two photon counters (G1 and G2) for the pion beam kinetic energy of 324 MeV. The solid line represents the SPA calculations by Liou and Nutt [39]. Figure is adopted from Ref. [38].

reaction were also measured by the Omicron collaboration using 300 GeV/c pion beam available at the CERN synchro-cyclotron [40] and for the $\pi^+ p \rightarrow \pi^+ p \gamma$ reaction were measured at SIN (at present Paul Scherrer Institut) at pion energy of 299 MeV [41].

An interesting result obtained in the measurements at SIN was the determination of the magnetic dipole moment of the $\Delta^{++}(1232)$ resonance. At energies near the $\Delta(1232)$ resonance, the bremsstrahlung photons are emitted not only by the pion or proton, but also by the Δ^{++} (see Fig. 1.13). This last contribution depends on the magnetic moment of the $\Delta(1232)$. The method of determining the Δ resonance magnetic moment was proposed by Kondratyuk and Ponomarev [42]. The magnetic dipole moment values of the $\Delta^{++}(1232)$ presented by the Particle Data Group [43] are based on the $\pi^+ p$ scattering data of SIN and LBL experiments.

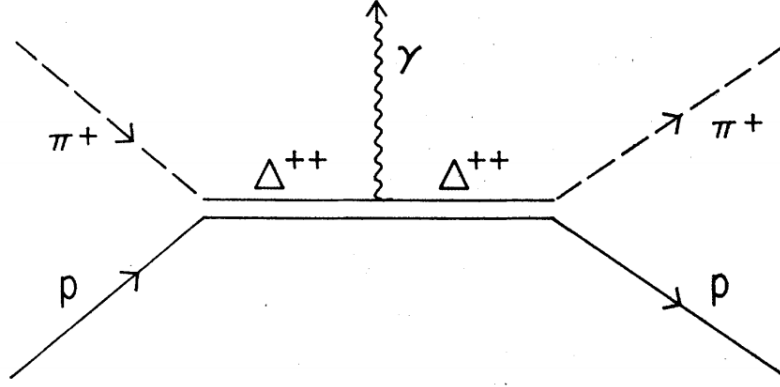


Figure 1.13.: Feynman diagram for the process $\pi^+ p \rightarrow \Delta^{++} \rightarrow \Delta^{++} \gamma \rightarrow \pi^+ p \gamma$. Figure is taken from Ref. [38].

1.5 Theoretical description of bremsstrahlung emission in hadron-hadron collisions

1.5.1 Soft-photon approximation

One of the important steps in the development of the theoretical description of the bremsstrahlung emission process was the derivation of the soft-photon theorem. This was done in 1958 by Low [44], who used the expansion of cross-section in power of the photon energy k :

$$\sigma = \frac{\sigma_{-1}}{k} + \sigma_0 + \sigma_1 k + \dots \quad (1.5.1)$$

where,

$$\sigma_{-1} = \lim_{k \rightarrow 0} (k \sigma), \quad (1.5.2)$$

$$\sigma_0 = \lim_{k \rightarrow 0} \frac{\partial}{\partial k} (k \sigma)_{x_i}, \quad (1.5.3)$$

$$\sigma_1 = \lim_{k \rightarrow 0} \frac{\partial^2}{\partial k^2} (k \sigma)_{x_i}, \quad (1.5.4)$$

Here x_i are various kinematic variables. They are held constant in carrying out the partial differentiation. The expansion coefficients are independent of k . The soft-photon theorem states that σ_{-1} and σ_0 are independent of the off-mass-shell effects and they can be evaluated from the knowledge of the two-body elastic scattering amplitude.

According to Ref. [39], the soft-photon approximation (SPA) for the bremsstrahlung cross-section is defined as:

$$\sigma_{SPA} = \frac{\sigma_{-1}(k)}{k} + \sigma_0(k). \quad (1.5.5)$$

The dependence of σ_{SPA} on k has the form of a hyperbola. Fig. 1.14 shows the differential cross-sections for the pion-proton bremsstrahlung ($\pi^\pm p \rightarrow \pi^\pm p \gamma$) measured at Lawrence Berkeley Laboratory [39, 45]. The dependence of the cross-sections on the photon energy is well described by the hyperbolic shape predicted by the SPA.

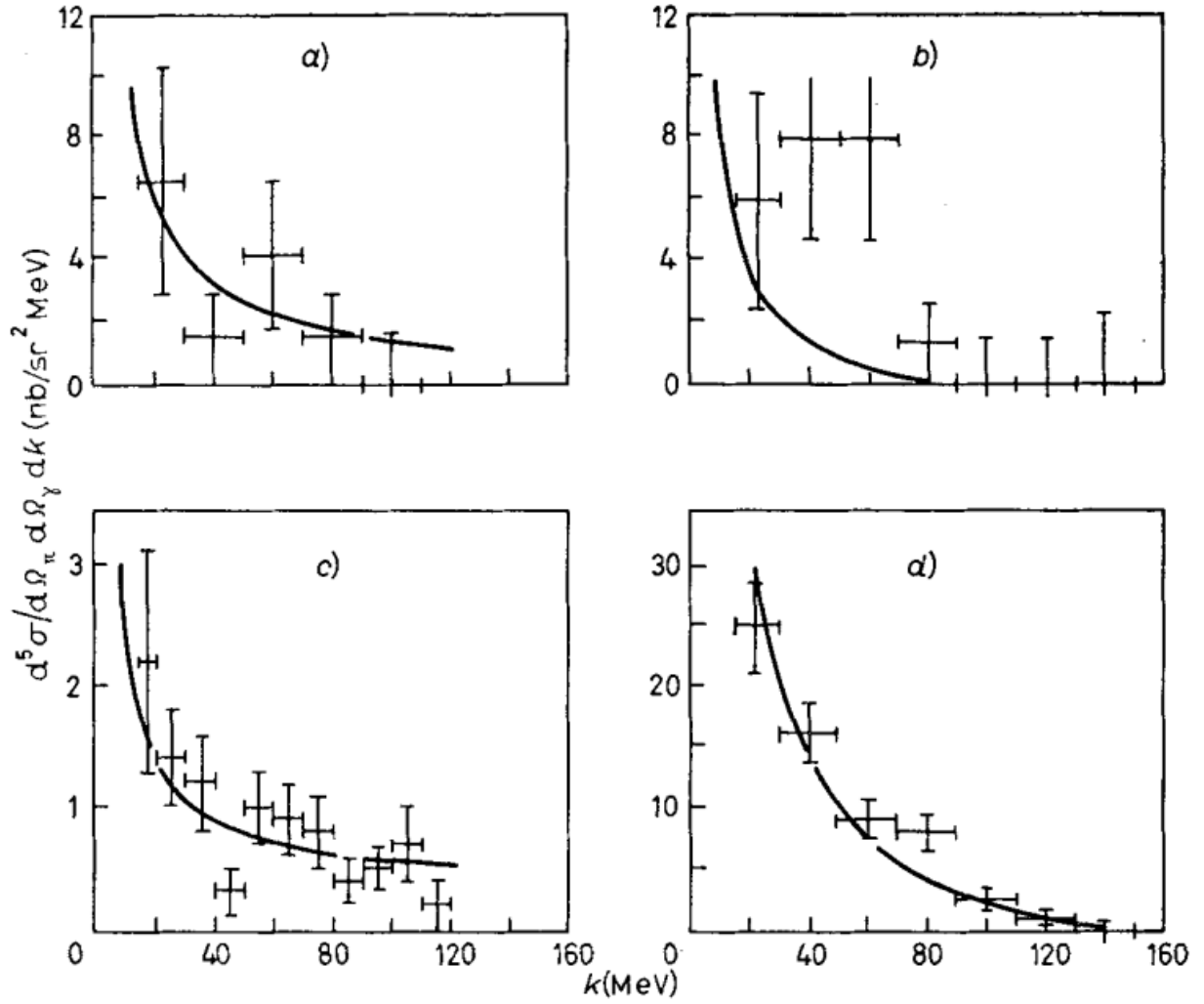


Figure 1.14.: The $\pi^\pm p \gamma$ cross-sections, $d^5\sigma/d\omega_\pi d\omega_\gamma dk$, in the laboratory system, where incident-pion energy is 298 MeV, the pion scattering angle is fixed at 50.5° and the gamma quantum scattering angle is: a) π^- , $\theta_\gamma = -120^\circ$ b) π^- , $\theta_\gamma = 120^\circ$ c) π^+ , $\theta_\gamma = -120^\circ$ d) π^+ , $\theta_\gamma = 120^\circ$. The solid curves represent the results of the soft-photon approximation given by eq. 1.5.5.

1.5.2 Dilepton production via virtual bremsstrahlung

In the soft-photon approximation, emission of bremsstrahlung occurs only from the external lines in the corresponding Feynman diagrams [46] as shown in Fig. 1.15. The radiation from the strongly interacting blob is negligible, till the energy of the photon is less than the inverse of the collision time related to the strong interaction $\tau \simeq 1\text{-}2 \text{ fm}/c$ which means $E_\gamma \leq 100\text{-}200 \text{ MeV}$ [47]. The main advantage of the soft-photon approximation, while estimating cross-section, is that the electromagnetic and strong interaction elements of the scattering matrix are separated. Besides, the strong interaction matrix is taken on energy shell. In the invariant cross-section for the emission of a real photon with four-momentum q^μ , the electromagnetic and strong interaction effects are represented by two separate factors [48]:

$$q_0 \frac{d^4\sigma^\gamma}{d^3q dx} = \frac{\alpha}{4\pi^2} \left(\sum_{pol\lambda} J \cdot \varepsilon_\lambda \cdot J \cdot \varepsilon_\lambda \right) \frac{d\sigma}{dx} \quad (1.5.6)$$

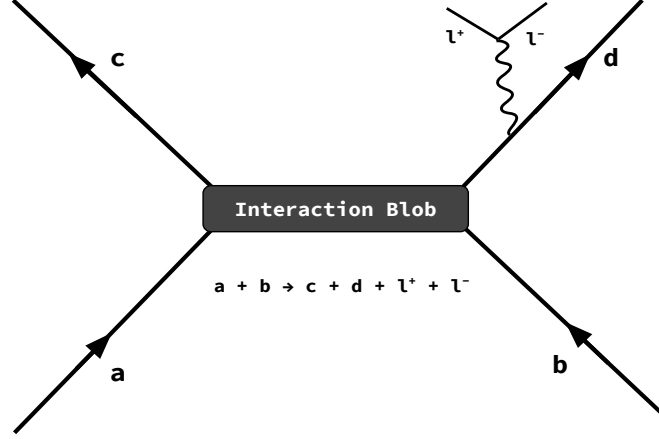


Figure 1.15.: Dilepton production via virtual bremsstrahlung in scattering of particles $a + b \rightarrow c + d + l^+ + l^-$. The shaded region indicates a strong interaction.

where ε_λ is emitted photon polarization, $d\sigma/dx$ is the strong interaction cross-section for the reaction $a + b \rightarrow c + d$, and

$$J^\mu = -Q_a \frac{p_a^\mu}{p_a \cdot q} - Q_b \frac{p_b^\mu}{p_b \cdot q} + Q_c \frac{p_c^\mu}{p_c \cdot q} + Q_d \frac{p_d^\mu}{p_d \cdot q}, \quad (1.5.7)$$

where Q 's and p 's are charge and four momentum of particles.

For virtual photons, leading to the emission of dileptons, the real photon cross-section can be extrapolated from $q^2 = 0$ to $q^2 = M$, where M is the invariant mass of the e^+e^- pair. According to Rückl [46] the result is:

$$E_+ E_- \frac{d^6 \sigma^{e^+e^-}}{dp_+^3 dp_-^3} = \frac{\alpha}{2\pi^2} \frac{1}{M^2} (\omega \frac{d^3 \sigma^{\gamma^*}}{d^3 q})_{\vec{q}=\vec{p}_+ + \vec{p}_-} \quad (1.5.8)$$

where $\vec{p}_+$ and $\vec{p}_-$ refers to three momentum of leptons, $E_\pm = (p_\pm^2 + \mu^2)^{1/2}$ is energy of lepton, $M^2 = (p_+ + p_-)^2$ is squared invariant mass of dilepton, ω is energy of emitted photon and α is fine structure constant. The above formula was studied in detail by Schafer et al. [49], Gale and Kapusta [48] and Wolf et al. [50], who generalized the formalism with additional phase-space correction.

1.5.3 Cross-section for the production of dileptons

In this work, for the simulation of dilepton production via virtual bremsstrahlung, we use the expression proposed in Ref. [50]. This expression contains a correction of the phase-space volume accessible for the colliding particles in their final state due to the emission of the virtual photon. It has the form:

$$\frac{d^4 \sigma_{e^+e^-}}{dy d^2 p_t dM} = \frac{\alpha^2}{6\pi^3} \frac{\widehat{\sigma}(s)}{ME^2} \frac{R_2(s_2)}{R_2(s)}, \quad (1.5.9)$$

$$R_2(s) = \sqrt{1 - \frac{(m_1 + m_2)^2}{s}}, \quad s_2 = s + M^2 - 2E\sqrt{s}, \quad (1.5.10)$$

$$\widehat{\sigma}(s) = \frac{s - (m_1 + m_2)^2}{2m_1^2} \sigma(s), \quad (1.5.11)$$

where m_1 and m_2 are masses of colliding particles, M is the invariant mass, E is the energy, p_t is the transverse momentum, y is the rapidity of dilepton pairs and $\sigma(s)$ is elastic cross-section of respective reaction channel. For our calculations, the above equation is reduced from fourfold to threefold differential equation as derived in Appendix A and is stated as:

$$\boxed{\frac{d^3\sigma}{dydp_t^2dM} = \frac{\alpha^2}{6\pi^2} \frac{\widehat{\sigma(s)}}{ME^2} \frac{R_2(s_2)}{R_2(s)}}. \quad (1.5.12)$$

Chapter 2

HADES spectrometer at GSI

"Science is the attempt to discover, by means of observation, and reasoning based upon it, first, particular facts about the world, and then laws connecting facts with one another and (in fortunate cases) making it possible to predict future occurrences."

Bertrand Russell

The experimental studies presented in this thesis were performed at the GSI research center using a pion beam and the HADES spectrometer. In Sections 2.1, 2.2 the accelerator complex in GSI, and the production and properties of the pion beam are described. In Section 2.3, the details of the HADES detection system are provided.

2.1 GSI accelerator complex

The accelerator facility in GSI Helmholtz Institute for Heavy Ion Research in Darmstadt, Germany [51], was designed to provide ion beams of all natural chemical elements, from hydrogen to uranium. The key components of this facility (Fig. 2.1) are ion sources, UNiversal Linear ACcelerator (UNILAC) and heavy-ion synchrotron SIS-18 (*Schwer-Ionen* Synchrotron). The ion sources are used to generate beams of ions with various intensities and ionization levels. The beams accelerated in UNILAC to maximum energy 11.5 MeV/u are delivered to the experimental setups in the low energy experimental area. They can be also transferred to SIS-18 for further acceleration to higher energies. SIS-18 has a circumference of 216 meters and consists of 24 dipole magnets, 12 triplet lenses, 12 sextupole lenses and 2 accelerating cavities. The maximum magnetic rigidity of SIS-18 magnets is 18 Tm. SIS-18 provides proton beam with maximum energy of 4.5 GeV and intensity 2.8×10^{13} /s. For heavy ions the achievable energies per nucleon and intensities are smaller and e.g. for U^{72+} , the maximum energy is 1.0 GeV/u and intensity is 10^9 ions/s. The beam extracted from SIS-18 is transferred to one of the external experiments, e.g. HADES. It is also used to produce secondary beams of pions as well as rare isotopes in collisions with external targets. The interesting rare isotopes are separated from other collision products in the fragment separator (FRS) and can be stored in the Experimental Storage Ring (ESR) for further investigations.

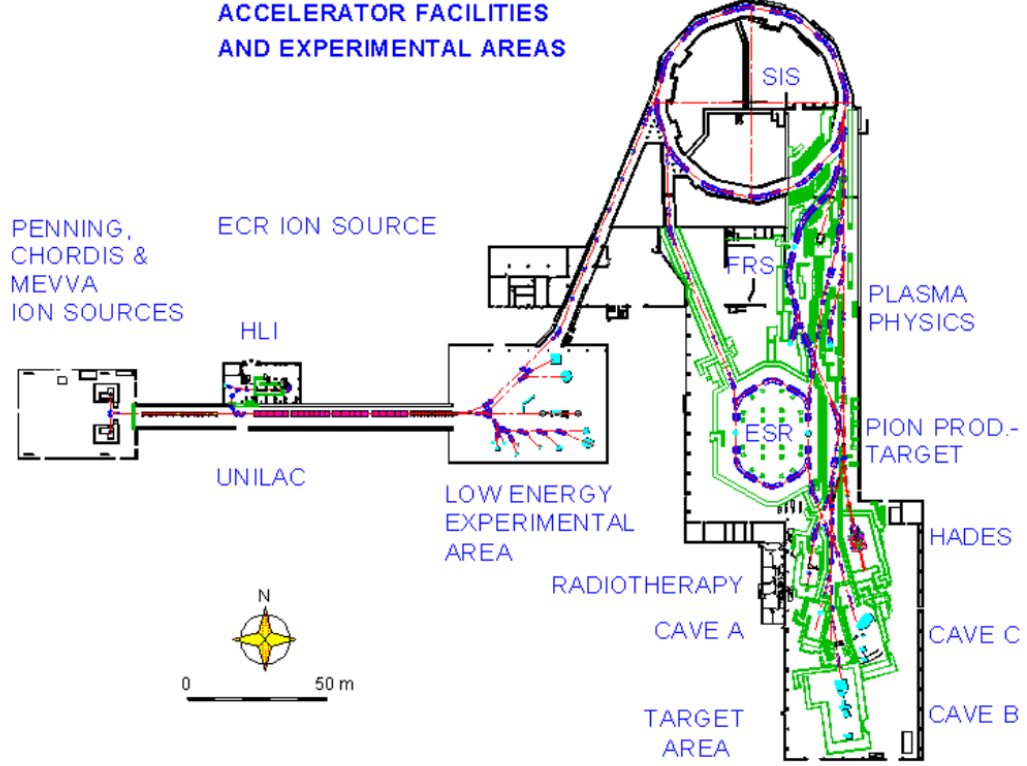


Figure 2.1.: Schematic view of the GSI accelerator complex. Ion sources, linear accelerator UNILAC, low energy experimental area, SIS-18, and the high energy experimental zone with various experiments, including the HADES spectrometer, are shown from left to right.

2.2 Production of secondary pion beam

The secondary π^- beam used in the present experiment was produced with the use of 2 AGeV nitrogen (^{14}N) beam from SIS-18, with an intensity of $0.8 - 1.0 \times 10^{11}$ ions / spill, hitting a 10 cm thick production target made of beryllium (^9Be). The produced pions were collected and transported to the HADES target in a dedicated beam line, hereinafter referred to as a pion beam line, shown schematically in Fig. 2.2. The pion beam line is 33 m long and consists of nine quadrupole magnets (Q1-Q9) and two dipole magnets (D1 and D2). Besides, it also contains a tracking system called CERBEROS [53] comprising two 0.3 mm thick double-sided silicon strip detectors (Det1 and Det2) that cover a $10 \times 10 \text{ cm}^2$ area and are divided into 128 horizontal and vertical strips. The detectors are placed in two dispersive planes of the pion beam line as shown in Fig. 2.2. The combination of the measured hit position (X , Y) from both silicon planes allows to reconstruct the pion momentum as discussed below in the second part of this section. The transmission of the pion beam line is highest for the pion momentum equal to a central momentum p_0 set in the beam line and it equals to about 56% with respect to the entrance solid angle [54]. The transmission decreases as the pion momentum deviates from the central momentum and it reaches zero for momenta of $p_0 \pm 6\%$. The transmission is approximately described by a Gaussian distribution with a variance of $\delta p/p_0 = 1.5\%$.

The dependence of the pion rate at the HADES target on the central momentum p_0 set in the pion beam line was investigated in a previous study for a primary carbon (^{12}C) beam as well as proton beam from SIS-18 [55]. For the energy of the ^{12}C beam equal to 2 AGeV, the largest pion rate was observed for $p_0 = 1 \text{ GeV}/c$ and decreased by a factor of 2 in the range $p_0 = 0.7 \text{ GeV}/c$ and $1.5 \text{ GeV}/c$ (see Fig. 2.3). Due to the same energy per nucleon of the primary ^{14}N beam used in the present measurements and a

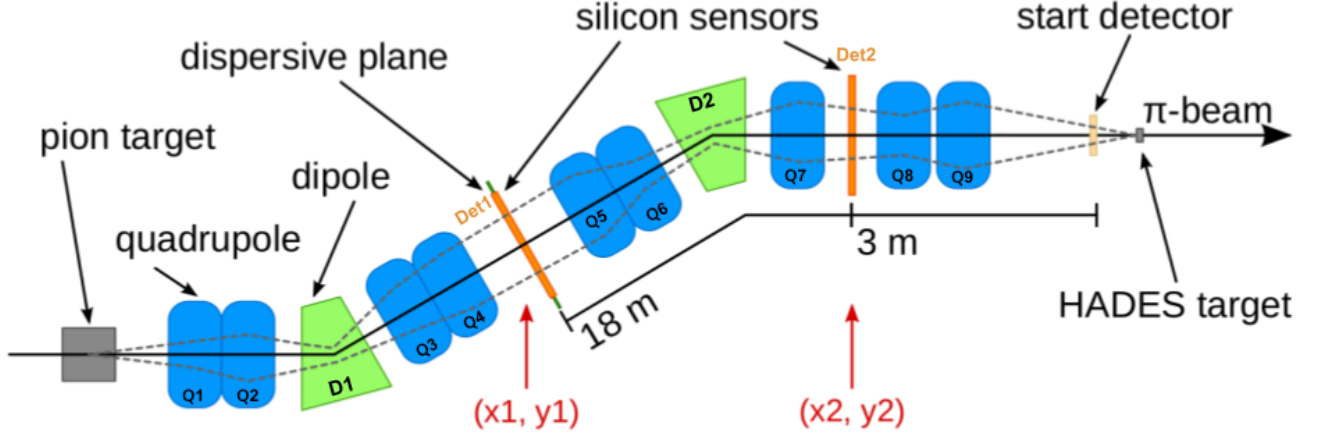


Figure 2.2.: Schematic top view of the pion beam line between the production target and the HADES target. The pions (dashed line) are guided by quadrupole (Q1-Q9) and dipole (D1 and D2) magnets.

similar value of the atomic number of the beam, one can expect a similar intensity distribution as in the cited measurements with the ^{12}C beam.

The present measurements were performed for 4 different central momenta of the pion beam: 0.656, 0.685, 0.748 and 0.800 GeV/c (see Section 5.1). The intensity of pions at the HADES target was around 2×10^5 per spill (1 second spills at three seconds long synchrotron cycles).

2.2.1 Determination of pion momentum

The pion track coordinates (X^{det1} , Y^{det1} , X^{det2} , and Y^{det2}) measured in the two silicon strip detectors are related to five parameters describing the pion track position, inclination and momentum at the production target, namely:

1. x_0 - the coordinate along the horizontal axis,
2. θ_0 - the angle in the horizontal plane, defined as $\tan\theta_0 = \frac{dx}{dz} = \frac{p_x}{p_z}$,
3. y_0 - the coordinate along the vertical axis,
4. ϕ_0 - the angle in the vertical plane, defined as $\tan\phi_0 = \frac{dy}{dz} = \frac{p_y}{p_z}$,
5. $\delta = \frac{p - p_0}{p_0}$ is the momentum offset with respect to the momentum p_0 of particles on the optical axis.

This relation was determined in the beam transport calculation [53]:

$$\begin{cases} X^{det1} = T_{11}^{det1} \cdot x_0 + T_{12}^{det1} \cdot \theta_0 + T_{14}^{det1} \cdot \phi_0 + T_{16}^{det1} \cdot \delta + T_{116}^{det1} \cdot x_0\delta + T_{126}^{det1} \cdot \theta_0\delta + T_{146}^{det1} \cdot \phi_0\delta + T_{166}^{det1} \cdot \delta^2, \\ X^{det2} = T_{11}^{det2} \cdot x_0 + T_{12}^{det2} \cdot \theta_0 + T_{14}^{det2} \cdot \phi_0 + T_{16}^{det2} \cdot \delta + T_{116}^{det2} \cdot x_0\delta + T_{126}^{det2} \cdot \theta_0\delta + T_{146}^{det2} \cdot \phi_0\delta + T_{166}^{det2} \cdot \delta^2, \\ Y^{det1} = T_{32}^{det1} \cdot \theta_0 + T_{33}^{det1} \cdot y_0 + T_{34}^{det1} \cdot \phi_0 + T_{36}^{det1} \cdot \delta + T_{336}^{det1} \cdot y_0\delta + T_{346}^{det1} \cdot \phi_0\delta + T_{366}^{det1} \cdot \delta^2, \\ Y^{det2} = T_{32}^{det2} \cdot \theta_0 + T_{33}^{det2} \cdot y_0 + T_{34}^{det2} \cdot \phi_0 + T_{36}^{det2} \cdot \delta + T_{336}^{det2} \cdot y_0\delta + T_{346}^{det2} \cdot \phi_0\delta + T_{366}^{det2} \cdot \delta^2, \end{cases} \quad (2.2.1)$$

where T_{ij} and T_{ijk} are the first and second order beam transport coefficients, respectively, calculated based on the known beam line optics.

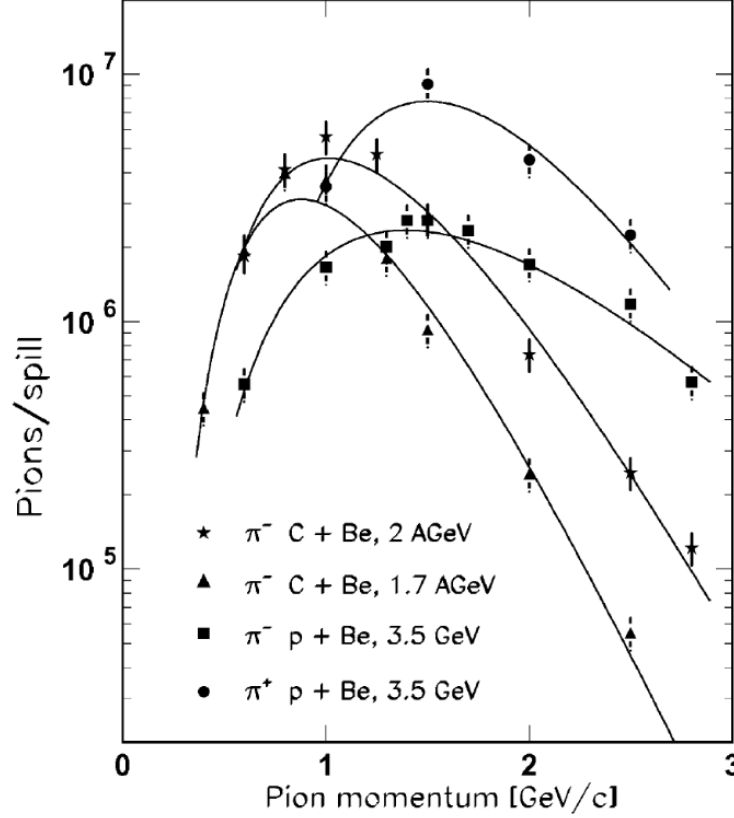


Figure 2.3.: The intensity of pions for carbon and proton primary beams at various energies as a function of the central beam momentum set in the pion beam line. Fits to the data shown by a solid line are described in Ref. [55].

The above 4 equations contain 5 variables and their solution is possible only with the imposition of an additional constraint. As this constraint, $x_0 = 0$ was set, which is justified by the weak dependence of the determined momentum on x_0 . Solving the above system of equations makes it possible to determine the pion momentum. Obtained resolution of the pion momentum determination is about 0.3%, [53].

Components of the pion momentum vector at the HADES target are calculated using the following formulae:

$$\begin{aligned}
 p &= p_{spec}(1 + \delta), \\
 p_x &= \frac{p \tan(\theta_H)}{\sqrt{1 + \tan^2(\theta_H) + \tan^2(\phi_H)}}, \\
 p_y &= \frac{p \tan(\phi_H)}{\sqrt{1 + \tan^2(\theta_H) + \tan^2(\phi_H)}}, \\
 p_z &= \frac{p}{\sqrt{1 + \tan^2(\theta_H) + \tan^2(\phi_H)}},
 \end{aligned} \tag{2.2.2}$$

where the pion track inclinations θ_H , ϕ_H at the HADES target are:

$$\begin{cases} \theta^H = T_{22}^H \cdot \theta_0 + T_{23}^H \cdot y_0 + T_{26}^H \cdot \delta + T_{226}^H \cdot \theta_0 \delta + T_{246}^H \cdot \phi_0 \delta + T_{266}^H \cdot \delta^2, \\ \phi^H = T_{42}^H \cdot \theta_0 + T_{43}^H \cdot y_0 + T_{44}^H \cdot \phi_0 + T_{46}^H \cdot \delta + T_{426}^H \cdot \theta_0 \delta + T_{436}^H \cdot y_0 \delta + T_{446}^H \cdot \phi_0 \delta + T_{466}^H \cdot \delta^2. \end{cases} \tag{2.2.3}$$

These components are used when calculating the kinematics of the $\pi^- p$ collisions in individual events.

2.2.2 Beam Contamination

The negative pion beam is partly contaminated with electrons, muons and negatively charged kaons. Electrons are mainly produced in the Dalitz decay of neutral pions ($\pi^0 \rightarrow \gamma e^+ e^-$, branching ratio = 1.2%) and in the conversion of gamma quanta near the production target ($e^+ e^-$ pair production), where the gamma quanta come from the neutral pion decay ($\pi^0 \rightarrow \gamma \gamma$, branching ratio = 98.8%). The electron contamination of the pion beam at the HADES target was determined to be about 10% for the beam momentum of 0.7 GeV/c.

The muon contamination is caused by an in-flight decay of pions ($\pi^- \rightarrow \mu^- \nu$, branching ratio = 99.99%, $c\tau = 7.8$ m). The simulated μ/π ratio at the HADES target for the pion beam momentum of 0.7 GeV/c is 0.65%. The contamination of the pion beam with electrons and muons does not affect the current experiment due to the low probability of their interaction with the target (the electromagnetic interaction, not the strong interaction as in the case of pions).

The negatively charged kaons are produced in associated strangeness creation process $NN \rightarrow NN K^+ K^-$, with a threshold kinetic energy of 2.5 GeV. The number of kaons reaching the target is significantly suppressed as most of the kaons decay before reaching the HADES target due to their small decay lengths ($c\tau = 5.25$ m at 0.7 GeV/c).

2.3 HADES Spectrometer

2.3.1 Overview :

The **H**igh **A**cceptance **D**i-**E**lectron **S**pectrometer (HADES), shown in Fig. 2.4, is a magnetic spectrometer operating since 2002 at the SIS-18 synchrotron in GSI Helmholtz Institute for Heavy Ion Research in Darmstadt, Germany [56]. It is used to study both hadron and rare dilepton production in collisions of pion, proton and heavy-ion beams with proton as well as nuclear targets in the beam energy range reaching up to 4.5 GeV for a proton beam and 1.5 GeV/nucleon for a heavy-ion beam.

The HADES spectrometer is built out of 6 identical sectors, which cover polar angles between 18° to 85° . The whole spectrometer is rotationally symmetric around the beam axis. The hadron-blind Ring Image CHerenkov (RICH) detector is the HADES innermost detector. It is the main component of HADES used for identification of electrons and positrons. The magnetic field created by the coils of a superconducting magnet and a set of four Multi-wire Drift Chambers (MDCs), two chambers in front of and the other two behind the field, are used for the momentum reconstruction of charged particles. The META detector (Multiplicity and Electron Trigger Array) is positioned on the spectrometer's outermost shell. It is used for triggering and particle identification. It consists of two time-of-flight detector systems (RPC and TOF) and an electromagnetic shower detector (Pre-Shower). For triggering purposes and to provide a reference time for time-of-flight measurements, the Start detector located in the target region, is used.

All of the different detector components of HADES are discussed in detail in the following sections.

The main advantages of the HADES spectrometer are:

1. large geometrical acceptance for detecting dielectron pairs, high detection efficiency $\epsilon_{pair} \approx 40\%$ for pairs with large opening angle (e.g from $\omega \rightarrow e^+ e^-$) and $\approx 10\%$ for close pairs with small opening angle (e.g from $\pi^0 \rightarrow \gamma e^+ e^-$),
2. high purity of electron and positron identification,
3. high mass resolution, $\Delta M_{inv} / M_{inv}$ of 2% in the light vector meson mass region,
4. high rate capability, up to 10^6 interactions per second.

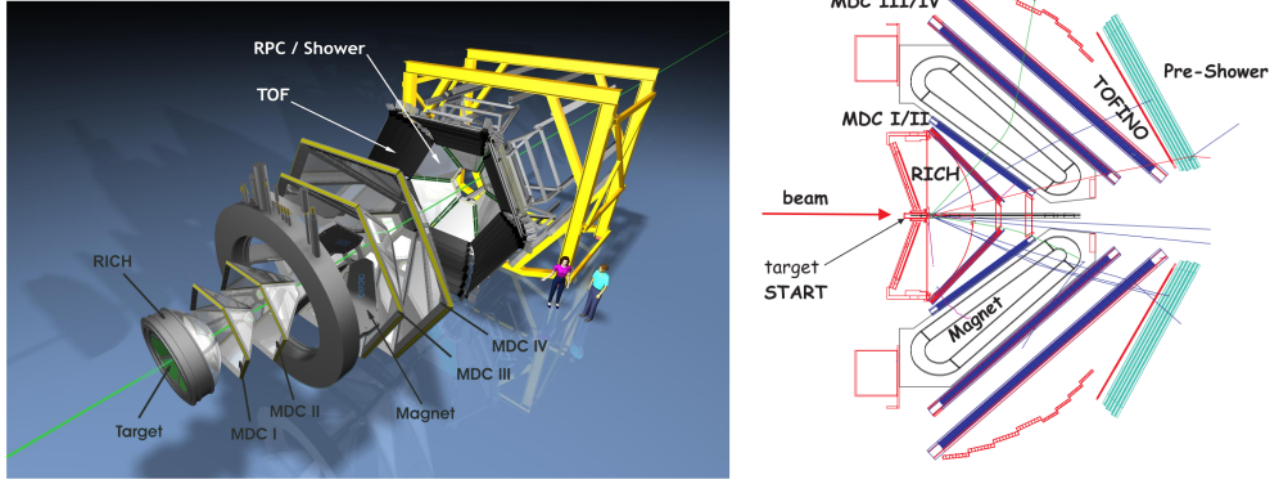


Figure 2.4.: The HADES detector (left panel), and its cross section view (right panel).

2.3.2 Target

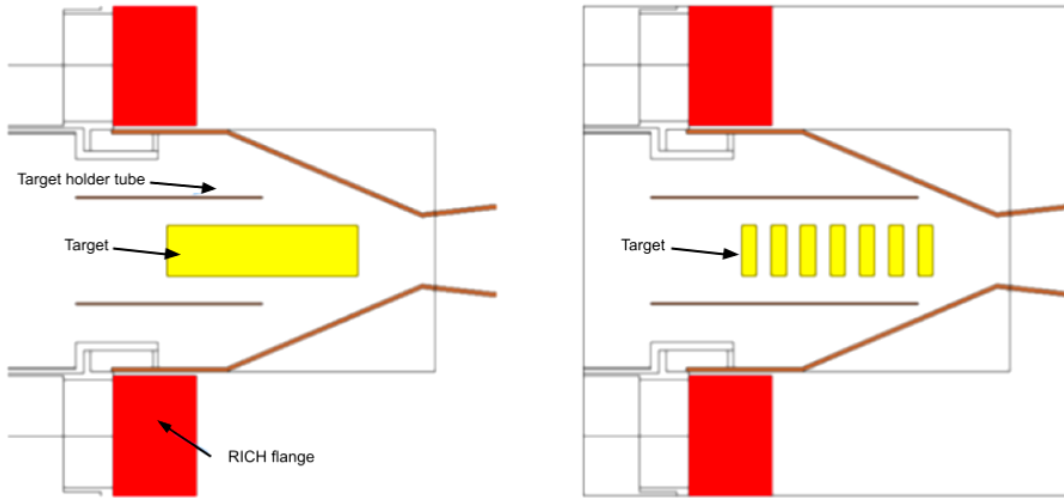


Figure 2.5.: Arrangement of the polyethylene target (left panel) and the carbon target (right panel) in the HADES vacuum pipe.

In the present experiment, a polyethylene (C_2H_4)_n target was used to study the reactions induced by π^-p collisions. Additionally, a carbon target was applied to measure the background related to the interaction of the pion beam with carbon nuclei in polyethylene. The polyethylene target was cylindrical in shape, 46 mm in length and 12 mm in diameter. The carbon target consisted of seven identical cylinders of the same diameter and occupying the same length along the beam direction as the polyethylene target, (see Fig 2.5).

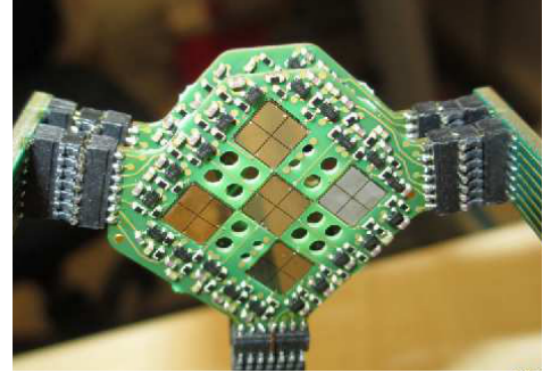
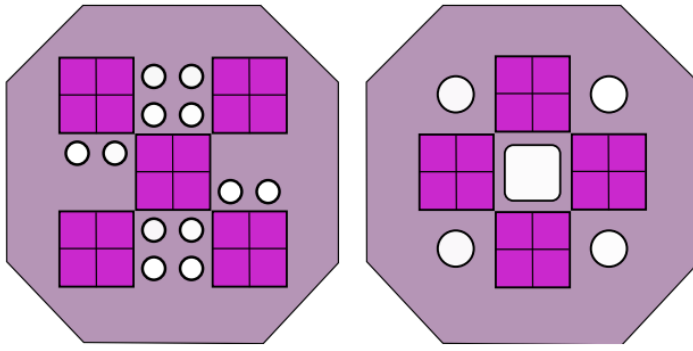


Figure 2.6.: Top view of five and four diamond sensor plates with fourfold segmentation on the diamond support Printed Circuit Boards (PCBs) (left panel). The nine diamond sensor plates are mounted on two printed circuit boards of the Start detector (right panel).

2.3.3 Start detector

The Start detector is used to determine the beam profile and generate fast timing signals for trigger purposes, as well as to measure time-of-flight of particles. The detector was created specifically for the pion beam experiment in order to achieve excellent timing precision.

A mono-crystalline Chemical Vapor Deposition (CVD) diamond semiconductor [57,58] has been developed for the 2nd generation of the detector. The Start detector, shown in Fig 2.6, is made up of nine diamond sensors, each $4.3 \times 4.3 \text{ mm}^2$ wide and $300 \text{ }\mu\text{m}$ thick, with signal/noise ratio of 30 and a time resolution of about 90 ps. Start detector was located 17 cm upstream from the target center. The detection efficiency of diamond sensors was found to be more than 95% [58].

2.3.4 Magnet

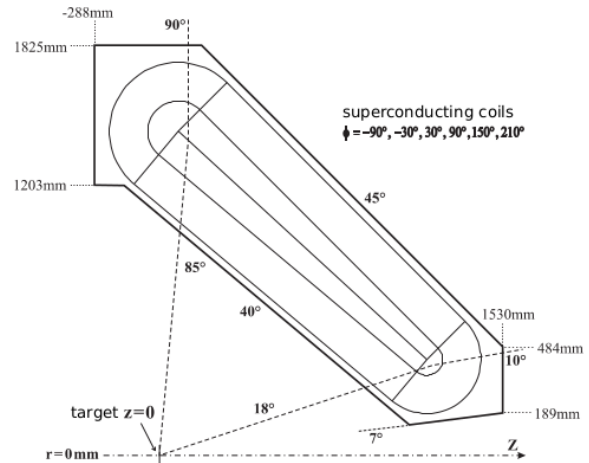
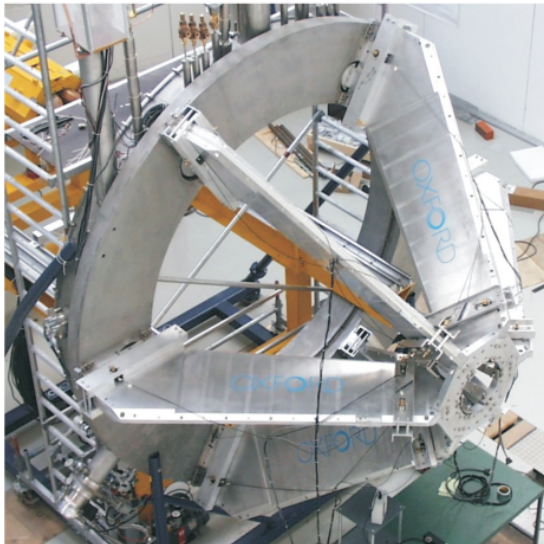


Figure 2.7.: Photography of the magnet during the mounting (left) and a drawing of a cross-section of one vacuum chamber with coil (right).

The magnetic field in the HADES spectrometer is produced by six superconducting coils arranged symmetrically around the beam direction (see Fig. 2.7, left panel) [51]. Each coil is housed in a vacuum chamber made of aluminum (see Fig. 2.7, right panel). The chambers are fixed at both ends on circular supporting structures. The materials used to build the magnet were chosen to minimize the creation of secondary particles. The weight of the magnet is 3.5 tones.

The coils produce magnetic field of a toroidal shape. The field is strongly inhomogeneous both in the polar and azimuthal angles. For the maximum current in the coils of 3566 A, it reaches 3.77 T in the coil and 0.8 T between the coils. The field map was measured using Hall probes and it is used in the reconstruction of particle momenta.

2.3.5 Mini Drift Chambers

In the HADES spectrometer, the deflection of particle trajectories in the toroidal magnetic field of the superconducting magnet is measured by a set of 24 Mini Drift Chambers (MDCs). The MDCs have trapezoidal shape and are arranged in 6 sectors defined by the coils of the magnet. In each sector, two drift chambers (MDC I, II) [59] are placed in front of the magnetic field and another two (MDC III, IV) behind the field (Fig. 2.8). The MDCs are arranged in relation to the magnet in such a way that the non-parallel sides of the trapezoidal chamber frames stay in the shadow of the magnet coils when viewed from the target point. To cover the same solid angle, the active area of the MDCs in one sector increases with the distance from the target and it ranges from 0.35 m² for MDC I to 3.2 m² for MDC IV. The MDC system covers polar angles in the range from 18° to 85°.

Each MDC contains six layers of sense (anode) wires interleaved with potential wires, oriented at different

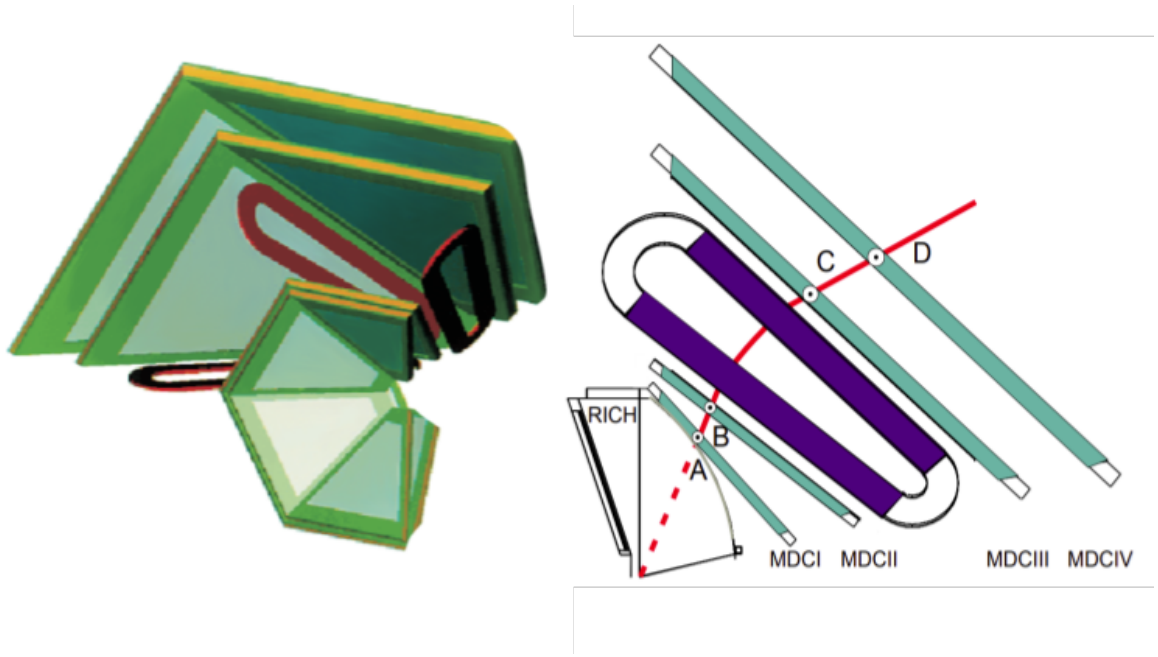


Figure 2.8.: Left: Arrangement of the HADES MDCs and the coils of the superconducting magnet. Right: Schematic view of a section of one sector including the RICH detector, four MDCs and coil of the superconducting magnet.

angles from the inner to the outer layer: +40°, -20°, 0°, 0°, +20°, +40° (see Fig. 2.9 left panel) to get high spatial resolution in the polar direction (the direction of particle trajectory deflection in the magnetic field) which is important for achieving high momentum resolution. The sense wire layers are separated by cathode wire layers. Cathode wires in all layers are set at an angle of 90°. The basic unit of the MDCs – a drift cell, comprising a single sense wire and the neighboring potential wires and cathode wires, is presented

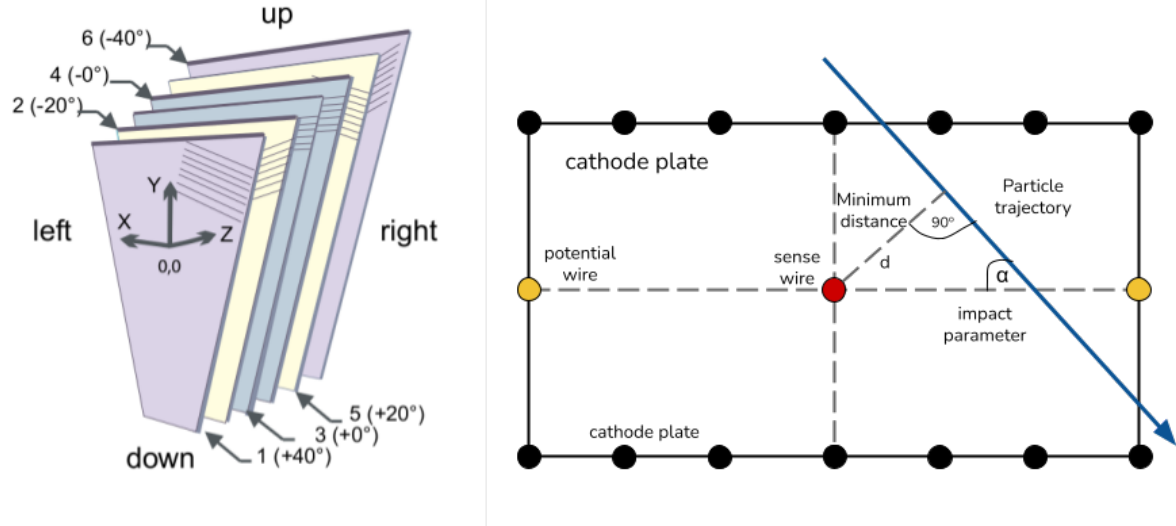


Figure 2.9.: Left: Schematic view of six anode wire layers in one MDC with indicated orientations of wires. The wire inclination angle is determined with respect to the x-axis of the indicated coordinate system. Right: A scheme of a single drift cell displaying the location of various wires. A charge particle track's minimum distance from the sense wire is also specified.

in the right panel of Fig. 2.9. The size of the cell increases from $5 \times 5 \text{ mm}^2$ in MDC I to $14 \times 10 \text{ mm}^2$ in MDC IV to achieve a similar cell occupancy. The sense wires are at 0 V potential and the cathode wires at -2 kV. The entire HADES MDC system contains 27000 drift cells. The chambers MDC I,II are filled with Ar:CO₂ (70:30) gas mixture and in MDC III, IV mixture of Ar and C₄H₁₀ (84:16) is used. In the past, the Ar:C₄H₁₀ mixture was also used in MDC I, II, however, due appearance of aging effects (deposits on wires, typically polymers), isobutene (C₄H₁₀) was replaced by carbon dioxide (CO₂) as the latter does not lead to polymer formation and is therefore more favorable in terms of drift chamber aging. The MDCs are equipped with preamplifier/shaper/discriminator cards as well as Time-to-Digital Converters (TDCs) that register the moments of time in which the rising and falling edges of the MDC pulses cross the discriminator threshold. The drift time of electrons in the cell is determined as a difference between the leading edge of the MDC pulse and a pulse from a reference detector e.g. the Start detector. The particle track distance from the sense wire is calculated from the drift time and the distance-drift time relation determined using Garfield simulation [60]. In turns, the time difference between the leading and trailing edge of the MDC pulse, referred to as time-over-threshold, is related to the energy loss of the particle in the drift cell and thus is used for particle identification. Track distances to anode wires in the drift cells through which the particle has passed, are used for the track reconstruction described in Section. 3.1.

2.3.6 Ring Imaging Cherenkov detector

The main purpose of the HADES RICH detector is to identify electrons and positrons in the huge background of charged hadrons – mainly protons, pions and kaons. As a radiator for the RICH, C₄F₁₀ gas at atmospheric pressure was chosen. The radiator has a refraction index of $n = 1.00151$ and the corresponding threshold velocity of charged particles to produce Cherenkov photons is $\beta = 0.9985$. The threshold momentum for electrons, pions and protons is 9.3 MeV/c, 2.55 GeV/c and 17.1 GeV/c, respectively. At the energies of the SIS-18 beams, only electrons produced in the interactions in the HADES target have sufficient velocity to radiate Cherenkov photons. Thus, the RICH remains completely insensitive to hadrons (it is “hadron-blind”).

The RICH is composed of two distinct gas chambers (see Fig. 2.10). The first one, surrounding the target

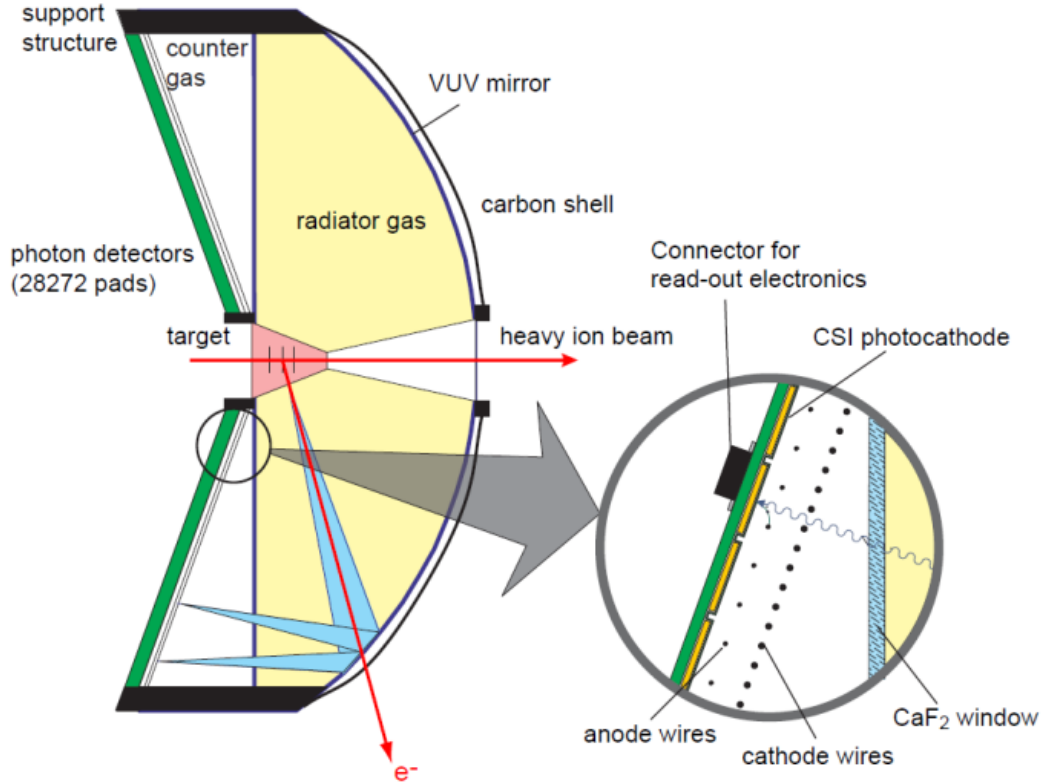


Figure 2.10.: The RICH detector cross section. The trajectory of the electron released from the target (marked in red) traveling through the radiator gas results in the emission of Cherenkov photons (blue cone), which are reflected further from the mirror and focused in the shape of a ring on the surface of the photo detector. The enlarged region at the right side displays the CaF_2 window and a scheme of the Multi Wire Proportional Chamber with CSI-coated photo-cathode pads.

area, is filled with C_4F_{10} gas radiator. Cherenkov photons emitted in this region by an electron/positron are reflected by an aluminized carbon spherical mirror and pass through CaF_2 crystalline window to the second chamber containing a photon detector based on Multi Wire Proportional Chambers (MWPCs). The second chamber is filled with methane, which is used as a working gas for the MWPCs. The photon detector consists of six independent segments, each covering an azimuthal angle interval of 60° and containing one MWPC. A scheme of the MWPC is shown on the right-hand side of Fig. 2.10. The MWPC consists of a grid of anode wires at a voltage of 2500 V resulting in a gas gain of 10^5 , placed between two cathode planes. One plane consists of cathode wires and the other contains an array of 4712 pads. The pads are coated with CsI which acts as a photon-electron converter. Photoelectron extracted from CsI pad by Cherenkov photon is multiplied in the strong electric field near the anode wire. Each pad is connected to individual readout electronics channel that allows to register the pulses caused by the movement of electrons and positive ions produced near the anode. Cherenkov photons produced by electrons/positrons in the radiator gas form a ring image on the photon detector surface with an almost constant radius (≈ 3.8 pads). Identification of electrons/positrons relies on the recognition of images of rings registered by the photon detector (see Section 3.4).

The main difficulty in the analysis of the RICH data results from a small number of registered photoelectrons. Electron passing through the HADES RICH radiator produces on the average 110 photons. Their wavelength spectrum lies mainly in the ultraviolet region. However, due to limited transmission coefficient in the C_4F_{10} gas radiator, the CaF_2 window and in methane, as well as due to limited quantum efficiency of

the CsI photo-cathode (see Fig. 2.11), number of registered photoelectrons is an order of magnitude smaller.

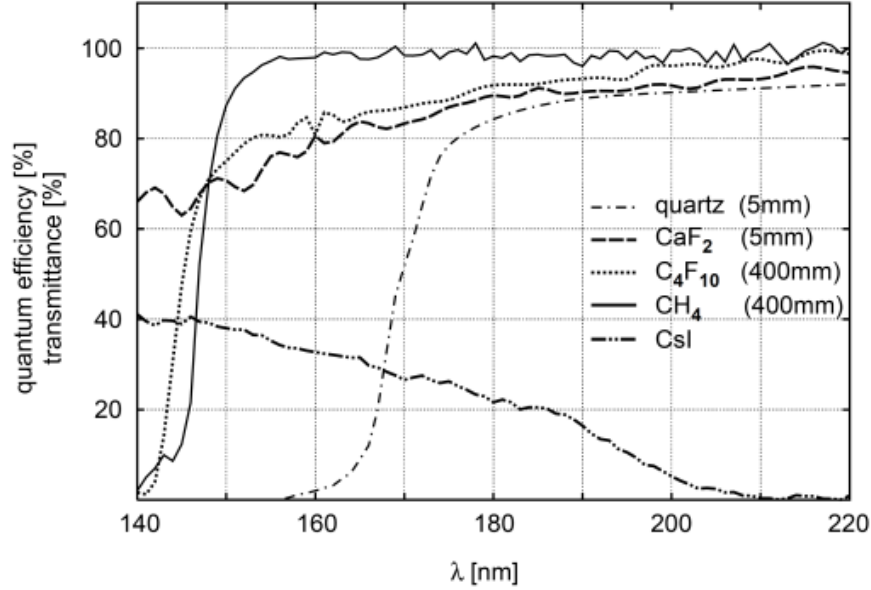


Figure 2.11.: Transmission of Cherenkov photons in 5 mm thick CaF_2 window and quartz (for comparison), in the 400 mm layer of C_2F_{10} and CH_4 gas as well as the quantum efficiency of the extraction of photoelectrons from the CsI pad plane coating.

2.3.7 META detector

For particle identification and triggering, the Multiplicity Electron Trigger Array (META) system is used. It comprises of two sets of time-of-flight detectors. One set is based on Resistive Plate Chambers (RPCs) and the other referred to as TOF detector, on plastic scintillator arrays. Besides, META includes an electromagnetic cascade detector (Pre-Shower). META system is positioned behind the outer MDCs.

TOF detector

The scintillation TOF detector covers the full range of azimuthal angles and the upper polar angles $45^\circ < \theta < 85^\circ$ of the HADES acceptance (Fig. 2.12, left panel). It is divided into six sectors, following a hexagonal geometry of the entire spectrometer. Each sector consists of eight modules and each module contains eight plastic scintillator rods readout from both ends by photomultiplier tubes (PMTs). The TOF detector provides the information about the time-of-flight of charged particles with a resolution of 150 ps [61], the hit location along the rod (x), with resolution $\sigma_x \approx 25$ mm, and the energy loss in the rod that can be used for the particle identification [62].

Resistive Plate Chambers

In 2008 Resistive Plate Chambers (RPCs) replaced the low granularity detector TOFINO, made of scintillators. In each of the six HADES sectors, one RPC covering polar angles $18^\circ - 45^\circ$ is used. The RPC has a trapezoidal shape and it contains two layers of individually shielded RPC counters. The counters have a strip-like shape and, in each of the two layers, are arranged in three columns and 31 rows, as shown in left panel of Fig. 2.13. A single counter consists of 3 aluminium electrodes interlaced with 2 glass electrodes (see right panel of Fig. 2.13). All the electrodes are 2 mm thick. The electrodes are separated by gaps

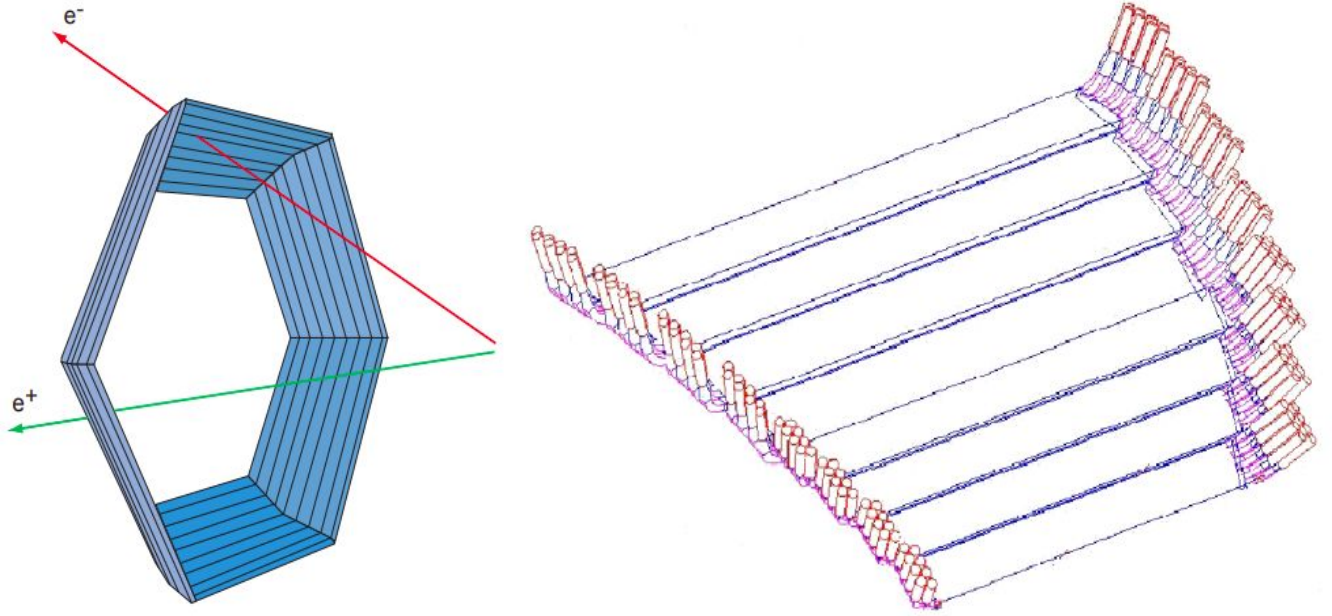


Figure 2.12.: Six sectors of the TOF detector (left panel) and a sketch of one TOF sector consisting of 8 modules each containing 8 scintillator rods (right panel).

defined by $270\ \mu\text{m}$ diameter PEEK fibers. The gaps are filled with $\text{C}_2\text{H}_2\text{F}_4:\text{SF}_6$ (90:10) gas mixture. A high voltage of 5600 V is applied to the central aluminium electrode and the outer electrodes are grounded. This leads to the creation of a high electric field in the gas. Charged particle passing through the counter ionizes the gas mixture in the gaps, leading to a discharge. The discharge is quenched due to the voltage drop on the glass electrodes. The discharge generates a fast electric pulse on the central electrode and its time of appearance as well as the charge are registered by the readout electronics. The HADES RPCs have a detection efficiency of about 95% and a high time resolution of about 70 ps (σ) [63–66].

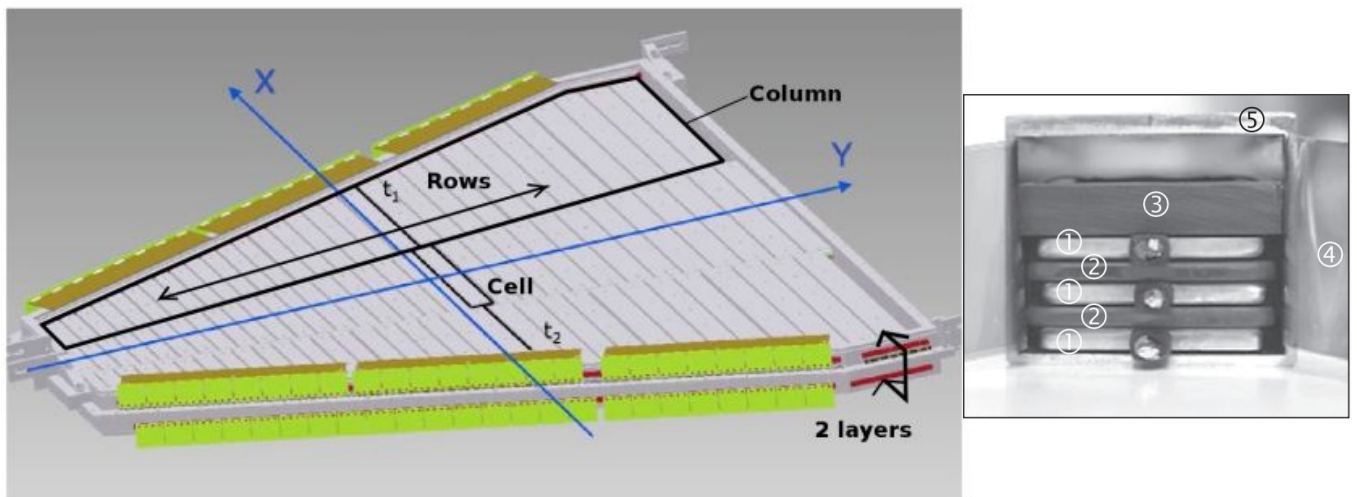


Figure 2.13.: Arrangement of RPC cells in one layer (left panel) and cell structure with indicated: 1) Al electrodes, 2) glass electrodes, 3) plastic pressure plate, 4) kapton insulation, 5) Al shielding (right panel). Presented picture is adapted from Ref. [66].

The Pre-Shower electromagnetic cascade detector is the outermost HADES spectrometer detector [67]. As separation between electrons and hadrons are difficult at lower angles and higher momenta, Pre-Shower detector is used to identify electrons and positrons for low polar angles ($\theta \leq 45^\circ$). The Pre-Shower consists of six identical sectors and every sector module has three trapezoidal multi-wire chambers (called pre-, post1-, and post2-chamber) separated by two lead converter plates each with a thickness of $2 \times$ radiation lengths ($= 2 \times 0.56$ cm). One chamber consists of an anode plane placed between two cathode planes (see Fig. 2.14, right panel). The anode plane contains a mesh of anode wires interleaved by field wires. One of the two cathode planes in each chamber is divided into 32×32 pads with individual readout, (see Fig. 2.14, left panel). The chambers are filled with a gas mixture of argon (30%) and isobutane (70%) with a small admixture of heptane as a quenching gas. Charged particles passing through the chamber ionizes the

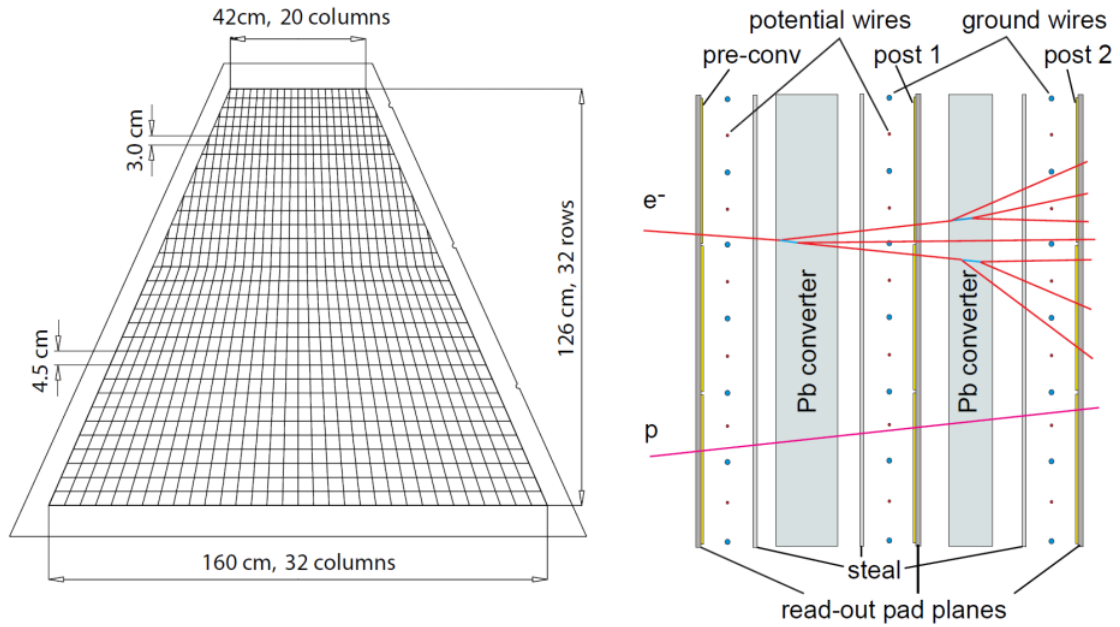


Figure 2.14.: Scheme of pad plane in one sector of Pre-Shower detector (left panel). The cathodes are set up in 32 rows, with 32 pads on one side and 20 pads on the other. The Pre-Shower detector (right panel) is made up of three gas chambers (pre-, post1-, and post2-chamber) separated by lead converters.

gas mixture and the ionization electrons are multiplied in avalanches occurring in the strong electric field near the anode wires. Positive ions produced in the avalanche drift towards the cathodes inducing electric pulses on the cathode pads. Electrons passing through the Pre-Shower detector produce electromagnetic shower in the lead absorber (see Fig. 2.14, right panel). This causes much higher ionization in the detector than in the case of charged hadrons, which leave single ionization traces and do not produce electromagnetic showers. Therefore, the amplitude of signals registered on the cathode pads is, on average, higher for electrons than for hadrons and this is the basis for distinguishing between electrons and hadrons in the Pre-Shower detector. Recently, this detector was replaced with a lead glass calorimeter, which is capable to register photons and thus allows the full reconstruction of neutral particle decays (e.g. π^0 , η) [68].

2.3.8 Data acquisition and trigger system

Data acquisition of HADES is based on Trigger Read-out Boards (TRBs) developed at GSI-Darmstadt [69, 70]. The main components of the TRB are Field-Programmable Gate Arrays (FPGAs) that can be programmed, among others as Time-to-Digital Converters (TDCs). The TRB also includes connectors for attaching add-on boards containing e.g. Analog-to-Digital Converters (ADCs) as well as optical links for data transmission, using TRBnet protocol.

For the event selection, HADES uses a two-level trigger system. The first level trigger (LVL1) is based on charged particle multiplicity in the META detector and the second level trigger (LVL2) requires a lepton candidate in the RICH detector, (see Fig. 2.15). In the present experiment, the interaction rate of the pion beam with the target was moderate and only the first level trigger based on a coincidence between a signal from the Start detector and a multiplicity of at least two hits in the META detector was used. This trigger is selective enough to reduce the data stream to a level allowing to write to the data storage.

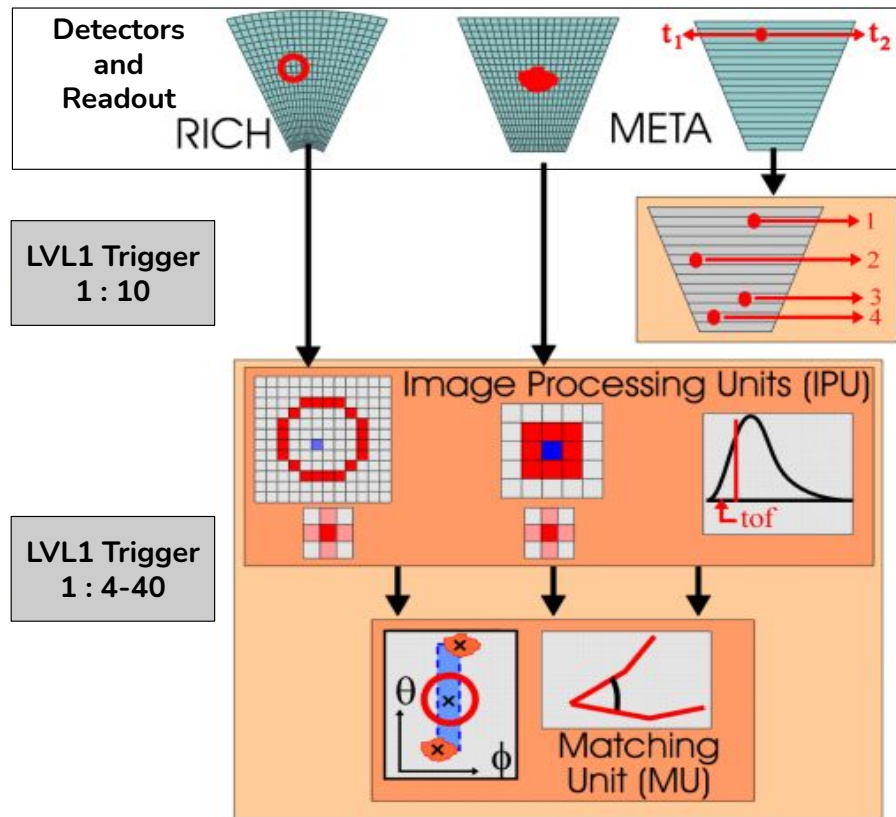


Figure 2.15.: General scheme of the HADES trigger system containing two levels: LVL1 and LVL2.

Chapter 3

Momentum reconstruction and particle identification

"..... I am inclined to think that scientific discovery is impossible without faith in ideas which are of a purely speculative kind, and sometimes even quite hazy"

Karl Raimund Popper

In this chapter, algorithms developed by the HADES collaboration for track and momentum reconstruction in the MDCs as well as for recognition of electrons / positrons in the RICH are presented. Also applied methods for identification of charged particles based on the time-of-flight information and on the energy losses are described.

3.1 Track Reconstruction

At HADES, the momentum of charged particles is determined, thanks to the magnetic field produced by the toroidal superconducting magnet. Deflection of particle trajectories in the magnetic field is measured with two pairs of Mini Drift Chambers (MDCs). One pair (MDC-I,II) is placed before the magnetic field and the other pair (MDC-III,IV) after the field (see Section 2.3.5). During the track reconstruction, particle trajectories in each of the pairs are approximated with straight lines. The procedure of reconstructing tracks in the MDCs begins with reconstruction of straight track segments in MDC-I,II. It consists of two main steps: track finding and track fitting. The track finding in MDC-I,II is done by projecting their hit information on a virtual plane between MDC-I and MDC-II (see Fig. 3.1).

The projection is done individually for all fired drift cells. For each cell, a plane containing the emission vertex in the target, parallel to the anode wire and spaced from the wire by a distance equal to the drift path (r) is determined. Due to the left-right ambiguity of the track position with respect to the anode wire, two such planes can be found. They are presented in Fig. 3.2 as planes TA_1B_1 and TA_2B_2 . The intersection of these two planes with the projection plane defines two straight lines A_1B_1 and A_2B_2 in Fig. 3.2.

In the next step, overlap of such lines corresponding to all fired drift cells in MDC-I and MDC-II is determined. An example of such overlap is shown in Fig. 3.3 (left panel), for an event with four tracks. The projection plane is divided into 2-dimensional array and for each element of array number of projection lines crossing this element is histogrammed, Fig. 3.3 (right panel). Local maxima in the array correspond to track candidates. In the second stage of the track reconstruction in MDC-I,II, a straight line fitting to the track candidate is performed and the χ^2 criterion is used to select satisfactory fits.

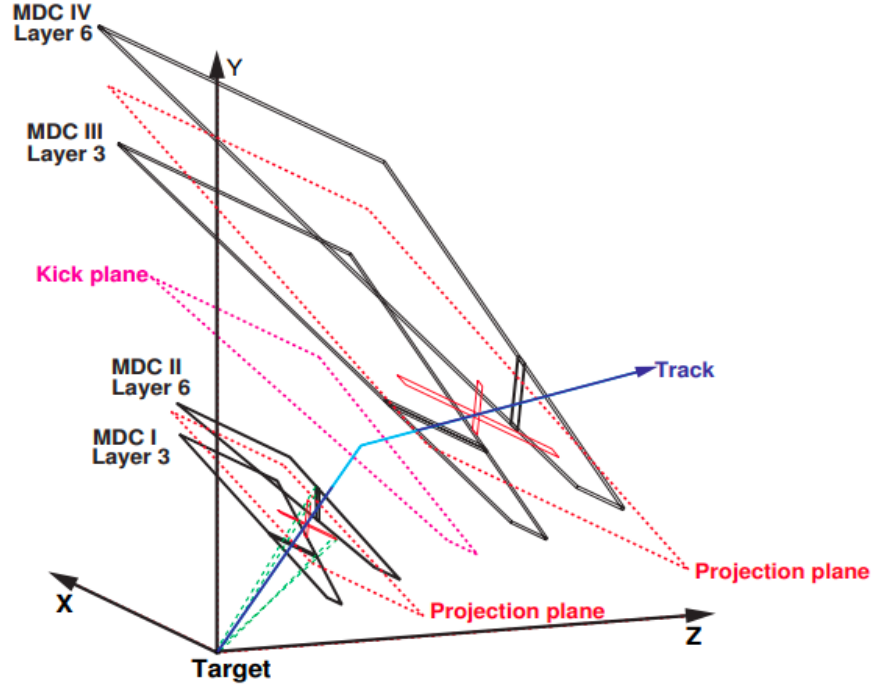


Figure 3.1.: Schematic layout of the four MDCs, each represented by only one of the six layers, and position of two projection planes, one between MDC-I and MDC-II and the other between MDC-III and MDC-IV. Also shown is kick plane between MDC-II and MDC-III.

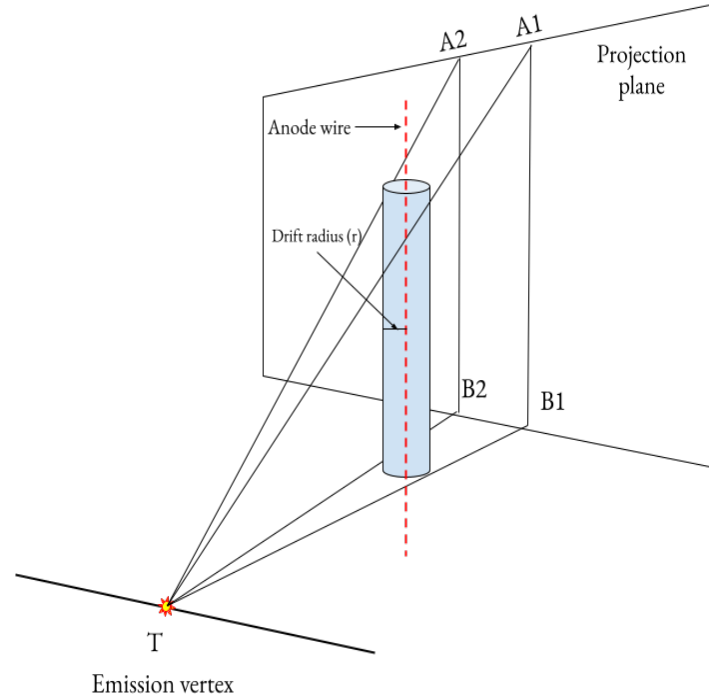


Figure 3.2.: Hit in one MDC drift cell represented by a cylinder with a radius equal to the track distance from the anode wire and ambiguity of the track candidate intersection with the projection plane represented by two straight lines A1-B1 and A2-B2.

Each track reconstructed in MDC-I,II is extended to the so-called kick plane, located between MDC-II and MDC-III and determined by simulations described in Ref [56]. The kick plane is also used for fast

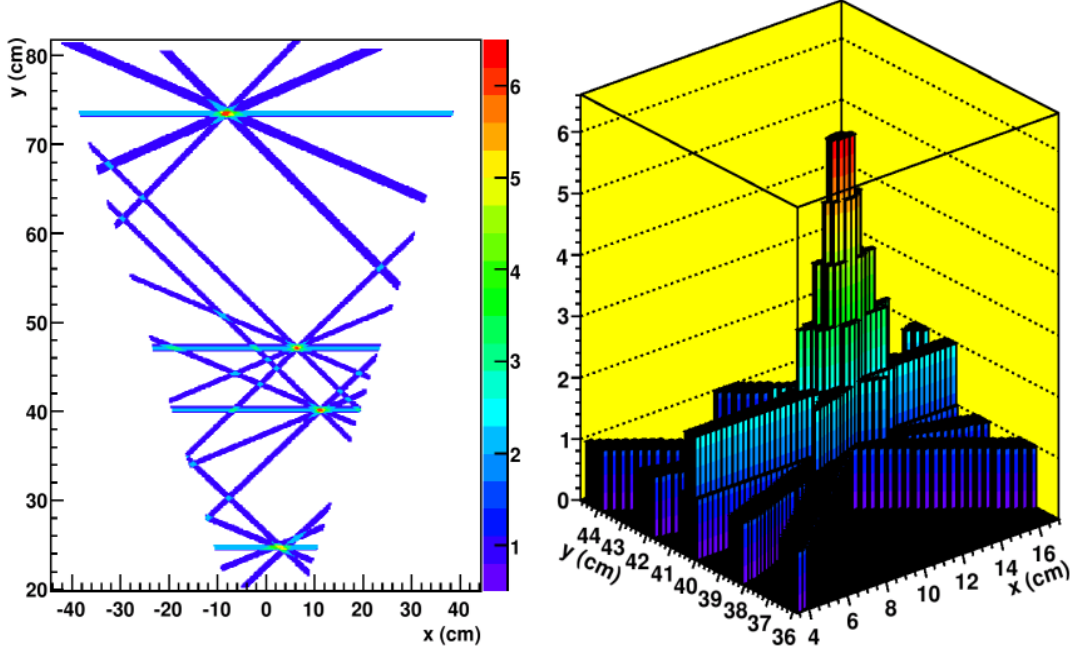


Figure 3.3.: Left: Projection of drift cells on $x - y$ detector coordinate space for an event with four tracks. Right: 2-dimensional histogram with a peak at the location where the drift cell projections have maximum overlap. In this example, the z axis indicates the peak height, corresponding here to a track totalling 12 hit layers in the inner drift chambers.

determination of particle momentum using procedure described in the next section. The crossing point of the MDC-I,II track with the kick plane is then used for reconstructing corresponding straight track segments in MDC-III,IV. The reconstruction is performed following the same algorithm as for MDC-I,II but using different projection plane which is between MDC-III and MDC-IV and as a reference point the MDC-I,II track crossing with the kick plane is used.

3.2 Momentum determination

The momenta of charged particles are determined based on the deflection of their trajectories in the magnetic field of the superconducting toroidal magnet. The change of three-momentum vector of particle moving in magnetic field $\vec{B}$ can be calculated using the formula:

$$\Delta\vec{p} = \vec{p}_{out} - \vec{p}_{in} = -q \int_{L_1}^{L_2} \vec{B} \times d\vec{L}, \quad (3.2.1)$$

where $\vec{p}_{in}$ and $\vec{p}_{out}$ represent incoming and outgoing particle momentum vectors, respectively, q is a particle charge and $d\vec{L}$ is an element of particle trajectory. The integration is performed along the particle's trajectory in the magnetic field. On the other hand, length of the vector $\Delta\vec{p}$ can be calculated from the deflection angle $\Delta\theta$ between the incoming and outgoing momentum vector:

$$|\Delta\vec{p}| = 2p \sin(\Delta\theta/2). \quad (3.2.2)$$

Particle momentum can be calculated combining Eqs. 3.2.1 and 3.2.2:

$$p = \frac{|q \int_{L_1}^{L_2} \vec{B} \times d\vec{L}|}{2 \sin(\Delta\theta/2)}. \quad (3.2.3)$$

From the above equation it follows that the momentum of charged particle can be determined based on the integral of the magnetic field along the particle's trajectory and the angle of deflection of the trajectory in the magnetic field. The HADES collaboration uses three different methods of the momentum determination which differ in precision and computing time: the kick plane, the spline and the Runge-Kutta method. For fast estimation of particle momentum, HADES uses the so called kick plane method. This method assumes that the momentum change of particle takes place at one point which is the intersection of linear track segment registered in MDC-I, II with the virtual kick plane, defined in Sec. 3.1. In this method the numerator of Eq. 3.2.3 is expanded in terms of $\sin(\Delta\theta/2)$, and the lowest three terms are taken into account. The particle momentum is then calculated as :

$$p = \frac{p_0}{2 \sin(\Delta\theta/2)} + p_1 + 2p_2 \sin(\Delta\theta/2), \quad (3.2.4)$$

with the coefficients p_0 , p_1 and p_2 stored in a tabular form as a function of the azimuthal and polar angles. The spline method of the momentum reconstruction uses a cubic spline for interpolation between straight track segments reconstructed before the magnetic field with MDC I and MDC II and after the field with MDC III and MDC IV. The field between MDC II and MDC III is integrated using 50 equally spaced points along the trajectory, see Fig. 3.4. The details of this method are given in Ref. [71]. Higher precision of momentum reconstruction is achieved by solving the equations of particle motion in the magnetic field, which is known in the form of a field map. Finding a solution of the equations of motion is done using the Runge-Kutta 4th order method. Initial input parameters for solving the equations of motion are provided by the spline method and the equations are solved recursively.

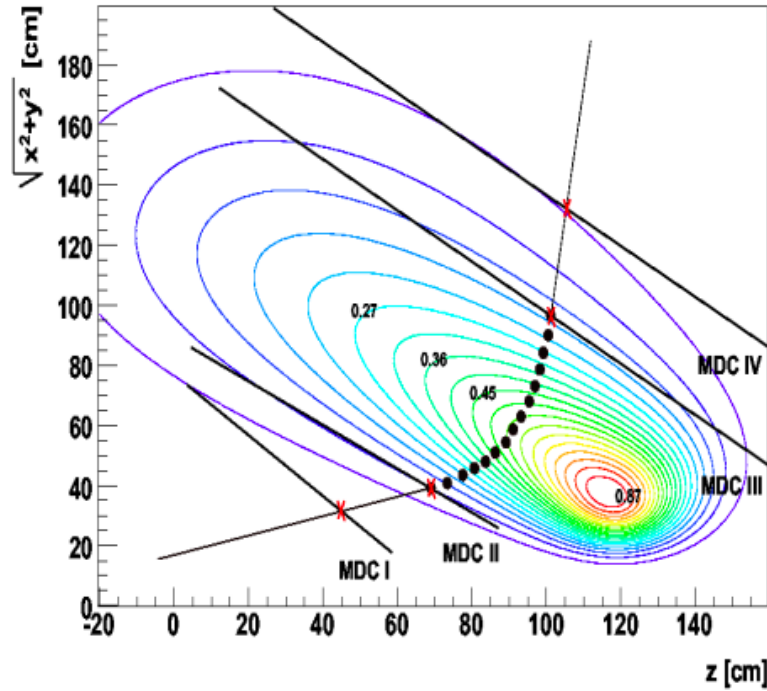


Figure 3.4.: Particle track as modelled by a cubic spline in the plane defined by the particle momentum vector at the target and the beam axis. The 50 points (only 15 are shown here for clarity) run from MDC II up to MDC III.

3.3 Hadron identification

The HADES spectrometer allows for the identification of charged particles in a wide momentum range. Hadrons are identified by combining the momentum, energy-loss and time of flight information from dedicated detector systems. Time-of-flight is measured between Start and TOF/RPC detector signals. Velocity

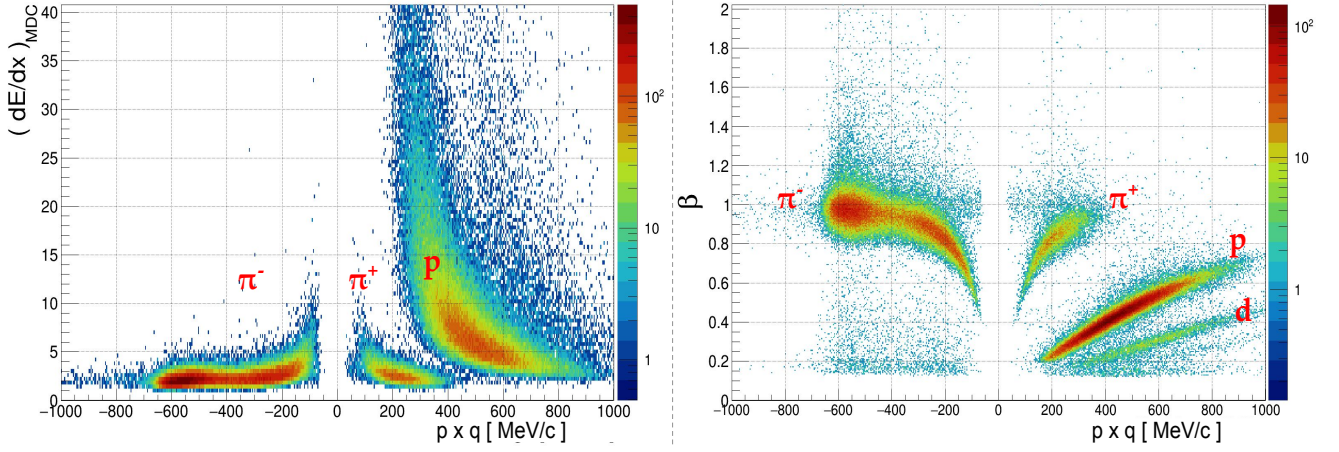


Figure 3.5.: Momentum dependence of the specific energy loss dE/dx in HADES Multiwire Drift Chambers (left panel), and of particle velocity β (v/c) determined based on the time-of-flight measurement (right panel).

of particle is determined by the time-of-flight measurement as follows:

$$\Delta t = t_{(TOF/RPC)} - t_{(Start)}, \quad (3.3.1)$$

$$\beta = \frac{v}{c} = \frac{s}{\Delta t \times c}, \quad (3.3.2)$$

where s represents path-length between Start detector and TOF/RPC detector calculated from track reconstruction. Fig. 3.5, right panel shows correlation of particle velocity and momentum times charge obtained in the experiment. Clear separation of proton and charged pions is visible.

Apart from the time-of-flight determination, measurement of energy losses in the layers of the MDCs is also used to identify charged particles. This is done by registration of the time-over-threshold (ToT) of the MDC pulses, which is correlated with the energy loss. Fig. 3.5, left panel shows the mean energy loss in MDCs as a function of the particle momentum. A good separation of protons and pions is visible. In the present data analysis, identification cuts were applied on the energy loss as well as the velocity, and the final identification spectra are shown in Chapter 5.6.3.

3.4 Lepton identification

The identification of leptons by means of a HADES spectrometer is mainly based on the observation of rings in the RICH detector. Two different algorithms were developed for finding the Cherenkov rings in the RICH: the pattern matrix (PM) algorithm and the backtracking (BT) algorithm. Due to limited efficiency of these algorithms for the ring finding, we combine them in our data analysis to reach higher efficiency. The two algorithms and the way of combining them are presented in the following subsections.

3.4.1 Pattern matrix algorithm

The pattern matrix (PM) algorithm was developed for standard offline selection of lepton production events. This procedure does not use the information about tracks reconstructed in the tracking system. The algorithm assumes a fixed radius of rings produced by leptons in the RICH. This assumption is well justified by the fact that velocities of registered leptons are close to the velocity of light and the Cherenkov angle is almost constant. This algorithm is based on overlapping (super-positioning) of 11×11 pads pattern matrix with fired pads on the pad plane.

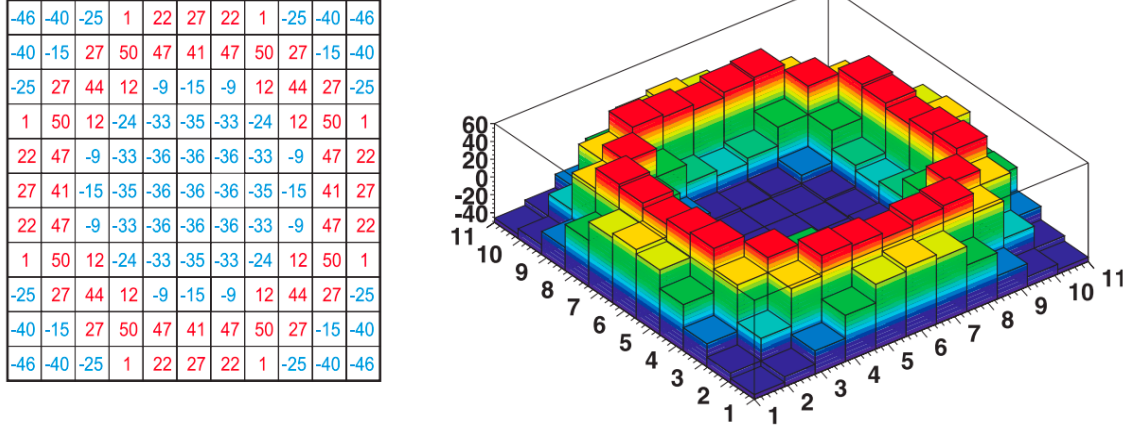


Figure 3.6.: Pattern matrix for the ring recognition (left) and the ring shape formed by the positive weights in the matrix (right).

The pattern matrix defines a mask with positive weights forming a circle as shown in Fig. 3.6. Every matrix cell corresponds to an individual pad on the RICH pad-plane. For a given position of the mask with respect to the pad plane, charge for fired pads is multiplied by the corresponding weights on the mask. The sum of all obtained values is used to characterize the quality of the ring identification and is called the pattern matrix quality (PM quality). This procedure is repeated for all pads in order to find local maxima of the PM quality which are qualified to be ring candidates. Minimum distance between two rings should be more than or equal to 4 pads. Detail description of the pattern matrix (PM) algorithm is given in Ref. [56]. This algorithm is also known as 'ring-finder' method (RF).

3.4.2 Backtracking technique

At low opening angles in the e^+e^- pair, the efficiency of pattern matrix algorithm is degraded, because two Cherenkov rings start to overlap and only one ring is identified. In order to improve the efficiency of lepton identification, another algorithm called backtracking method was developed. In the first step of this algorithm, potential lepton candidates are selected based on combining the information from TOF measured in the RPC and the TOF system and the energy loss measured in the MDCs. For each reconstructed track pre-selected as lepton it is checked whether this track intersects the RICH mirror. If this is the case, the expected ring image in the RICH pad plane is determined. The ring image is also referred to as region of interest. The ring position, radius and width is calculated from the known track vertex position as well as the polar angle and the azimuthal angle, using a parametrization obtained in simulations of the Cherenkov photons propagation in the RICH detector. In the next step of the backtracking algorithm, fired pads matching the area of interest are selected. In the RICH detector, Cherenkov photons are often registered not by individual pads, but several adjacent pads grouped into clusters. In each cluster, pads corresponding to local maxima are selected. A 2D Gaussian function is used to fit the obtained two-dimensional charge

distribution, which allows to get more accurate center of maximum, (see Fig. 3.7). Ring with at least one such maximum is accepted as a lepton candidate. The detailed description of the backtracking algorithm is given in Refs. [72, 73].

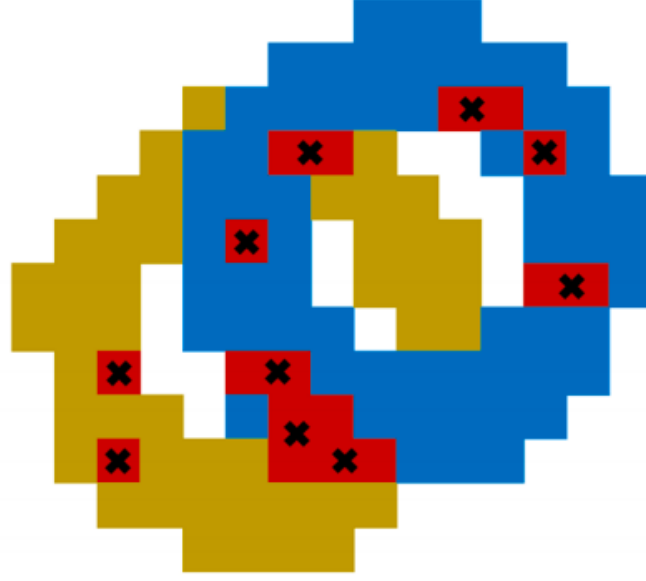


Figure 3.7.: An example from simulation with two partly overlapping rings. Regions of interest are shown in blue and yellow. Fired pads are shown in red and centers of maxima are marked by black crosses.

A spatial similarity in polar (θ) and azimuthal (ϕ) angle is explored to find electron tracks by creating all possible correlations between the track directions observed by the RICH detector and the MDC's tracking system. Figure 3.8 shows $\Delta\phi$ and $\Delta\theta$ differences in the azimuthal and the polar angle, respectively, registered in the RICH and MDC systems, plotted in function of particle momentum. Factor $\sin(\theta)$ in the case of the azimuthal angles difference is employed for maintaining the same solid angle. Cone structure of distributions in Fig. 3.8 results from the multiple scattering in low momentum region.

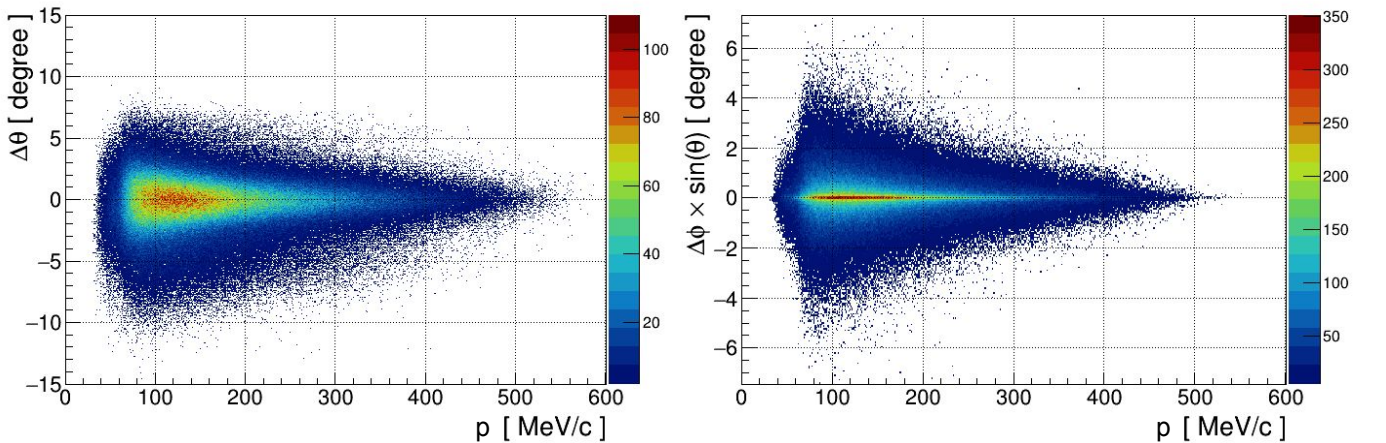


Figure 3.8.: Spatial correlation for polar and azimuthal angles between RICH hits and inner MDC's sectors.

Due to low particle multiplicity in the measurement, full correlation of $\Delta\theta$ and $\Delta\phi$ angles are taken into consideration and such correlation are calculated separately for each sector of the spectrometer.

The distribution of the product of momentum and electric charge sign (polarity) of electrons and positrons identified in the experiment with the PM and BT algorithms are shown in Fig. 3.9. The number of identified electrons is larger than the number of positrons and this is related to the different deflection of these particles in the magnetic field of the HADES magnet, which affects the momentum acceptance. Positrons experience lack of acceptance towards the forward angle in the direction of the beam axis while electrons bend in the opposite outward direction. Figure 3.9 also shows that the efficiency of lepton identification using the BT algorithm is about a factor 2 higher than for the PM algorithm.

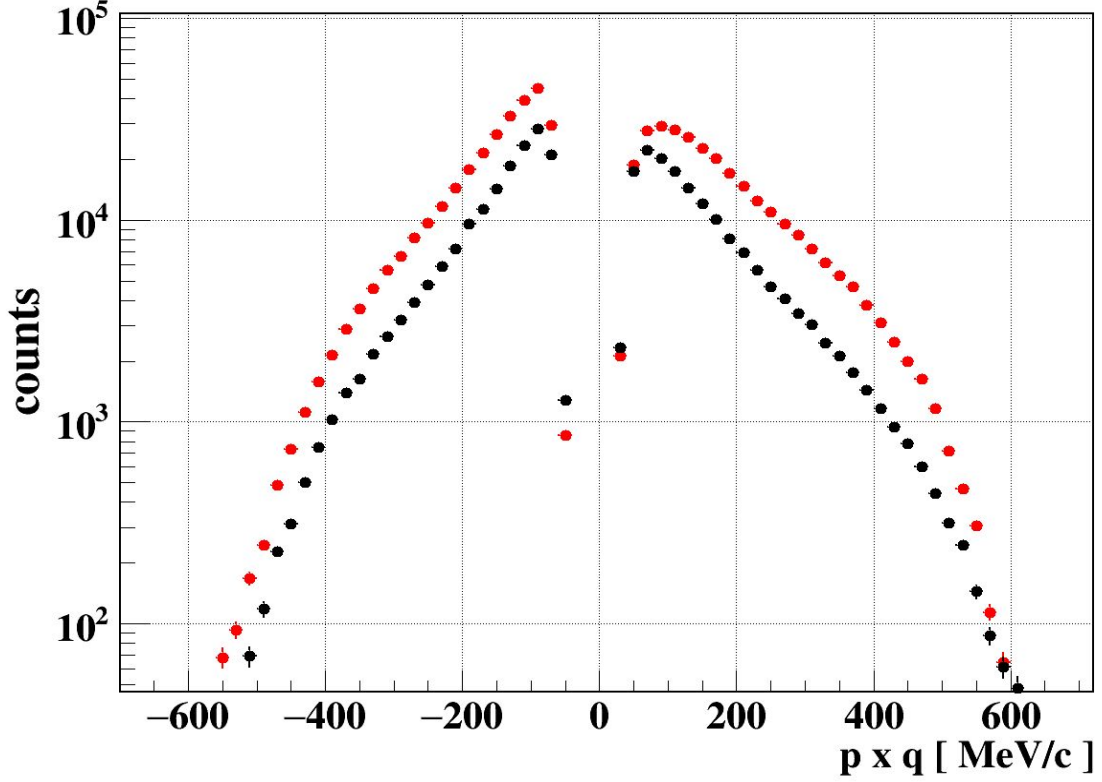


Figure 3.9.: Distribution of the product of momentum and polarity for leptons reconstructed with the pattern matrix (black) and backtracking (red) algorithms.

3.4.3 Efficiency of lepton identification with backtracking and pattern matrix technique

In order to improve the efficiency of lepton identification with the RICH detector, the two ring finding methods - the pattern matrix algorithm (PM) and the backtracking (BT) are both applied and the data sets corresponding to leptons identified with any of these algorithms are used. This is illustrated in Fig. 3.10 for the case of identifying single electrons. As shown in the figure, the set of identified electrons consists of three disjoint sets grouping electrons identified exclusively with the PM and BT as well as electrons identified with the both methods ($\text{PM} \cap \text{BT}$).

In the data analysis the combination of PM and BT methods was applied to the identification of e^+e^- pairs. In this case the set of identified pairs can be divided into 3×3 separate subsets as shown in Fig. 3.11. We represent the number of counts in each subset by an element of 3×3 matrix A_{ij} , where i and j represent rows and columns. Electrons (positrons) identified exclusively by the BT method correspond to index $i(j) = 1$. Analogously, index $i(j) = 3$ represents identification by the PM method. In turns, electrons (positrons) identified by both methods correspond to $i(j) = 2$.

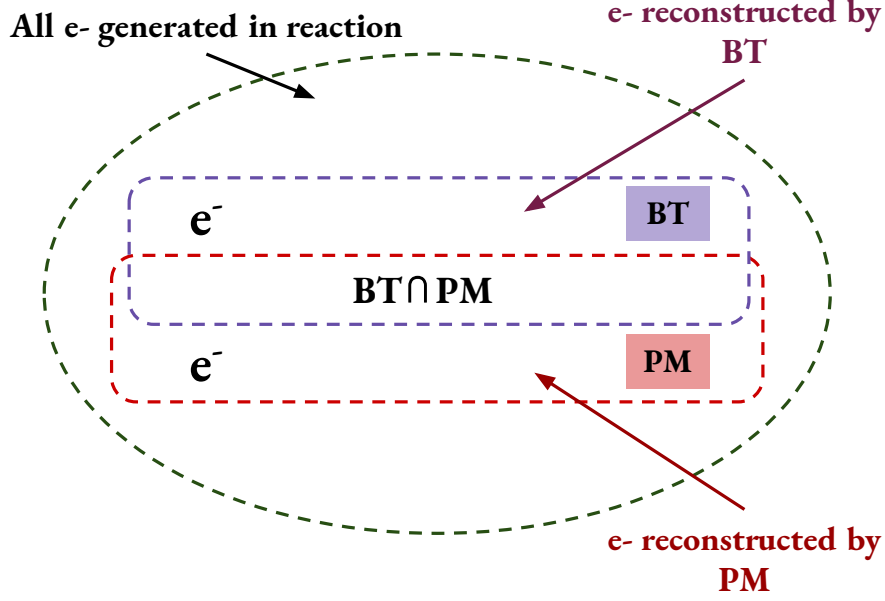


Figure 3.10.: Graphical illustration of a set of all produced electrons (green line) and subsets of electrons identified with the pattern matrix algorithm (red line) and the backtracking algorithm (blue line). The same methodology is applied to e^+ identification.

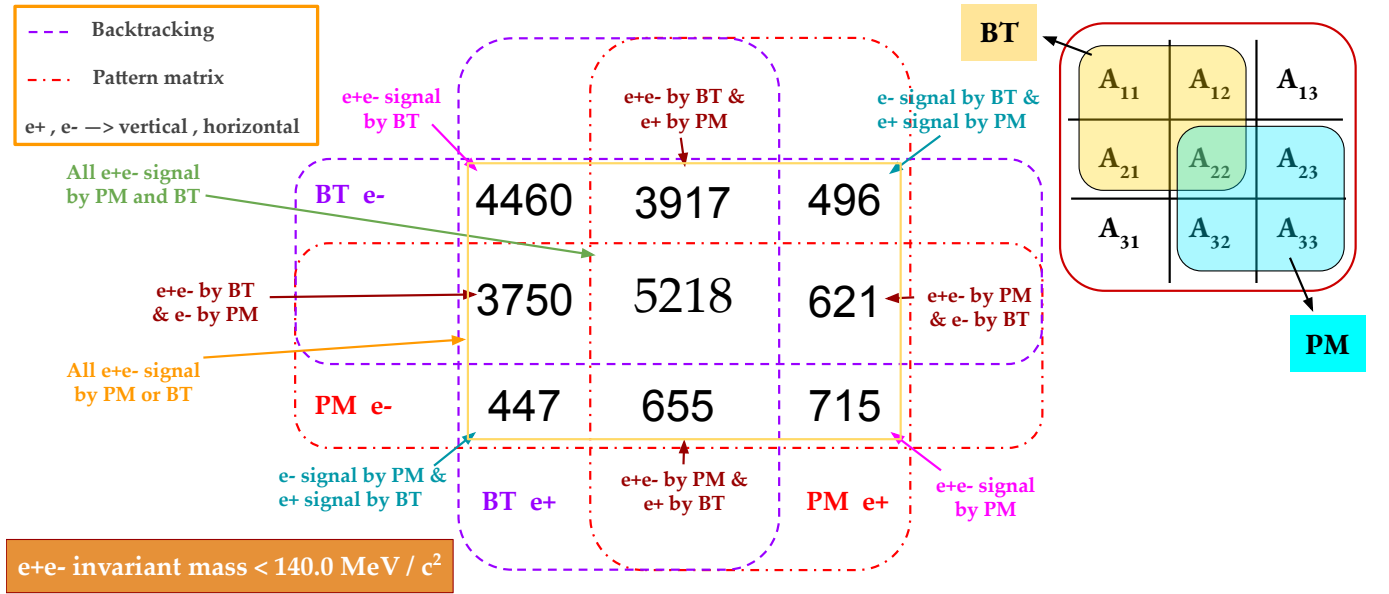


Figure 3.11.: Graphical presentation of a set of all identified $e^+ - e^-$ pairs. Subsets corresponding to electrons identified with PM and BT method are represented by horizontally oriented rectangular areas limited by red and violet line, respectively. Subsets corresponding to positrons identified with PM and BT method are represented by vertically oriented rectangular areas limited by red and violet line, respectively.

The numbers of identified dileptons represented by different A_{ij} elements were determined from finally reconstructed $\pi^- p \rightarrow e^+ e^- X$ events with the $e^+ e^-$ opening angle larger than 9° and the $e^+ e^-$ invariant mass spectra within π^0 region ($M_{inv}(e^+ e^-) < 140 \text{ MeV}/c^2$). The number of dileptons identified exclusively with the BT method ($A_{11} + A_{12} + A_{21} + A_{22}$) is equal to 17345 and with the PM method ($A_{22} + A_{23} + A_{32} + A_{33}$) equals 7209. The combination of both methods ($\sum A_{ij}$) provides 20279 dilepton

events. By using both identification methods the number of identified dileptons is 17% higher than for the BT method alone.

Chapter 4

Monte Carlo simulations

"We learnt a lot in the last 25 – 35 years. Its quite amusing when you look back how much we did not know. But its even more amusing to consider how much we do not know yet !"

David Gross

To compare experimental distributions with theoretical cross-sections, it is necessary to take into account the acceptance and efficiency of the HADES detection system. This is done by performing Monte Carlo simulations of the conducted measurements.

Besides simulation of the studied bremsstrahlung process in π^-p collisions ($\pi^-p \rightarrow \pi^-p e^+e^-$) dominant background processes were also simulated including:

1. $\pi^-p \rightarrow \pi^-p \pi^0(\gamma e^+e^-)$ (π^0 Dalitz decay),
2. $\pi^-p \rightarrow \pi^- \Delta^+(p e^+e^-)$ (Δ^+ Dalitz decay),
3. $\pi^-p \rightarrow n \eta(\gamma e^+e^-)$ (η Dalitz decay),
4. $\pi^-p \rightarrow N(1520) \rightarrow n e^+e^-$ ($N(1520)$ Dalitz decay).

The processes 1 and 2 make the main contribution to the background in the exclusive measurements of the bremsstrahlung process. In the inclusive measurements, processes 3 and 4 additionally contribute to the background. The simulations of the bremsstrahlung process and the background reactions are discussed in Sections 4.1 and 4.2, respectively.

For the carbon target, only the bremsstrahlung process was simulated. This was done by assuming the quasi-free interaction of beam pions with protons in carbon nucleus, as described in details in Section 4.3.

The procedure of comparing simulation results with experimental data is described in Section 4.4.

4.1 Simulation of π^-p bremsstrahlung

4.1.1 Simulation procedure and tools

In the simulation of the bremsstrahlung process the PLUTO event generator [75, 76] was used to generate $\pi^-p \rightarrow \pi^-p e^+e^-$ events distributed over the available four-body phase space. Each event was assigned a weight w_{SPA} equal to the differential cross-section calculated according to the soft-photon approximation (SPA) given by Eq. 1.5.12. The cross-section of the SPA model depends only on the kinematic variables describing the dilepton pair i.e. invariant mass M , transverse momentum p_t and rapidity y . It does not describe the distributions of hadrons present in the final state i.e. π^- and p in the studied process. Therefore, additional weights w_{ang} equal to the differential cross section $d\sigma/d\Omega$ for the elastic π^-p scattering are used. The implementation of these weights allows us to describe the experimental angular distributions

of pions and protons, as presented in Chapter 6. In this way, each PLUTO-generated event is assigned the product of two weights: $w_{SPA} \times w_{ang}$. Verification of the calculated values of the bremsstrahlung weights w_{SPA} , by comparing them with published results as well as with the predictions of the GiBUU model, is discussed in the next subsection, and in the following subsection, a parametrization of the π^-p differential cross sections used for calculating the weights w_{ang} , is presented.

4.1.2 Cross check of bremsstrahlung calculation

In order to verify if Eq. 1.5.12 is properly implemented in the simulation software, results of simulations of the π^-p bremsstrahlung process were compared with corresponding results obtained using the GiBUU transport code simulation [77], which contains the same SPA model for the description of bremsstrahlung as we use. This transport code is not used in the present analysis of experimental data because it does not provide the kinematically complete information for the exclusive bremsstrahlung process. It provides only the kinematic observables for the e^+e^- pair. To make the aforementioned comparison, we generated

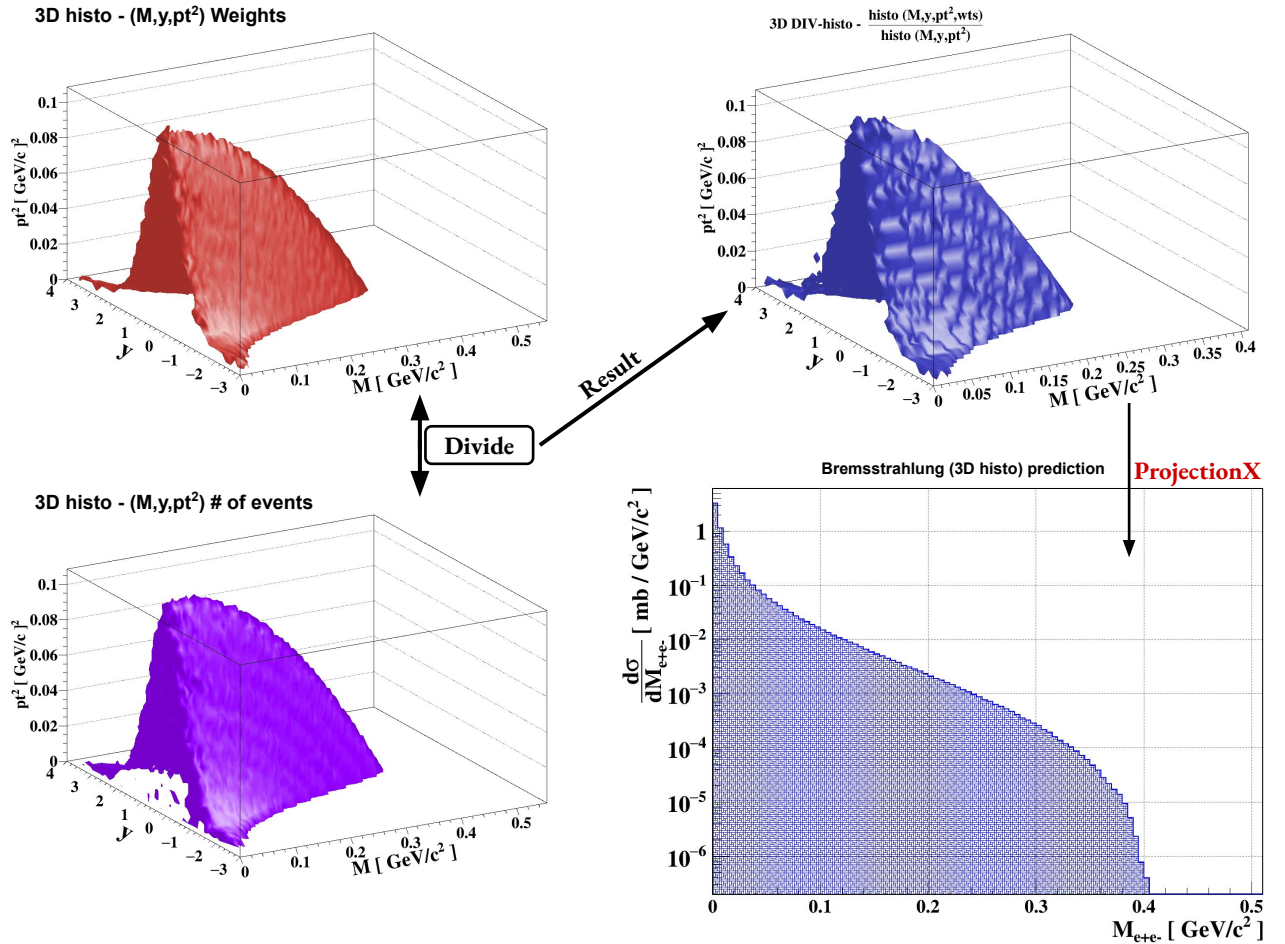


Figure 4.1.: Distributions in 3D space spanned by M , p_t and y , containing SPA weights (upper-left panel), number of generated events (lower-left panel), and ratio of the two distributions (upper-right panel) corresponding to the SPA differential cross-section. A projection of cross-section on the e^+e^- invariant mass axis is shown in lower-right panel.

$\pi^-p \rightarrow \pi^-p e^+e^-$ events at pion beam momentum 0.685 GeV/c using PLUTO and we calculated corresponding weights using Eq. 1.5.12 with the π^-p total elastic cross-section 18.3 mb taken from the SAID database [78]. These weights are calculated in function of three kinematic variables: invariant mass (M),

transverse momentum (p_t) and rapidity (y) of the e^+e^- pair, and are histogrammed in a 3D histogram (see upper left corner of Fig. 4.1). Another 3D histogram is used to store the number of events (see lower left corner of Fig. 4.1).

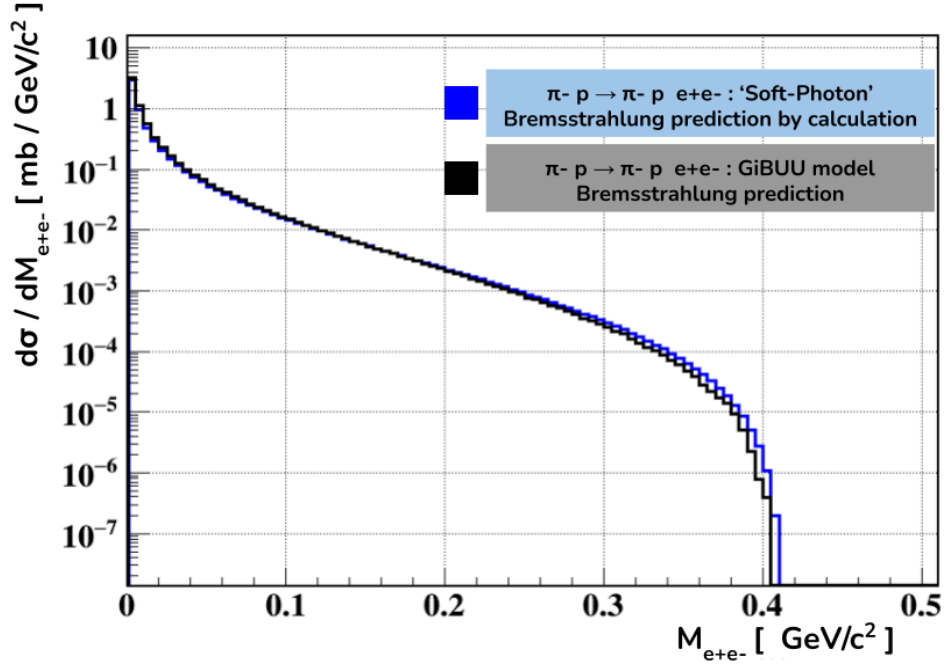


Figure 4.2.: Cross-section for π^-p bremsstrahlung at 0.685 GeV/c pion beam momentum calculated in function of e^+e^- invariant mass using Eq. 1.5.12 (blue line) and GiBUU (black line).

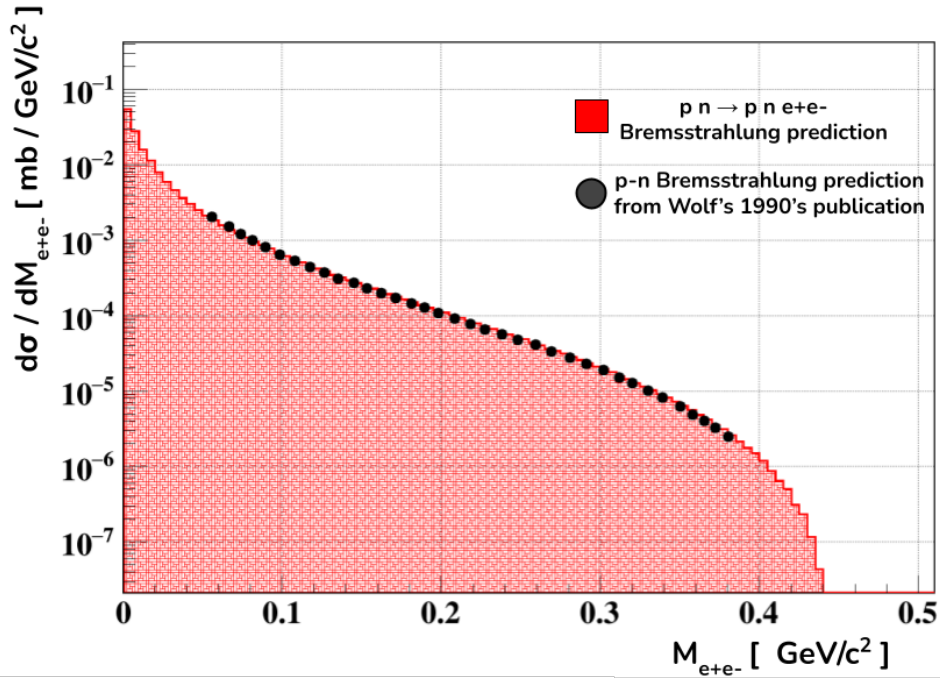


Figure 4.3.: Cross-section for $p - n$ bremsstrahlung at kinetic energy 1 GeV of proton beam calculated in function of e^+e^- invariant mass using Eq. 1.5.12 (red line) and digitized points (black dots) extracted from Ref. [50].

Finally, the 3D histogram with weights is divided by the histogram with the number of events and the obtained distribution represents the differential cross section for the bremsstrahlung as a function of the kinematic variables M , p_t and y (see upper right corner of Fig. 4.1). Projections of this 3-D distribution can be used to determine 2-D or 1-D distributions of the cross-sections. For example, Fig. 4.1, lower right corner, shows a distribution of the cross-section as a function of the e^+e^- invariant mass. In Fig. 4.2 this distribution is compared with the result obtained from the GiBUU transport code. A good agreement between the distributions is observed except for the range of highest invariant masses ($M_{e^+e^-} > 0.3 \text{ GeV}/c^2$) where the current bremsstrahlung prediction exceeds the GiBUU calculation.

For an independent check of our prediction, the calculation of the proton-neutron bremsstrahlung was performed and compared with corresponding e^+e^- invariant mass distribution calculated according the SPA formalism given in [50]. The present calculation of the $p - n$ bremsstrahlung was done at a kinetic energy of the proton beam at 1 GeV and for the elastic $p - n$ cross-section of 18 mb taken from [48]. We got a good agreement with results published in [50], as shown in Fig. 4.3.

4.1.3 Angular modeling

For the simulations of the bremsstrahlung process, a parametrization of the differential cross-section for the π^-p scattering was taken from Ref. [79]. This parametrization is based on the best fit to elastic π^-p data at the center of mass energy 1496 MeV, equivalent to the pion beam momentum 679 MeV/c, with the 5th order series expansion of the Legendre polynomials:

$$\frac{d\sigma}{d\Omega} = \sum_n A_n P_n(\cos\theta). \quad (4.1.1)$$

The numerical values of coefficients A_n are given in Table. 4.1.

Energy (C.M)	A_0	A_1	A_2	A_3	A_4	A_5
1.496 GeV	1.52 ± 0.06	2.23 ± 0.15	2.42 ± 0.21	0.41 ± 0.24	0.07 ± 0.19	0.20 ± 0.15

Table 4.1.: Legendre polynomial coefficients in cross-section expansion: $d\sigma/d\Omega = \sum_n A_n P_n(\cos\theta)$. The cross-section is calculated in units [mb/sr].

The HADES detection system has a limited angular acceptance and, therefore, the measured yields depend not only on the luminosity and the total cross-section of studied process, but also on angular distributions of the reaction products. In Fig. 4.4, angular distributions of pions and protons obtained in the simulation of the bremsstrahlung events generated with PLUTO over the available phase space are compared to ones obtained by ascribing to each event a weight equal to the differential π^-p cross-section. The distributions for the two cases - the phase space distribution and the elastic-like events - are normalized to the same total number of events. The difference between phase space (red) and elastic-like (blue) angular distribution for the both hadrons can be seen within the HADES acceptance (18° to 85°).

The simulated bremsstrahlung events are further passed through the full analysis and reconstruction chain as developed for experimental hits. Figure 4.5, left panel presents the e^+e^- invariant mass distribution for events with registered $\pi^-e^+e^-$ tracks, assuming the phase space distribution and elastic-like distribution of the produced particles. The observed ratio of the number of phase space and elastic-like events is about 3.

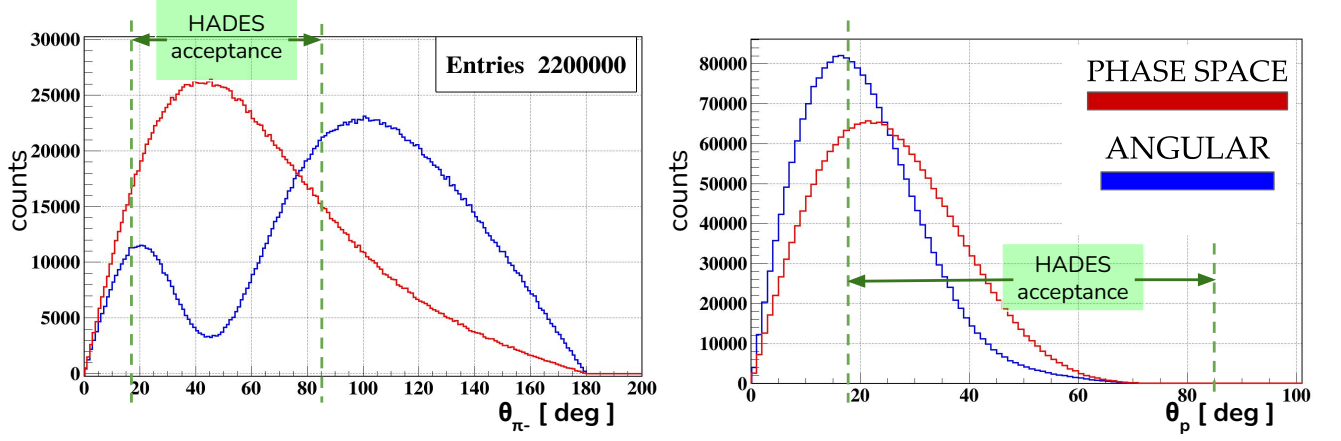


Figure 4.4.: Angular distributions for pions (left panel) and protons (right panel) obtained in simulation of bremsstrahlung events assuming phase space distribution (red line) and elastic-like distribution (blue line) of the reaction products. Green line with arrows defines the HADES acceptance region.

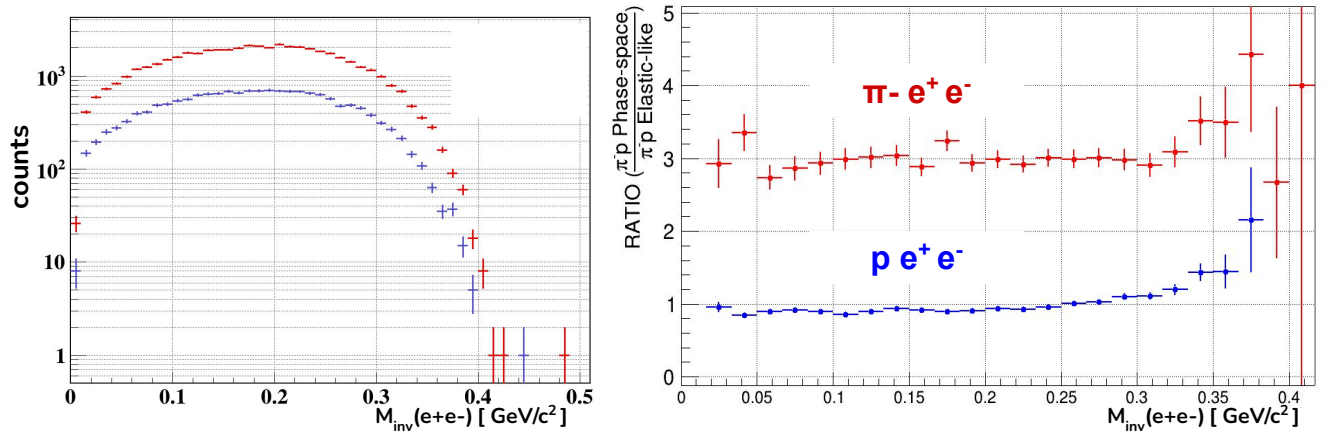


Figure 4.5.: Left panel: yields for registered $\pi^- e^+ e^-$ events in function of $e^+ e^-$ invariant mass for uniform phase space distribution (red line) and elastic-like (blue line) of the final state particles in $\pi^- p \rightarrow \pi^- p e^+ e^-$ reaction. Right panel: ratio of registered yields for phase space and elastic-like distributions in case of registered $\pi^- e^+ e^-$ events (red points) and $p e^+ e^-$ events (blue points).

The dependence of this ratio is shown as a function of the $e^+ e^-$ invariant mass in the right panel of Fig. 4.5. For events with registered $p e^+ e^-$ tracks this ratio is close to 1.

4.2 Background channels

This section is divided into two parts: the first describes simulations of the π^0 and the Δ^+ Dalitz decay contributing to the experimental background in the exclusive measurements, and the second one the simulation of the η and $N(1520)$ Dalitz decay included, in addition, in the description of background in the inclusive channel.

4.2.1 $\pi^- p \rightarrow \pi^- p \pi^0 (\gamma e^+ e^-)$ and $\pi^- p \rightarrow \pi^- \Delta^+ (p e^+ e^-)$

For the simulation of π^0 Dalitz decay, we used PLUTO to generate $\pi^- p \rightarrow \pi^- p \pi^0$ events. Each event was assigned a weight equal to the corresponding differential cross-section. The cross-section value was taken from the partial wave analysis (PWA) of one- and two-pion production data from pion-nucleon scattering and photo-production experiments [80]. The PWA was based on the approach developed by Bonn-Gatchina group [82, 83]. In the analyzed data for the two pion production, listed in Table 4.2, the HADES data for $\pi^- p \rightarrow \pi^- p \pi^0$ and $\pi^- p \rightarrow \pi^- n \pi^+$ were included. These data were collected in the same measurement as the current data for the $e^+ e^-$ pair production, only using different triggering conditions. In the present simulations, the neutral pions (π^0) generated using PLUTO, with the weights obtained from the PWA, were subsequently subject to the Dalitz decay in $\gamma e^+ e^-$ channel.

The PWA was also used in the simulation of the $\pi^- p \rightarrow \pi^- \Delta^+$ reaction channel, providing the corresponding differential cross-sections. The Δ^+ Dalitz decay in $p e^+ e^-$ was modeled using the PLUTO generator.

Reaction	Observable	W (GeV)	Experiment
$\gamma p \rightarrow \pi^0 \pi^0 p$	DCS, Tot	1.2 - 1.9	MAMI
$\gamma p \rightarrow \pi^0 \pi^0 p$	E	1.2 - 1.9	MAMI
$\gamma p \rightarrow \pi^0 \pi^0 p$	DCS, Tot	1.4 - 2.38	CB-ELSA
$\gamma p \rightarrow \pi^0 \pi^0 p$	P, H	1.45 - 1.65	CB-ELSA
$\gamma p \rightarrow \pi^0 \pi^0 p$	T, P_x , P_y	1.45 - 2.38	CB-ELSA
$\gamma p \rightarrow \pi^0 \pi^0 p$	P_x , P_x^c , P_x^s (4D)	1.45 - 1.8	CB-ELSA
$\gamma p \rightarrow \pi^0 \pi^0 p$	P_y , P_y^c , P_y^s (4D)	1.45 - 1.8	CB-ELSA
$\pi^- p \rightarrow \pi^0 \pi^0 n$	DCS	1.29 - 1.55	Crystall Ball
$\pi^- p \rightarrow \pi^+ \pi^- n$	DCS	1.45 - 1.55	HADES
$\pi^- p \rightarrow \pi^0 \pi^- p$	DCS	1.45 - 1.55	HADES

Table 4.2.: Two-pion production data used in the PWA: reactions, observables, energy ranges and experiments. In the list of observables, DCS stands for differential cross-section, Tot for total cross-section and the other symbols corresponding to different polarization observables are explained in [84].

4.2.2 $\pi^- p \rightarrow N(1520) \rightarrow n e^+ e^-$ and $\pi^- p \rightarrow n \eta (\gamma e^+ e^-)$

In the inclusive analysis of the bremsstrahlung process, besides the above discussed contributions to the background from the Dalitz decay of π^0 and Δ^+ , additional contributions originate from the excitation of intermediate $N(1520)$ resonance decaying to $n e^+ e^-$ final state, as well as from the intermediate $n \eta$ pair with subsequent η Dalitz decay to $\gamma e^+ e^-$. The simulations of the latter two processes were performed by F. Scozzi for the $\pi^- p \rightarrow e^+ e^- X$ measurement at HADES and details of this simulations are presented in his Ph.D thesis, Ref. [33]. Results of this simulations are adopted in the present studies.

4.3 Bremsstrahlung in π^- C collisions

4.3.1 Interaction of π^- with proton in carbon nucleus

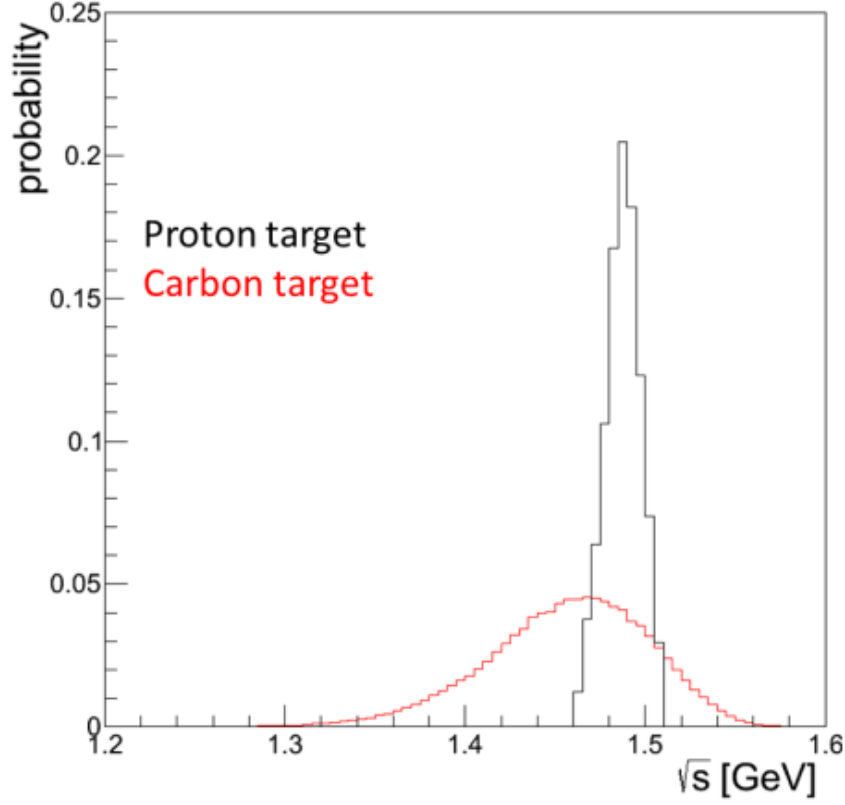


Figure 4.6.: The probability density as a function of $\sqrt{s}$ for the reaction on a free proton (black) and a bound proton (red) using the participant-spectator model. According to the measurements, the incident pion beam dispersion was taken into consideration in both circumstances, the figure is adopted from [33].

For the description of the bremsstrahlung process in π^- C collisions we used a simplified model assuming that the process occurs in the quasi-free interaction of the pion with a proton in carbon nucleus. Furthermore, we assume that the proton moves in the carbon nucleus reference frame with the Fermi momentum (p_F). On its mass-shell, the rest of the carbon nucleus is considered as ^{11}B , retaining its initial momentum (p_F). As a result, in the laboratory frame, the participant proton is off-shell, with a total energy E_{part} and a mass M_{part} related by:

$$M_{12C} = E_{part} + \sqrt{(M_{11B})^2 + (p_F)^2}, \quad (4.3.1)$$

$$E_{part}^2 = (M_{part})^2 + (p_F)^2, \quad (4.3.2)$$

where M_{12C} and M_{11B} are the ^{12}C and ^{11}B masses, respectively. The three components of the Fermi

momentum of the participant proton are sampled in the PLUTO event generator for each event. Using Eq. 4.3.2, one can calculate total center-of-mass energy for the reaction of pion with participant proton:

$$\sqrt{s_{\pi-part}} = \sqrt{(E_{\pi} + E_{part})^2 - (\vec{p}_{\pi} + \vec{p}_F)^2}. \quad (4.3.3)$$

In the simulation of the quasi-free bremsstrahlung process $\pi^- p \rightarrow \pi^- p e^+ e^-$ we use the SPA weights calculated in the same way as for the bremsstrahlung on the free proton. For all background processes, events for carbon target were not simulated. Only a pre-scaling of $\pi^- p$ interaction was used and is described in the next section 4.3.2.

4.3.2 Scaling π N cross section to carbon

In the collision of pion with carbon nucleus, the production of bremsstrahlung may arise as a result of quasi-free scattering on a proton as well as on a neutron:

$$\begin{aligned} \pi^- p &\rightarrow \pi^- p e^+ e^-, \\ \pi^- n &\rightarrow \pi^- n e^+ e^-. \end{aligned}$$

Both processes contribute to the studied exclusive $\pi^- e^+ e^- X$ and the inclusive $e^+ e^- X$ channel. In the SPA model, the bremsstrahlung cross-section is proportional to the elastic scattering cross-section for the colliding particles. According to the SAID database [78], the cross-section for the elastic scattering of negatively charged pions on a proton and on a neutron target at $p_{beam} = 0.69$ GeV/c is $\sigma_{\pi-p} = 18.13$ mb and $\sigma_{\pi-n} = 9.77$ mb, respectively. We calculate the cross-section for the quasi-elastic scattering of pions with one of nucleons in a carbon nucleus using the formula:

$$\sigma(C) = [(W_1 \times \sigma_{\pi-p}) + (W_2 \times \sigma_{\pi-n})] \times A^{2/3}, \quad (4.3.4)$$

where W_1 and W_2 are weights calculated as:

$$W_1 = \frac{\sigma_{\pi-p}}{\sigma_{\pi-p} + \sigma_{\pi-n}}, \quad (4.3.5)$$

$$W_2 = \frac{\sigma_{\pi-n}}{\sigma_{\pi-p} + \sigma_{\pi-n}}, \quad (4.3.6)$$

and A is the mass number of carbon nucleus (12). The factor $A^{2/3}$ represents the number of nucleons in carbon which are "seen" by the pion beam and take part in the $\pi^- N$ quasi-free scattering. The ratio of the quasi-elastic cross-section on carbon calculated using Eq. 4.3.4 and the elastic $\pi^- p$ cross-section is equal to 4.4. We take this factor as the ratio of the bremsstrahlung cross-section on carbon and on proton:

$$\frac{\sigma_{brem}(\pi^- C)}{\sigma_{brem}(\pi^- p)} = 4.4. \quad (4.3.7)$$

The corresponding ratio of the bremsstrahlung cross-section on polyethylene and on proton target is:

$$\frac{\sigma_{brem}(\pi^- (C_2H_4)_n)}{\sigma_{brem}(\pi^- p)} = \frac{\sigma_{brem}(\pi^- p) + \frac{1}{2}\sigma_{brem}(\pi^- C)}{\sigma_{brem}(\pi^- p)} = 3.2. \quad (4.3.8)$$

Figure 4.7 presents the $e^+ e^-$ invariant mass spectrum obtained in the simulation of bremsstrahlung process in $\pi^- p$ and $\pi^- C$ collisions. The bremsstrahlung cross-section for carbon target was scaled by the factor

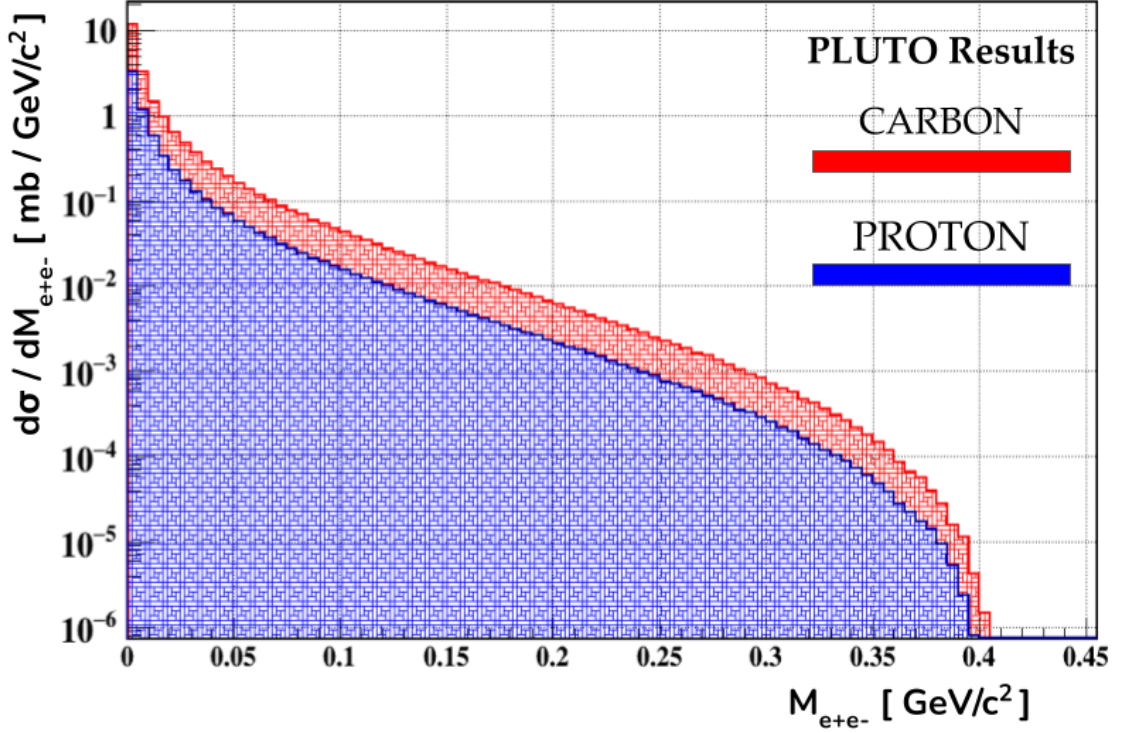


Figure 4.7.: e^+e^- invariant mass distribution obtained in the simulation of bremsstrahlung process for collisions of 0.685 GeV/c pion beam with proton target (blue histogram) and with carbon target (red histogram).

4.4 with respect to the π^-p bremsstrahlung cross-section.

In the case of the exclusive $p e^+e^- X$ and $\pi^-p e^+e^-$ analyses, only collisions of pions with protons in a carbon nucleus contribute to the observed yield. The cross-section for the quasi-elastic π^- scattering on one of the protons in the carbon target contains only the first expression from Eq. 4.3.4 and, as a result, it amounts to:

$$\sigma(C) = (W_1 \times \sigma_{(\pi^-p)}) \times A^{2/3}. \quad (4.3.9)$$

The ratio of the bremsstrahlung cross-section on carbon and on proton is in this case:

$$\frac{\sigma_{brem}(\pi^-C)}{\sigma_{brem}(\pi^-p)} = 3.4. \quad (4.3.10)$$

For the determination of cross-sections for the background processes in the π^-C interaction we used the corresponding cross-sections obtained in the simulations of the π^-p collisions and we multiplied them by the factor $Z^{2/3}$, where Z is the atomic number of a carbon ($Z = 6$). This factor we took from Ref. [33], where it was obtained from an analysis of inclusive e^+e^- spectra in collisions of pions with polyethylene and carbon target:

$$\frac{\sigma_{\pi^-C}}{\sigma_{\pi^-p}} = 3.3. \quad (4.3.11)$$

4.4 Comparison of data to models

For the comparison of experimental yield with theoretical cross-section, the standard relation was used:

$$N = N_0 n \sigma_T A \epsilon, \quad (4.4.1)$$

where N is the number of registered events, N_0 is the number of beam particles, n is the target areal density, σ_T is the total cross section for the studied process, A and ϵ is the detector acceptance and efficiency, respectively. The product of the number of beam particles and the target areal density ($N_0 n$) is the integrated luminosity in the conducted measurements. It was determined by the simultaneous measurement of the elastic $\pi^- p$ angular distributions and normalizing them to the corresponding differential cross-sections taken from the literature, as described in Chapter 5.4. The acceptance of the HADES detection system was determined in Monte Carlo simulations of the studied process, using events generated by the PLUTO generator. For each generated event i , a weight w_i was assigned equal to the differential cross section for the studied process in the corresponding phase-space coordinate. The acceptance was calculated as:

$$A = \frac{\sum_A w_i}{\sum_{4\pi} w_i}, \quad (4.4.2)$$

where the sum in the numerator runs over all the events that are within the angular acceptance of HADES and the sum in the denominator runs over all generated events and corresponds to the full solid angle (4π). Thus, the acceptance A is equal to the integral of the differential cross section within the angular acceptance of HADES divided by the total cross section (integral in the range of 4π). The detection efficiency was calculated as:

$$\epsilon = \frac{\sum_D w_i}{\sum_A w_i}, \quad (4.4.3)$$

where the sum in the numerator runs over all events that are within the angular acceptance and are registered by the HADES detectors, while the sum in the denominator is the sum of weights of events within the HADES acceptance and is identical to the numerator of equation 4.4.2.

In the present work, the experimental yields are mainly presented as a function of selected kinematic variables e.g. the invariant mass of the e^+e^- pair. In this case, the experimental yield ΔN in a range ΔX of kinematic variable X can be written as:

$$\frac{\Delta N}{\Delta X} = N_0 n \sigma_T A \epsilon \frac{1}{W} \frac{\Delta W}{\Delta X}, \quad (4.4.4)$$

where ΔW is equal to the sum of weights of registered events with X lying in the range ΔX and W is equal to the numerator of Eq. 4.4.3. In the measurement with the polyethylene target, the areal density of hydrogen atoms n^p is two times higher than the density of carbon atoms ($1/2 n^p$). The measured yields for the studied process originating from the interaction of the pion beam with protons in hydrogen atoms and with carbon nuclei are equal to:

$$\frac{\Delta N^p}{\Delta X} = N_0 n^p \sigma_T^p A^p \epsilon^p \frac{1}{W^p} \frac{\Delta W^p}{\Delta X}, \quad (4.4.5)$$

and

$$\frac{\Delta N^C}{\Delta X} = N_0 \frac{1}{2} n^p \sigma_T^C A^C \epsilon^C \frac{1}{W^C} \frac{\Delta W^C}{\Delta X}, \quad (4.4.6)$$

where the superscripts p and C refer to the scattering on proton and carbon, respectively. After adding the equations 4.4.5 and 4.4.6, we get the expression for the yields on the polyethylene target N^{PE} ($= N^p + N^C$):

$$\frac{\Delta N^{PE}}{\Delta X} = N_0 n^p \left[\sigma_T^p A^p \epsilon^p \frac{1}{W^p} \frac{\Delta W^p}{\Delta X} + \frac{1}{2} \sigma_T^C A^C \epsilon^C \frac{1}{W^C} \frac{\Delta W^C}{\Delta X} \right]. \quad (4.4.7)$$

In Chapter 6, the experimental distributions $\Delta N/\Delta X$ are compared with the predictions of theoretical models. For this comparison, the experimental distributions are divided by the number of beam particles N_0 and the areal density of the target n . For the scattering on protons in the polyethylene target we take n as equal to the areal density of protons in the target ($n = n^p$) and for the scattering on the carbon target we take the areal density of the carbon target ($n = n^C$). However, for the scattering on the polyethylene target we take n as equal to the areal density of protons in this target ($n = n^p$). This corresponds to dividing both sides of Eq. 4.4.7 by the factor $N_0 n^p$ leading to:

$$\frac{\Delta N^{PE}}{\Delta X} \frac{1}{N_0 n^p} = \left[\sigma_T^p A^p \epsilon^p \frac{1}{W^p} \frac{\Delta W^p}{\Delta X} + \frac{1}{2} \sigma_T^C A^C \epsilon^C \frac{1}{W^C} \frac{\Delta W^C}{\Delta X} \right]. \quad (4.4.8)$$

In Chapter 6, normalized in this way distributions are compared to the theoretical cross-sections determined within the HADES acceptance. In the case of the scattering on the polyethylene target, the experimental distributions correspond to the left side of Eq. 4.4.8 and the theoretical cross sections are calculated according to the expression on the right side of the equation. This expression contains theoretical values of the total cross section for the studied process as well as the acceptances, efficiencies and distributions of kinematical variable X ($\Delta W/W \Delta X$) determined in the simulations taking into account the distributions of the theoretical cross sections over the phase space.

Theoretical values of the total cross section for the bremsstrahlung process $\pi^- p \rightarrow \pi^- p e^+ e^-$ and for the studied background reactions are collected in Table 4.3. The corresponding total cross sections for the scattering on carbon nuclei were calculated using the scaling factors discussed in the previous section. In the simulations of the bremsstrahlung process, the acceptance and efficiency for the $\pi^- p$ collisions and $\pi^- C$ collisions are determined independently. In the latter case, the Fermi momentum of protons in carbon nucleus is taken into account. Simulations of the background processes were performed only for the case of $\pi^- p$ scattering, providing the determination of the acceptance and the efficiency. For the $\pi^- C$ scattering, the same acceptance and efficiency values were used as for the $\pi^- p$ scattering.

Reaction	Branching ratio	Cross-section	Model
$\pi^- p \rightarrow \pi^- p e^+ e^-$	–	0.028 mb	SPA
$\pi^- p \rightarrow p \pi^- \pi^0$	$\pi^0 \rightarrow \gamma e^+ e^-$ (1.174×10^{-2})	3.69 mb	PWA
$\pi^- p \rightarrow \pi^- \Delta^+$	$\Delta^+ \rightarrow p e^+ e^-$ (4.2×10^{-5})	0.57 mb	PWA
$\pi^- p \rightarrow N(1520) \rightarrow n e^+ e^-$		30.1 mb	VDM
$\pi^- p \rightarrow n \eta$	$\eta \rightarrow \gamma e^+ e^-$ (6.9×10^{-3})	0.63 mb	PWA

Table 4.3.: Cross-sections of $\pi^- p$ simulated reaction channels obtained from various model predictions. VDM stands for the Vector Dominance Model in which the e^+e^- contribution comes from the off-shell ρ production (for more details see Ref. [33]).

Chapter 5

Reconstruction of signals

"..... a knowledge of the effects is what leads to an investigation and discovery of causes"

Galileo Galilei

The aim of this analysis is to extract bremsstrahlung process in the π^-p collisions, within the kinematically complete final states reconstruction and as a unique contribution, not as a background process, as it was in the case of the proton-proton [85] or neutron-proton measurements [86, 87]. For this purpose, a good understanding of contributing processes, where resonances are produced, is mandatory. The identification of resonance contributions in the case of π^-p collisions was completed in the two-pion analysis [80], where the hadronic two-pion final states were extracted and described with the partial wave analysis. In the present chapter, all dedicated data analysis steps taken to reconstruct π^-p bremsstrahlung signal are described.

5.1 Pion beam

The present experiment was performed in August 2014 with a total of two weeks of π^- beam collisions measured with the HADES spectrometer. The aim of the experiment was to study the production of pion pairs as well as electron-positron pairs in π^-p collisions, in the second resonance region [80]. The measurements were performed by using polyethylene $((C_2H_4)_n)$ target. In order to subtract the carbon contribution in the collected data, additional measurements with pure carbon (C) target were conducted as well. The experimental data were collected at four beam momenta to give four energy points for the partial wave analysis (PWA) study of hadronic channels. The beam momenta measured with CERBEROS system (see Chapter 2.2) and corresponding number of collected events in the measurements with $(C_2H_4)_n$ and C target are given in Table 5.1.

The pion beam momentum distributions are presented in Fig. 5.1. The widths of these distributions correspond to a momentum resolution of $\delta p_{beam}/p_{beam} \approx 1.7\%$. The central beam momenta determined from the CERBEROS measurements were based on the position of neutron peak in the $\pi^+\pi^-$ missing mass spectra, as described in Ref. [80]. The obtained beam momenta are 0.6501 ± 0.002 GeV/c, 0.6853 ± 0.0025 GeV/c, 0.7332 ± 0.003 GeV/c, and 0.786 ± 0.0035 GeV/c, respectively. These values are lower by $0.005 - 0.015$ GeV/c as compared to the nominal beam momentum values.

To search for the bremsstrahlung signal in the collected data, only events for beam momentum of $p_{beam} = 0.685$ GeV/c were selected as statistics for the data taken at the remaining three beam momentum settings is much lower. This is why the pion beam momentum in the further part of the analysis is quoted to be equal to 0.685 GeV/c.

Target	$p_{beam}^{\pi^-}$ momentum [GeV/c]	Total number of Events
Polyethylene ((C ₂ H ₄) _n)	0.656	42.4M
	0.690	774.7M
	0.748	76.5M
	0.800	52.4M
Carbon (C)	0.656	41.9M
	0.690	115.7M
	0.748	42.2M
	0.800	41.2M

Table 5.1.: Pion beam momenta and total number of collected events in measurements with (C₂H₄)_n and C targets.

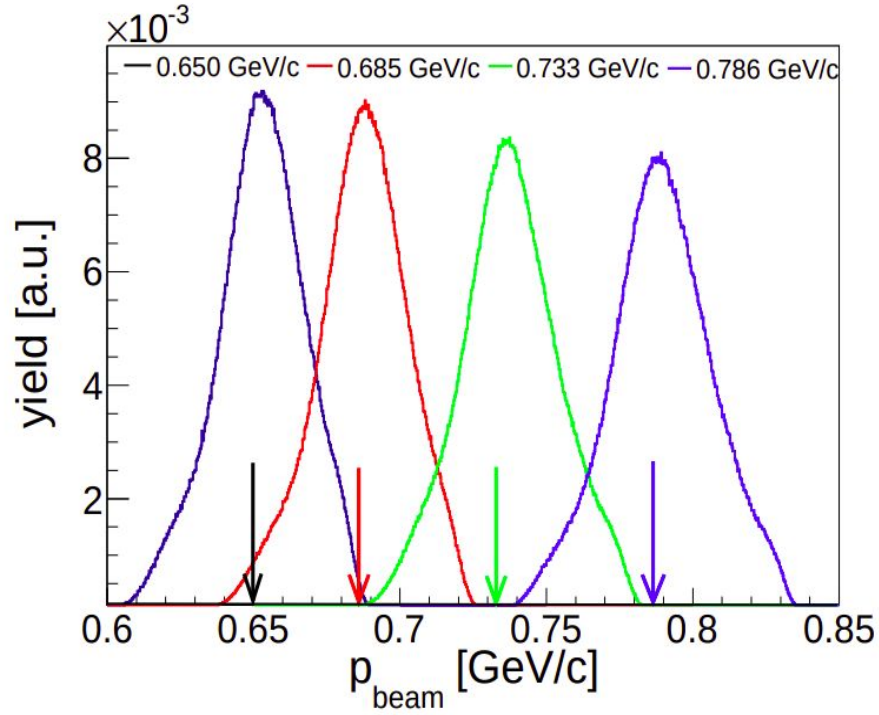


Figure 5.1.: Distribution of the pion beam momentum measured with the CERBEROS system. The distributions have been normalized to the same area. The arrows represent the corrected values of the beam momenta as described in the text.

5.2 Analysis Flow

The HADES data analysis is carried out using the HYDRA (**H**ades **sY**stem for **D**ata **R**eduction and **A**nalysis) framework [89], which is written in C++ with small admixture of FORTRAN, and is based on the ROOT package [90].

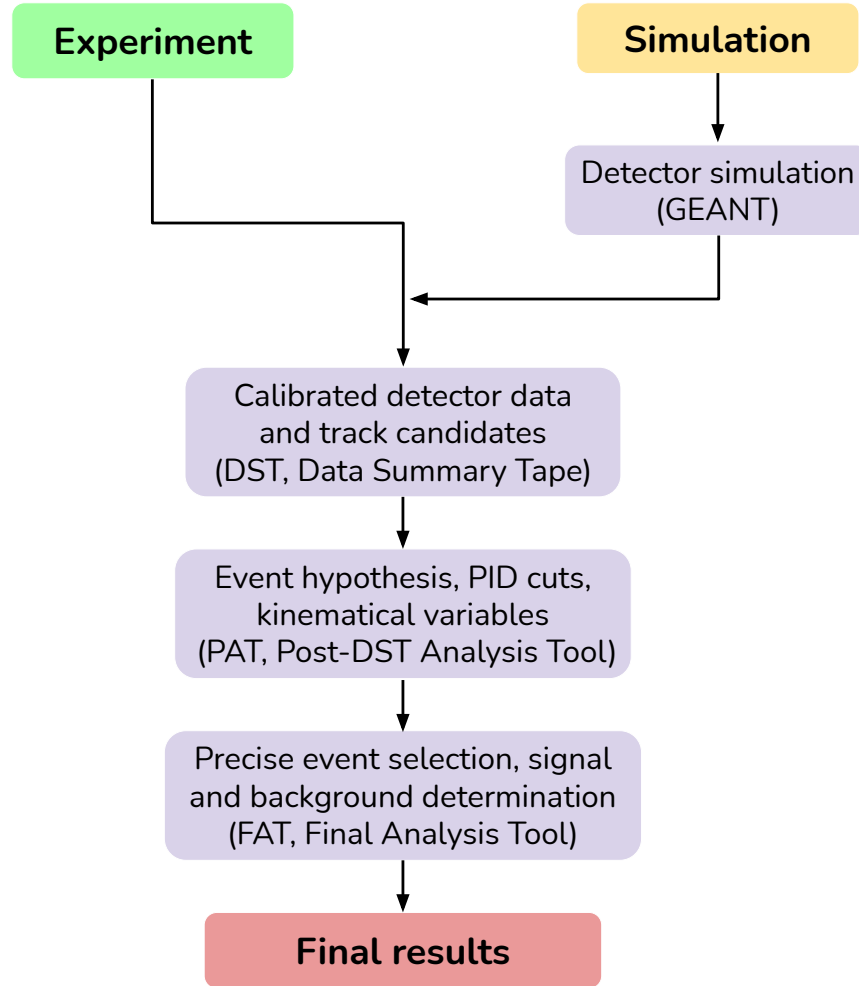


Figure 5.2.: The diagram of the experimental data and Monte Carlo simulation analysis workflow.

A flow diagram for simulation and experimental data analysis is shown in Figure 5.2. The investigation techniques include raw data processing, calibration, track reconstruction, particle identification, and reaction channel selection. In addition to the procedure used for the analysis of experimental data, the simulation procedure includes a detailed modelling of detector response and trigger conditions using HGEANT framework. The HGEANT [91] is a simulation package for HADES written in FORTRAN and built upon the GEANT [92] program from CERN.

At the time of experiment, the HADES data acquisition system writes HLD (**H**ades **L**ist **D**ata) files with information accepted by the trigger conditions. They include raw data acquired from various detectors read-out electronics: ADC or TDC values, as well as hardware addresses that allow a single module of the appropriate detector to be identified. As a next step of the analysis, essentially after the calibration of the detectors, the data relevant for physics analysis are written to the DST (**D**ata **S**ummary **T**ape) files. They contain, among others, physical properties like identified hits in RICH, MDCs and TOF/RPC with corresponding parameters like hit coordinates, energy loss, time of flight values, and also reconstructed

tracks in the MDCs as well as reconstructed momentum vectors of charged tracks. Creation of DST files is a standard part of the analysis for both experimental and simulation data.

The input for the Monte Carlo simulation is computed with the PLUTO [75, 76] and GiBUU [77] event generators. Then the GEANT package [93] is used for simulating particle interactions with the detector material. The remaining part of the analysis chain is the same as in the experimental case, including all selection criteria.

PAT (**P**ost**D**ST **A**nalysis **T**ool) framework takes DST files as input. In this step, particle identification (PID) is performed and events of interest are selected based on the track multiplicity and PID information.

FAT (**F**inal **A**nalysis **T**ool) provides the final stage in the analysis chain by extracting physical observables such as invariant mass, total energy, angular distributions, missing mass *etc.*

5.3 Carbon subtraction from π^- (C_2H_4)_n

To study the π^-p collisions, in measurement with the (C_2H_4)_n target, the π^-C contribution has to be subtracted. This was done based on the observation of quasi-elastic scattering of pions on protons in carbon nuclei. The π^-p events were first pre-selected using a cut on coplanar pion and proton reconstructed tracks ($\pm 5^\circ$) and a $\tan\theta_{\pi^-} \times \tan\theta_p > 1$ selection (see Fig. 5.3, left panel). The π^-p missing mass spectra obtained after these selection cuts for measurements with (C_2H_4)_n and C target are shown in Fig. 5.3, right panel. The distribution for the (C_2H_4)_n target (black histogram in Fig. 5.3, right panel) shows a prominent peak around zero missing mass corresponding to the π^-p elastic scattering.

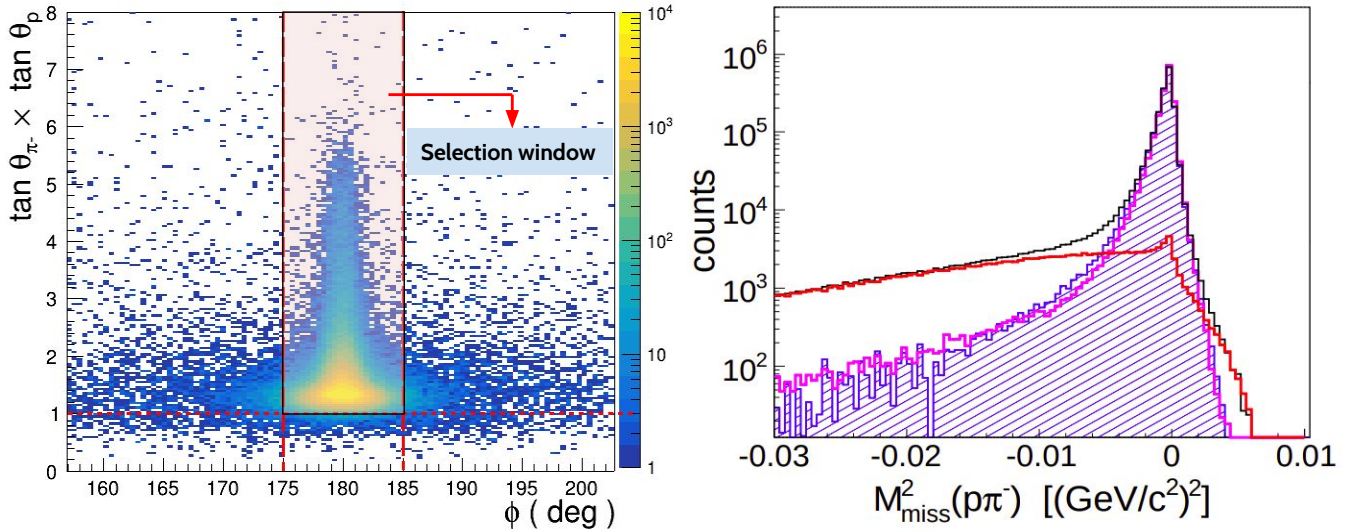


Figure 5.3.: Left panel presents cuts selecting pion-proton quasi-elastic scattering, where red vertical dashed lines show the condition of coplanarity ($\pm 5^\circ$) and horizontal dotted line shows $\tan\theta_{\pi^-} \times \tan\theta_p > 1$ selection window. Right panel presents π^-p missing mass squared for (C_2H_4)_n target (black histogram) and C target (red histogram) for pion beam momentum of 0.685 GeV/c. Blue hatched area presents the π^-p contribution, obtained as a subtraction of carbon contribution from polyethylene target. Simulation of π^-p elastic scattering is shown as a magenta histogram.

The tail at negative values of missing mass occurs due to the quasi-free elastic scattering of pions on a bound proton in carbon nucleus. The contribution is shifted towards negative values due to the binding energy of the protons in carbon and is very broad due to the Fermi momentum distribution. The fraction of π^-p events measured with the carbon target (red histogram) was scaled to match the negative tail of the missing mass squared distribution for the polyethylene target. The corresponding scaling factor was

then used in analyses of various π^-p inelastic channels for subtracting carbon contribution measured with the carbon target from data registered with the polyethylene target (for details see Ref. [80]). For the elastic π^-p scattering, the subtraction of the carbon contribution results in a π^-p missing mass distribution presented with blue hatched histogram in Fig. 5.3, right panel. This distribution is very well reproduced by Monte Carlo simulations of π^-p elastic scattering.

5.4 Normalization of cross-sections

The π^-p elastic scattering was measured in the pion beam experimental run to serve as an absolute normalization to the cross-sections. Acceptance corrections for the measured elastic counts were determined based on simulations. Elastic π^-p events were generated using the PLUTO event generator [94] with an angular parametrization taken from Ref. [79]. The simulation reproduces the shape of angular distribution of the experimentally registered elastic yields. The acceptance was calculated as the ratio of reconstructed to simulated Monte Carlo events as a function of scattering angle in the center-of-mass frame [95]. After applying the acceptance correction, the angular distribution of the elastic scattering was normalized to the world data (averaged data points) in the $\theta_{\pi^-}^{CM}$ range of $59.5^\circ - 110.5^\circ$ (Ref. [80]). Figure 5.4 shows the angular distribution of the elastic events measured for the central pion beam momentum of 0.685 GeV/c, together with the world data selected for a pion beam momentum window $\delta p = \pm 10$ MeV/c around the central momentum. The angular distribution of HADES data are in agreement with the distribution of the world data. Figure 5.4 shows also angular distributions of the SAID solution [78] which is about 10% lower than the HADES data. The systematic error of the normalization is dominated by the errors of the world data and is equal to 4 %.

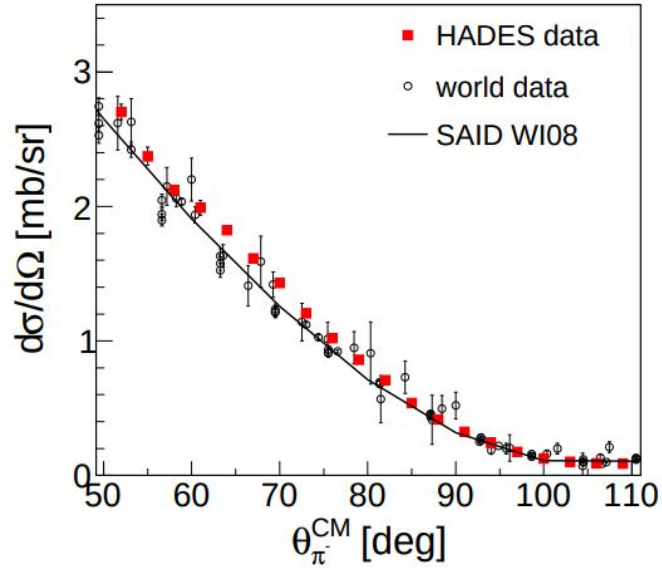


Figure 5.4.: Angular distribution of π^-p elastic-scattering cross-section at 0.685 GeV/c, including the HADES data, the world data and SAID solution WI08.

5.5 Combinatorial background

An important step in the present data analysis is determination of the combinatorial background in the measurement of dileptons. The combinatorial background is formed in two different scenarios:

a) The uncorrelated background (Fig. 5.5, left) is due to the combination of electrons and positrons coming from decay of different particles. The lack of a specific kinematical correlation between the leptons characterizes this kind of background. Dilepton pairs may be produced either directly from a virtual photon at the electromagnetic vertex or by converting real photons in detector material;

b) The correlated background (Fig. 5.5, right) is due to the decay of a single particle into 2γ or into $\gamma e^+ e^-$ (e.g. $\pi^0 \rightarrow \gamma\gamma$ which has branching ratio $\sim 99\%$ or $\pi^0 \rightarrow \gamma e^+ e^-$ which has branching ratio $\sim 1.2\%$) and the subsequent conversion of one or two γ .

Because electrons and positrons are always emitted in pairs, no difference between them is anticipated in complete phase space, e.g. without taking into account detector acceptance and efficiency. Therefore, the contribution of lepton pairs with the same sign can be used to estimate the combinatorial background.

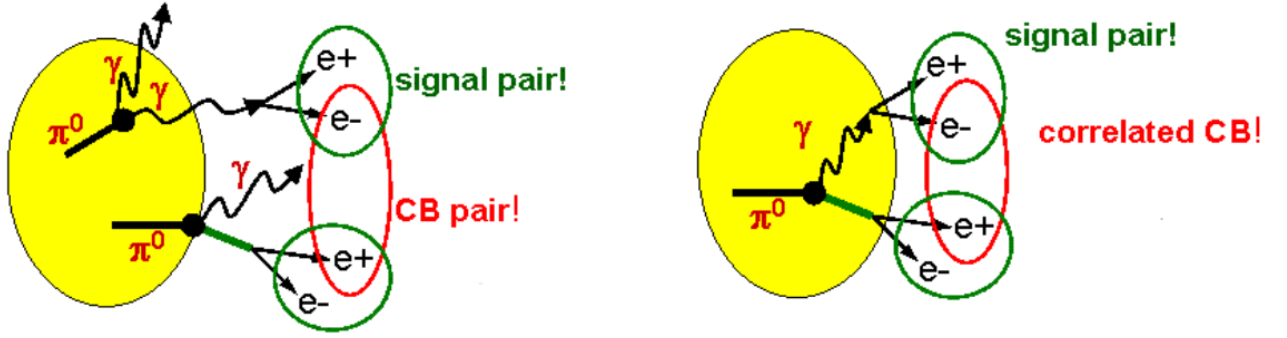


Figure 5.5.: Uncorrelated (left) and correlated (right) combinatorial background.

In analysis of high statistics data for e^+e^- dilepton production measured at HADES, the combinatorial background is typically estimated by the geometrical mean of e^+e^+ (N_{++}) and e^-e^- (N_{--}) the like-sign pairs, [96–99]:

$$N_{+-}^{CB} = 2 \times \sqrt{N_{++}N_{--}}. \quad (5.5.1)$$

To determine the e^+e^- signal, combinatorial background is subtracted from the total number of reconstructed e^+e^- :

$$N_{+-}^{Sig} = N_{+-}^{Tot} - N_{+-}^{CB}. \quad (5.5.2)$$

In the present analysis, due to low statistics of collected e^+e^- events, one or both numbers of like-sign pairs (N_{++} or N_{--}) may be zero and the geometrical mean leads to an underestimation of the combinatorial background. Therefore, the geometrical mean is replaced by the arithmetic mean which is justified by the following consideration:

$$\sqrt{N_{++}N_{--}} = \{N_{--} = N_{++} + \epsilon\} = \sqrt{N_{++}^2 + \epsilon N_{++}}. \quad (5.5.3)$$

By using Maclaurin's series expansion,

$$\sqrt{N_{++}N_{--}} \approx N_{++} \times \left(1 + \frac{1}{2} \frac{\epsilon}{N_{++}}\right) = \frac{N_{++} + N_{--}}{2}. \quad (5.5.4)$$

5.6 Selection Cuts

In this section, cuts applied for the identification of final state particles are described. There are three basic groups of such cuts, one is applied to the RICH detector signals, second to the opening angle between reconstructed tracks and the third one to the TOF information used for particle identification.

5.6.1 Cuts on RICH detector data

For the lepton signal identification in the RICH detector, two alternative approaches, the backtracking (BT) and the pattern matrix (PM) are used (see Section 3.4). Figure 5.6 presents distributions of ring candidate characteristics with selection cuts applied. The backtracking approach is based on the matching a reconstructed track with ring candidates in the RICH. The idea of BT is to accept any signature of a ring. Therefore, a track correlated with a ring pattern containing at least one (or more) local maximum in the distribution of the fired pads is considered as a lepton track candidate (see Fig. 5.6, left panel). In the case of the PM algorithm, at least four pads must contribute to the accepted ring candidate (Fig. 5.6, right panel). Both approaches are based on the minimum bias cuts.

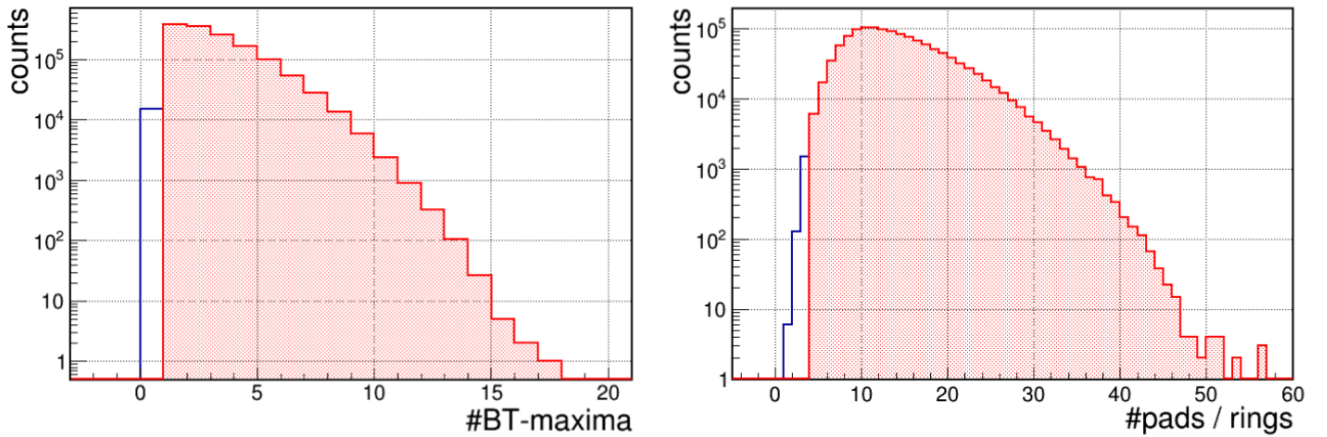


Figure 5.6.: Left: distribution of number of local maxima attributed to a ring candidate in the backtracking approach. Right: number of pads contributing to a ring candidate in the pattern matrix approach. In both cases, the accepted candidates are marked in red on histograms.

5.6.2 Close-neighbour track and opening angle cut

The main source of background in the measurement of dileptons is due to the photon conversion in the target and RICH radiator. The e^+e^- pairs created in such a conversion have, in most cases, low opening angle, and one of tracks holds low momentum. Particles with low momenta are in most cases extracted in the magnetic field out of the spectrometer. Therefore, only their inner track fragments, reconstructed with the inner MDC I and II chambers, remain. In order to tackle with such a contamination, every track identified in the RICH as a lepton candidate is tested with respect to the presence of another track (so called close-neighbour track), within a given opening angle. Figure 5.7, left presents the distributions of the opening angle between a lepton track candidate and its close-neighbour track. Negative values in the figure are assigned to close-neighbour tracks reconstructed only within the inner MDC. In the analysis, only fully reconstructed tracks (with positive values in histogram) are considered and are suppressed by the minimum opening angle condition, which is at 4° . In Fig. 5.7, right, a distribution of the opening angle between two fully reconstructed lepton tracks is shown. Effect of the close neighbour cut ($>4^\circ$) is

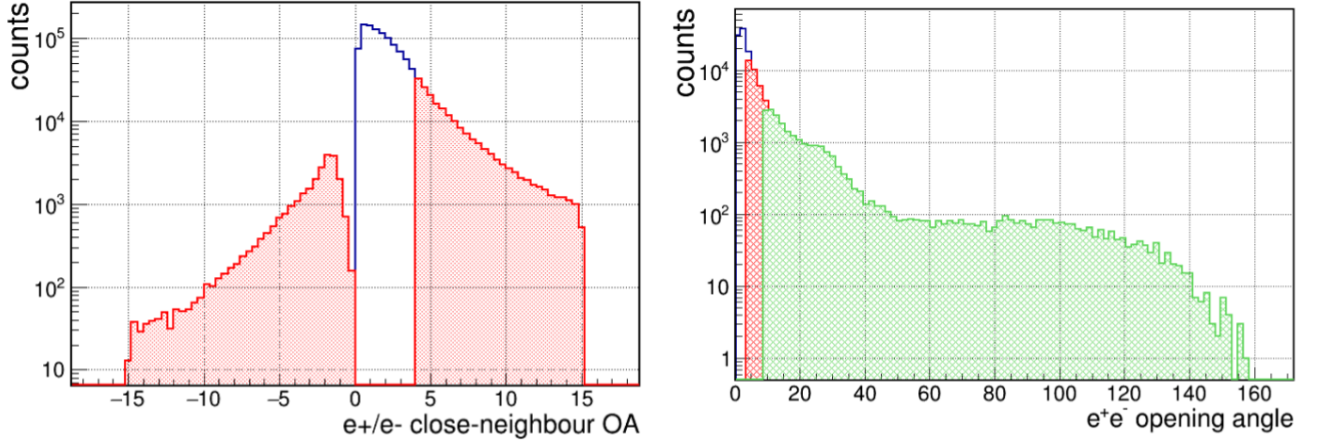


Figure 5.7.: Left panel: distribution of opening angle between electron/positron track candidate and the close neighbour track. Neighbour tracks reconstructed only in the inner MDCs are attributed with a negative values of the opening angle and the cut suppressing pairs is set to opening angle $> 4^\circ$ (red shaded area for this angular range represents the accepted tracks). Right panel: opening angle distribution for two fully reconstructed lepton tracks. For the e^+e^- pairs, two cuts of the opening angles were investigated : $>4^\circ$ (red hatched) and $>9^\circ$ (green hatched). In the analysis, the later cut was used as described in text.

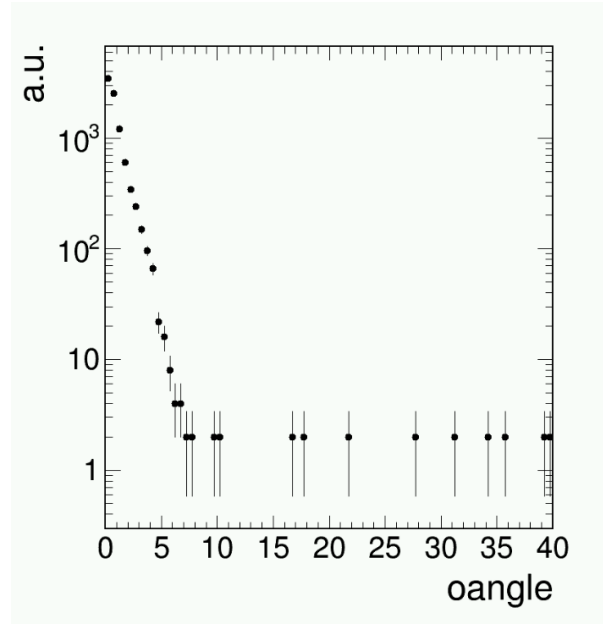


Figure 5.8.: e^+e^- opening angle distribution from Monte Carlo simulation of photon conversion at π^- beam momentum of 0.685 GeV/c, where photons are produced by the Dalitz decay of π^0 produced in reaction $\pi^- p \rightarrow n \pi^0$.

shown with red region. For the further suppression of the contribution from the conversion in the measured dilepton pairs, a requirement on the minimum opening angle of 9° was introduced and is shown with green area in Fig. 5.7, right. The justification of this requirement comes from simulations of the conversion of gamma quanta originating from the $\pi^- p \rightarrow n \pi^0 \rightarrow n \gamma e^+ e^-$ reaction channel, presented in Ref. [33]. As one can see in Fig. 5.8, the opening angle distribution for e^+e^- tracks produced in the conversion process is dominated by angles smaller than 9° .

5.6.3 Particle identification

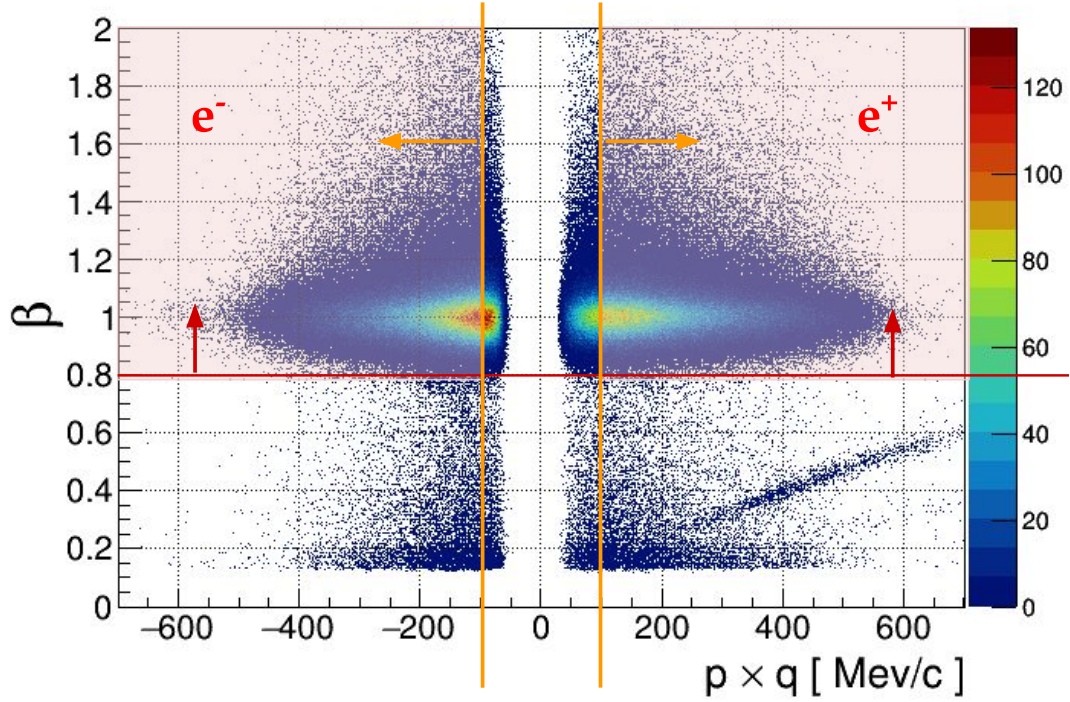


Figure 5.9.: β (v/c) of e^+ or e^- versus momentum $\times$ charge with the cuts applied to the velocity (red horizontal line) and the momentum (orange vertical lines).

The TOF information from the META detector system is used to improve the identification of leptons done with the RICH detector and to identify pions and protons. Figure 5.9 presents velocity versus momentum distribution for tracks identified as electrons/positrons in the RICH detector. In order to separate leptons from pions and protons, the selection cut $\beta > 0.8$ was applied. In addition, due to difficulties in the reconstruction of lepton tracks with very low momenta, a cut on the momentum (p) > 100 MeV/c was introduced. For the identification of pions and protons, particle mass was calculated based on the velocity measured with the META system and the momentum reconstructed with the MDCs:

$$m = \sqrt{p^2 \left(\frac{1}{\beta^2} - 1 \right)}. \quad (5.6.1)$$

In Fig. 5.10, presenting particle mass vs momentum $\times$ charge, pions and protons are clearly separated from each other.

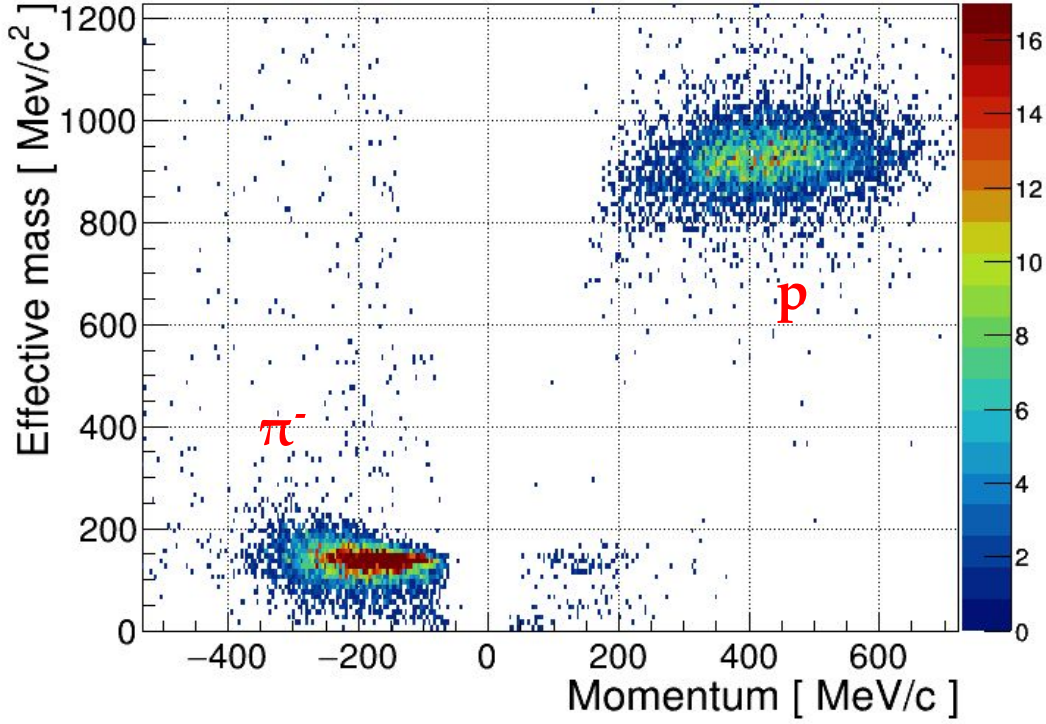


Figure 5.10.: Particle mass versus momentum times charge.

5.7 Inclusive and exclusive channels

The objective of the analysis is to extract the bremsstrahlung process occurring, when a charged particle under acceleration radiates a virtual photon which subsequently decays into a dilepton pair. The idea is to tackle the problem within the inclusive and exclusive channels which are possible to be reconstructed. In this thesis, the inclusive $e^+e^- X$ channel and the exclusive $p \pi^- e^+e^-$ channel are investigated. The exclusive channel is investigated using three different analysis procedures. They are listed in Fig. 5.11.

In the inclusive dilepton channel analysis only e^+e^- in the final state is identified and remaining reaction products are unmeasured. As this analysis takes into account only two particles in the final state, therefore, it is also called the two-prong hypothesis. This analysis includes all possible channels contributing to the e^+e^- final state in the respective energy regime.

In the exclusive analysis three different (complementary) groups of registered events were analysed:

- (1) registered three tracks with identified $\pi^- e^+e^-$ and the remaining 4th particle (proton) is identified by the missing mass technique.
- (2) registered three tracks with identified $p e^+e^-$ and the remaining 4th particle (π^-) is identified by the missing mass technique.
- (3) registered four tracks with all final state particle ($p \pi^- e^+e^-$) identified in the detection system.

In the listed above exclusive analysis channels, 1 and 2 are referred to as the three-prong hypotheses, while the 3rd case is called the four-prong hypothesis. In the following chapters, the analyses of the 2-, 3- and 4-prong hypotheses is described in details.

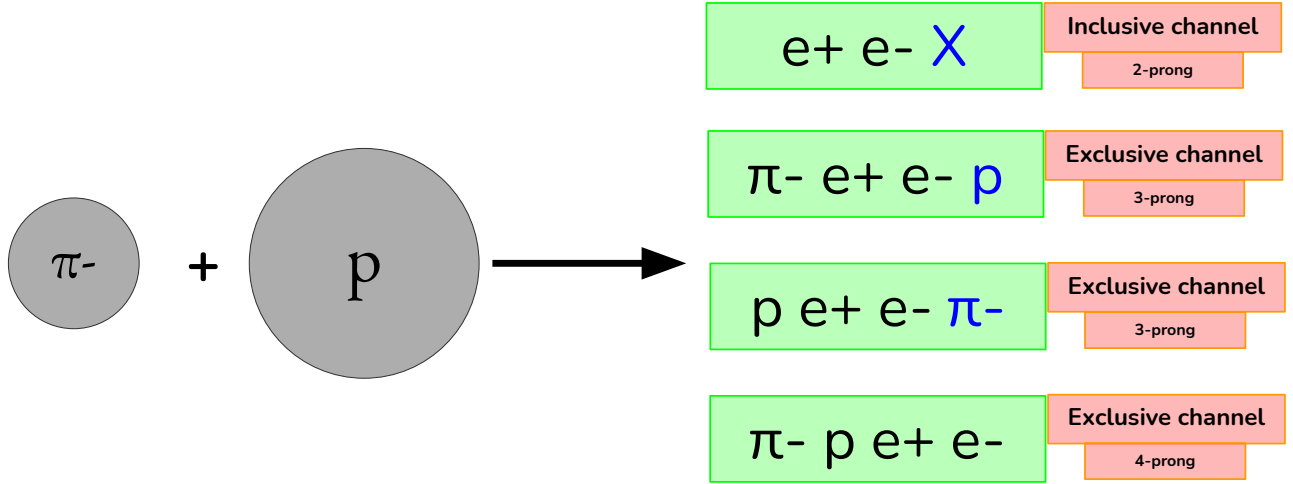


Figure 5.11.: Analyzed inclusive and exclusive channels. The measured particles are denoted in black, whereas missing (unmeasured) particles are marked in blue.

5.7.1 Four particle ($p \pi^- e^+ e^-$) analysis

In the first step of the 4-prong analysis (p , π^- , e^+ and e^- tracks measured and identified), the missing mass squared of the $p \pi^- e^+ e^-$ system is determined. Figure 5.12 (left panel) shows the $(p \pi^- e^+ e^-)$ missing mass squared distribution for the $(C_2H_4)_n$ target, combinatorial background and the signal obtained after the CB subtraction. The prominent peak located around the value 0, represents either no particle remained as unmeasured or one undetected gamma photon $M_{inv}(\gamma) = 0$. In the right panel of

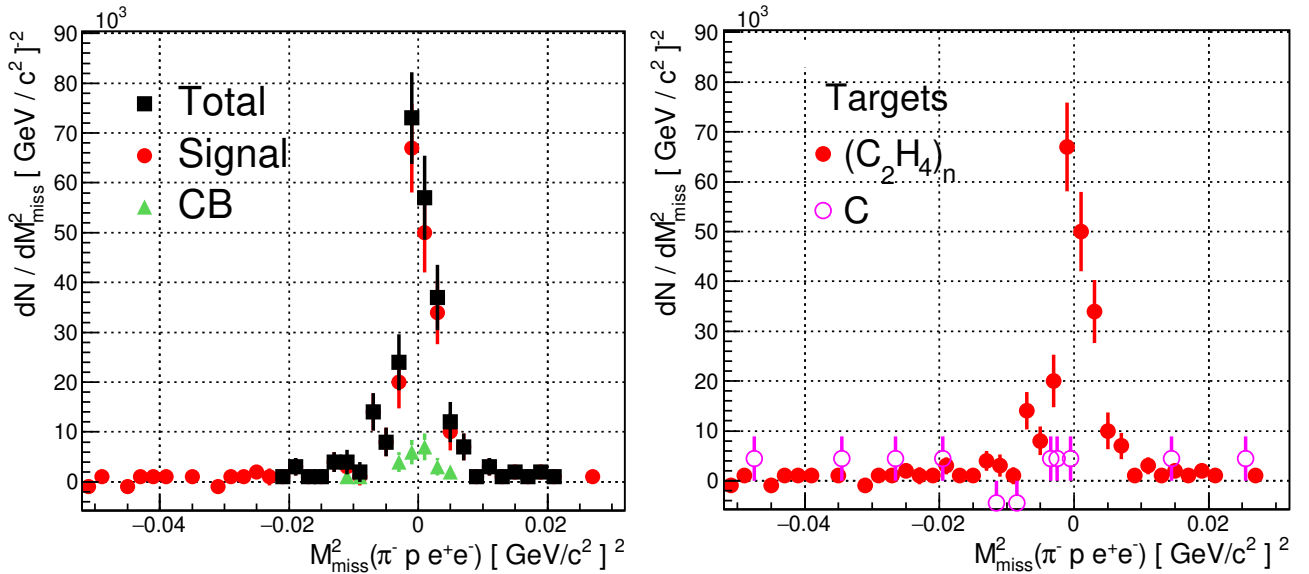


Figure 5.12.: Four-particle missing mass squared distributions, $M_{miss}^2(p \pi^- e^+ e^-)$. Left panel: all $e^+ e^-$ events (black squares), combinatorial background (green triangles), and signal extracted after CB subtraction (red full dots) for measurements with polyethylene target. Right panel: signal extracted from two different targets, $(C_2H_4)_n$ (red full dots) and C (magenta open dots). The distributions are normalized to the bin width equal to $0.002 \text{ (GeV/c}^2\text{)}^2$.

Fig. 5.12, the missing mass squared spectrum obtained with the polyethylene target is compared to the corresponding distribution for the carbon target. In later case no peak at zero mass is visible. In order

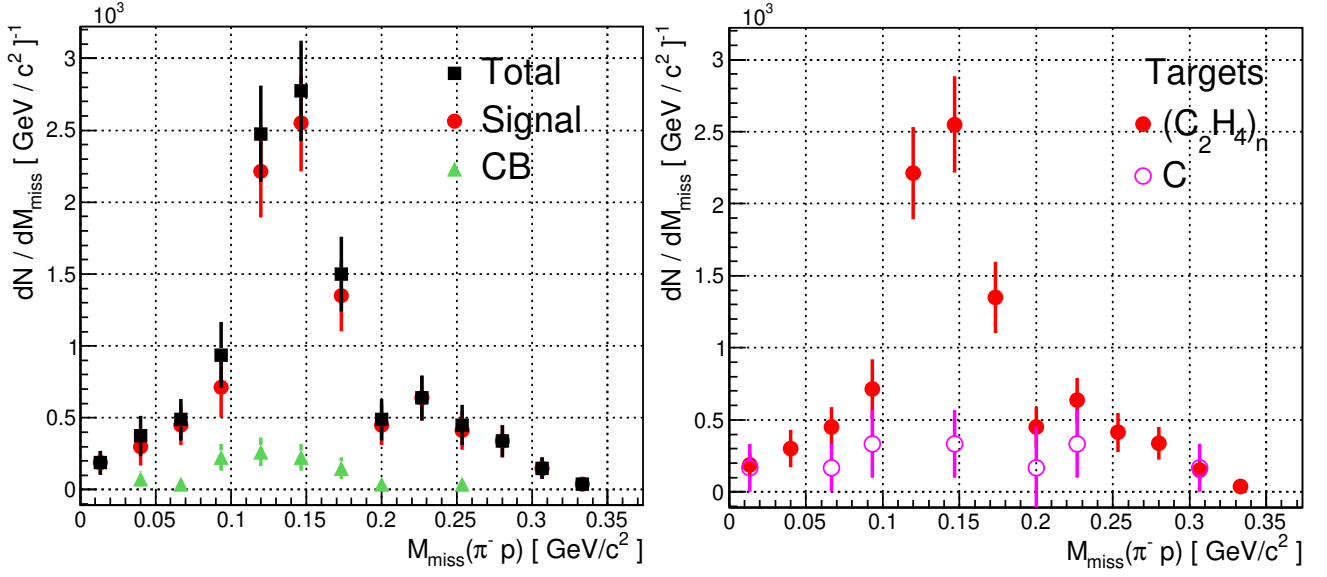


Figure 5.13.: $\pi^- p$ missing mass distributions. Legend as in Fig. 5.12. For all e^+e^- pairs, an opening angle $> 9^\circ$ cut was applied and distributions are normalized to the bin width of the size equal to $0.027 \text{ GeV}/c^2$.

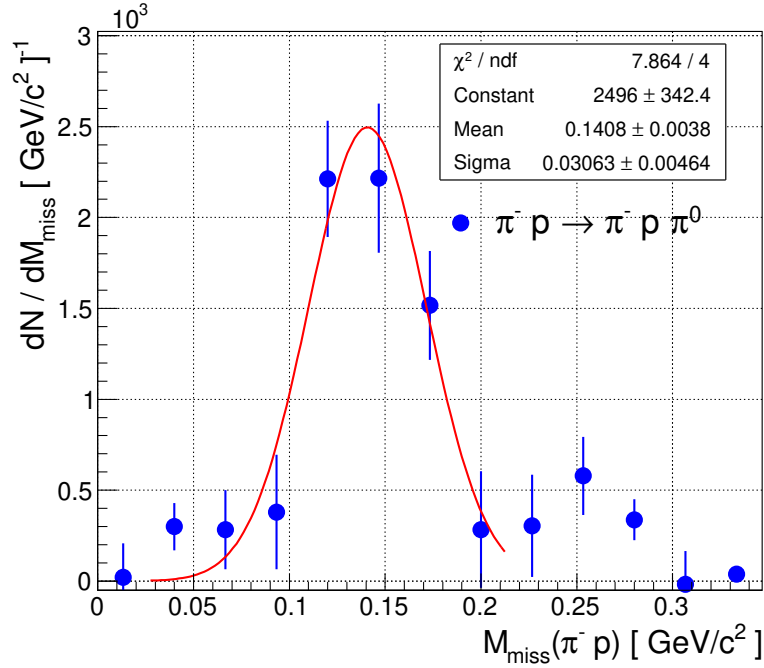


Figure 5.14.: $p \pi^-$ missing mass distribution for measurements with polyethylene target after subtracting the carbon contribution. The parameters of the gaussian fit shown with the red curve are presented in the legend box.

to check possible contribution of the π^0 Dalitz decay in the analyzed events, a missing mass spectrum of the $\pi^- p$ system was determined and is shown in Fig. 5.13. The peak in the data taken with the $(\text{C}_2\text{H}_4)_n$ target is clearly seen at the π^0 invariant mass.

The peak in the missing mass spectrum for the polyethylene target after subtraction of the carbon contribution was fitted with a Gauss function, as shown in Fig. 5.14, resulting in the maximum value at $0.140 \pm 0.004 \text{ GeV}/c^2$ which is consistent with the π^0 mass ($0.135 \text{ GeV}/c^2$). One can conclude that the π^0 Dalitz

decay dominates the analyzed exclusive channel.

Figure 5.15 presents the e^+e^- invariant mass distributions. Due to the π^0 Dalitz decay contribution, the

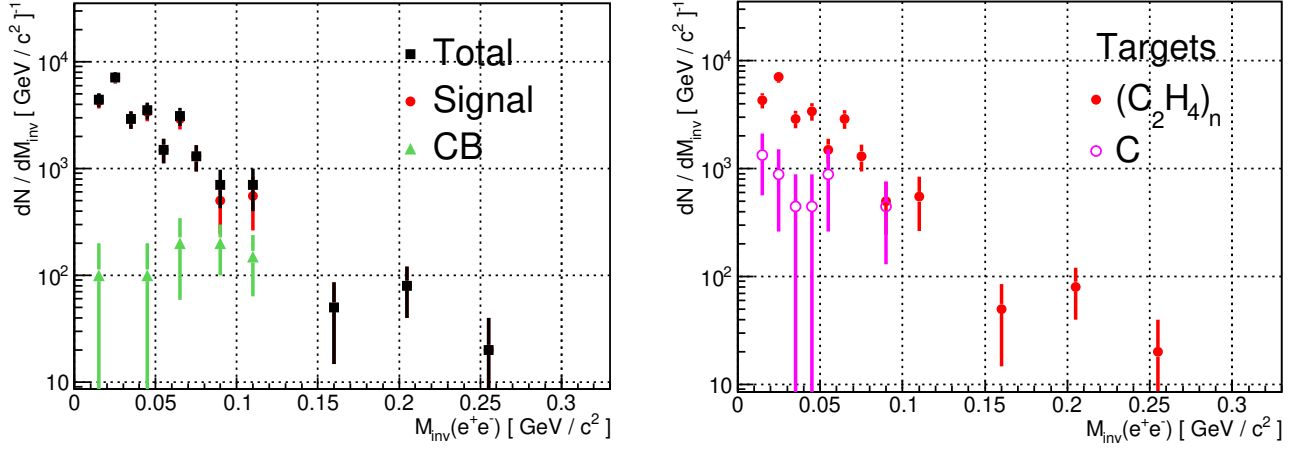


Figure 5.15.: e^+e^- invariant mass distribution. The legend is the same as in Fig. 5.13. The spectra are normalized to the variable bin width size.

distribution is dominated by values smaller than the π^0 mass.

In the exclusive channel analysis, the statistics of reconstructed $p \pi^- e^+ e^-$ is very low as presented in Table 5.2. For the measurements with the $(C_2H_4)_n$ target and the e^+e^- pairs identified in the RICH using the backtracking algorithm, the number of reconstructed events is only 261. By including the pattern matrix algorithm for the dilepton identification, this number increases to 307. Figure 5.16 presents missing mass squared distribution $M_{miss}^2(\pi^- p e^+ e^-)$ above π^0 mass region, where the number of reconstructed events is only 9. Due to the small statistics and dominating role of the π^0 Dalitz decay in the pair production, the exclusive 4-prong channel is not well suited to search for bremsstrahlung events. Substantially higher statistics was collected for the 3-prong events ($\pi^- e^+ e^-$ and $p e^+ e^-$) and, therefore, these events were used for the search of bremsstrahlung signal. Their analysis is presented in the following sections.

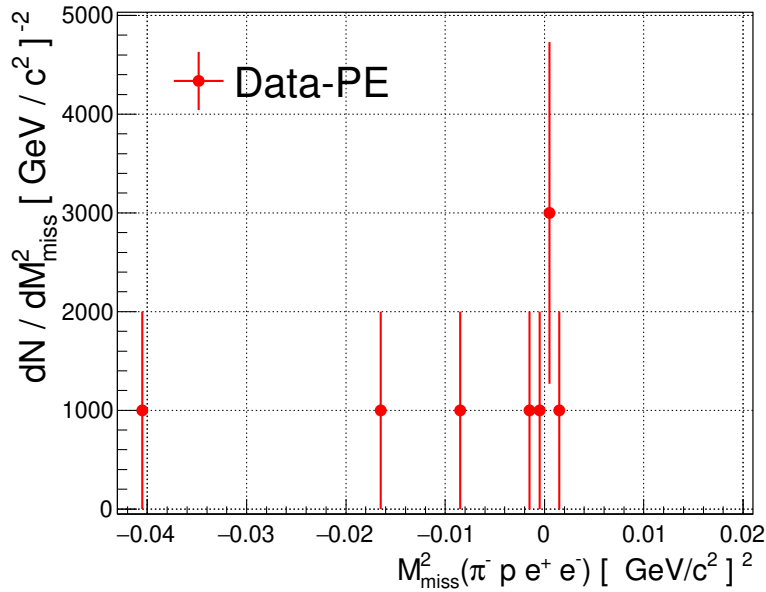


Figure 5.16.: $p \pi^- e^+ e^-$ missing mass squared distribution for $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$ in measurements with polyethylene target.

	$(\text{C}_2\text{H}_4)_n$		Carbon	
Type	Signal	CB	Signal	CB
BT method	261	28	146	21
BT PM method	307	37	172	20

Table 5.2.: Statistics of reconstructed 4-prong events ($\pi^- p e^+ e^-$) in measurements with $(\text{C}_2\text{H}_4)_n$ and C targets. The signal is obtained after the combinatorial background subtraction. The first row in table corresponds to the number of events with dileptons reconstructed in the RICH by means of the backtracking (BT) algorithm and in the case of the second row the pattern matrix (PM) algorithm was used in addition to the backtracking as described in Chapter 3.4.3.

Even though the 4-prong analysis is not very suitable for the study of the bremsstrahlung process, it was used for a consistency check with the previous HADES analysis of the $\pi^- p \rightarrow \pi^- p \pi^0$ channels (Ref. [80]). This check is presented in Chapter 6.

5.7.2 Three particle $\pi^- e^+ e^-$ and $p e^+ e^-$ analysis

Within the present studies, two types of 3-prong analysis were conducted: one with reconstructed $\pi^- e^+ e^-$ tracks and the other with $p e^+ e^-$ tracks. The remaining particle was identified by using the missing mass technique. Figure 5.17 shows the $\pi^- e^+ e^-$ missing mass (left panel) and $e^+ e^-$ invariant mass distributions (right panel) obtained in the measurements with the polyethylene target. The $e^+ e^-$ invariant mass distribution is dominated by data contributing below π^0 mass. This suggests that the reconstructed events are dominated by the π^0 Dalitz decay. This could explain the shift of the peak visible in the missing mass spectrum towards values above the proton mass (in the case of bremsstrahlung channel, the expected peak position is located at the proton mass).

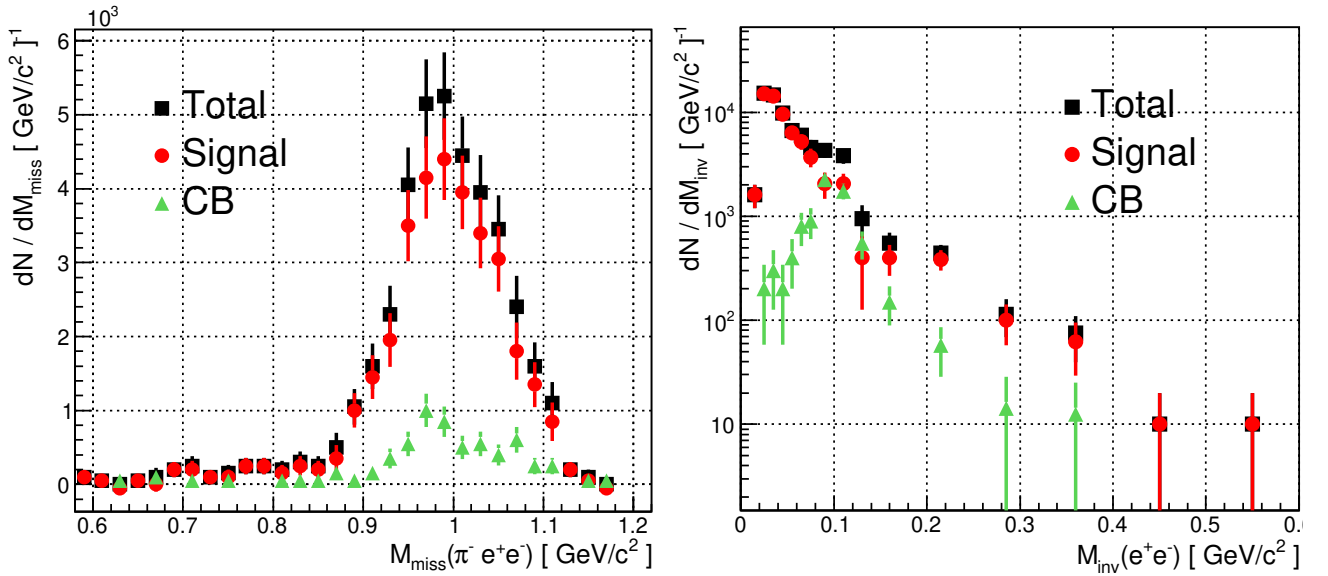


Figure 5.17.: $\pi^- e^+ e^-$ missing mass distribution (left panel) and $e^+ e^-$ invariant mass (right panel) in measurements with $(C_2H_4)_n$ target; black squares represent all events, green triangles - CB_{arithm} , and red dots - signal obtained after the CB subtraction.

In figure 5.18, the missing mass (left panel) and invariant mass distribution (right panel) obtained in measurements with polyethylene and carbon target, respectively, are compared. The striking difference, as compared to the four-particle analysis (see Figs. 5.13, 5.12), is the increasing contribution from events measured with the carbon target.

In order to eliminate the π^0 Dalitz decay, the invariant mass cut: $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ was applied. With this selection, the $\pi^- e^+ e^-$ missing mass distributions for polyethylene and carbon target are compared in Fig. 5.19, left panel. Both distributions contain a clear peak located around the nucleon mass. The increased carbon contribution makes it difficult to extract a statistically significant $\pi^- p$ bremsstrahlung signal in the missing mass spectrum obtained by subtracting the carbon contribution from the polyethylene target spectrum, presented in Fig. 5.19, right panel.

Table 5.3 provides statistics for the three-particle identification (π^- , e^+ , e^-), for both targets, $(C_2H_4)_n$ and C. The first row presents the statistics of events with dileptons reconstructed only by the backtracking (BT) algorithm. In the measurement with $(C_2H_4)_n$ target, the number of events are almost three times higher than in the case of 4-prong analysis (see Table 5.2). However the carbon contribution is relatively

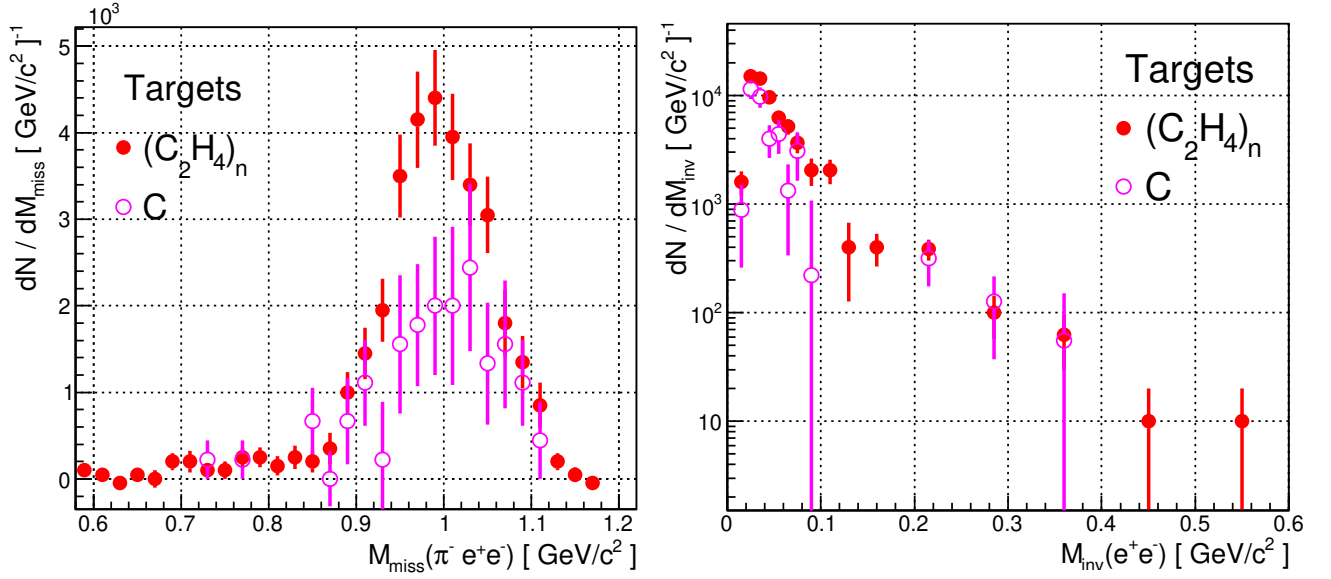


Figure 5.18.: The contribution from the measurement with polyethylene target (red full dots) and carbon target (magenta open dots) in $\pi^- e^+ e^-$ missing mass spectrum (left panel) and $e^+ e^-$ invariant mass spectrum (right panel).

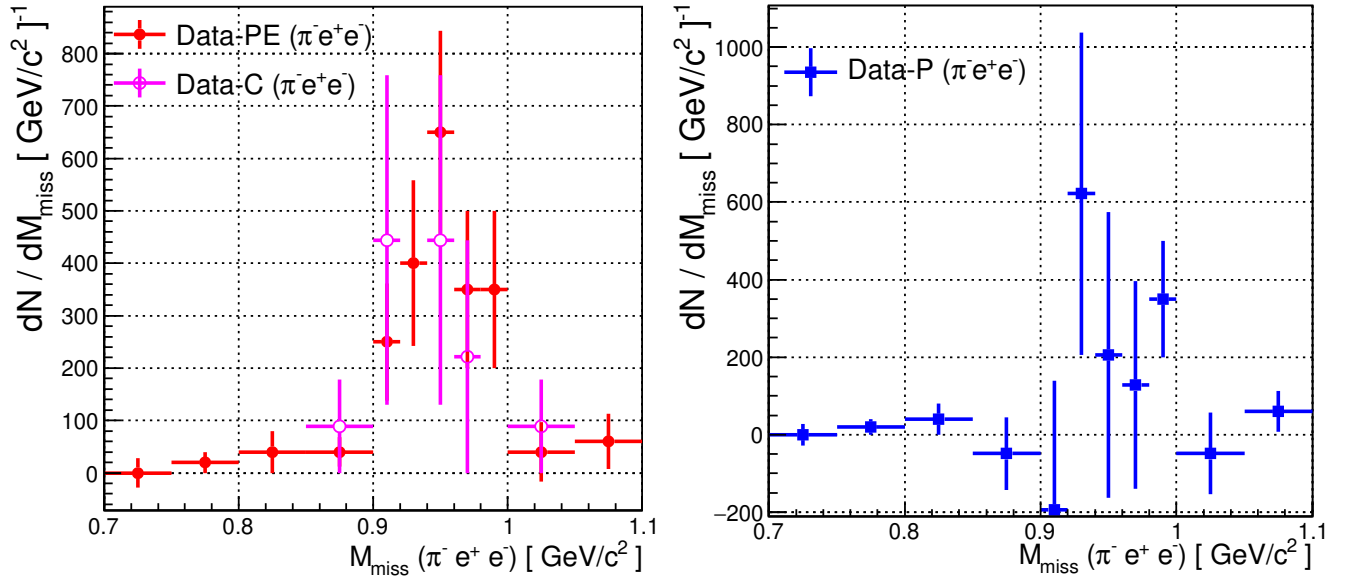


Figure 5.19.: $\pi^- e^+ e^-$ missing mass distribution, after applying the cut $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$, for measurement with polyethylene target and carbon target - left panel: red and magenta dots, respectively, and after subtracting carbon contribution from polyethylene - right panel.

large and it amounts to about 50% of the $(\text{C}_2\text{H}_4)_n$ signal. The second row provides the statistics of events selected with the cut on the invariant mass, $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$. Unfortunately the number of events collected with $(\text{C}_2\text{H}_4)_n$ target is not far above the carbon contribution and, therefore, a search of a statistically significant signal of the bremsstrahlung process is very difficult, as it will be presented in Chapter 6.

	$(C_2H_4)_n$		Carbon	
Type	Signal	CB	Signal	CB
BT method	706	131	387	111
BT method $M_{inv}(e^+e^-) > 0.140 \text{ GeV}/c^2$	57	12	35	9

Table 5.3.: Statistics of reconstructed 3-prong events (π^- , e^+ , e^-) in measurements with $(C_2H_4)_n$ and C targets. The signal is obtained after the combinatorial background subtraction. The first row in table corresponds to the number of events with dileptons reconstructed in the RICH by means of the backtracking (BT) algorithm and in the case of the second row the cut on invariant mass $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$ was used in addition.

Another possibility to reconstruct the 4-prong final state is to identify three particles in the final state: $p e^+ e^-$, with π^- left as unmeasured particle. However, as it will be demonstrated, the HADES spectrometer acceptance for protons is lower than the acceptance for pions. In figure 5.20, similarly to contributions presented in Fig. 5.17, there is $p e^+ e^-$ missing mass squared (left panel) and e^+e^- invariant mass (right panel) presented. The missing mass squared distribution shows a maximum at around zero, and has a long tail towards negative values. Due to its limited width the peak at zero squared mass is consistent with π^0 signal expected at $0.018 (\text{GeV}/c^2)^2$. The distribution of the e^+e^- invariant mass drops rapidly with increase in mass and its shape is similar to one obtained in the $\pi^-e^+e^-$ analysis.

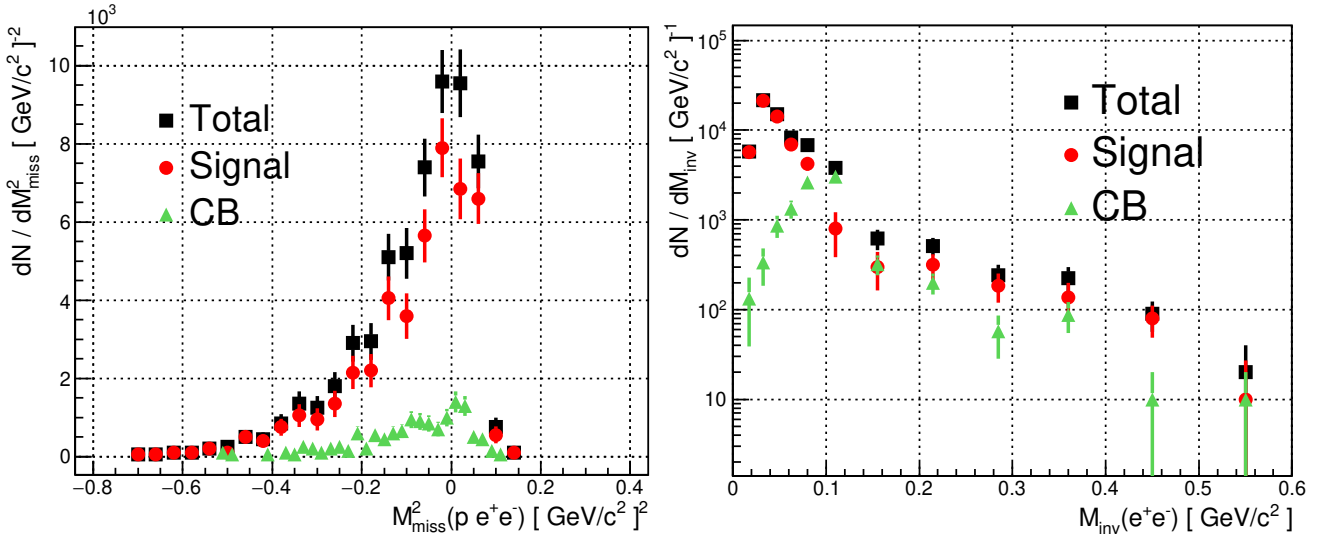


Figure 5.20.: Distribution of $p e^+ e^-$ missing mass squared (left panel), and e^+e^- invariant mass (right panel) for the three-particle hypothesis. The description of contributions is as in Fig. 5.17.

Figure 5.21 shows the distributions from measurements with the polyethylene (red dots) and the carbon (violet open dots) target, after subtracting combinatorial background. The negative tail visible in the

$p e^+ e^-$ missing mass squared distribution for the $(C_2H_4)_n$ target is consistent with one for the carbon target, therefore, it can be explained as a result of the carbon contribution in the $(C_2H_4)_n$ target. This tail may originate from the smearing of the momenta of the measured particles due to the presence of the Fermi-motion of nucleons inside the carbon nucleus.

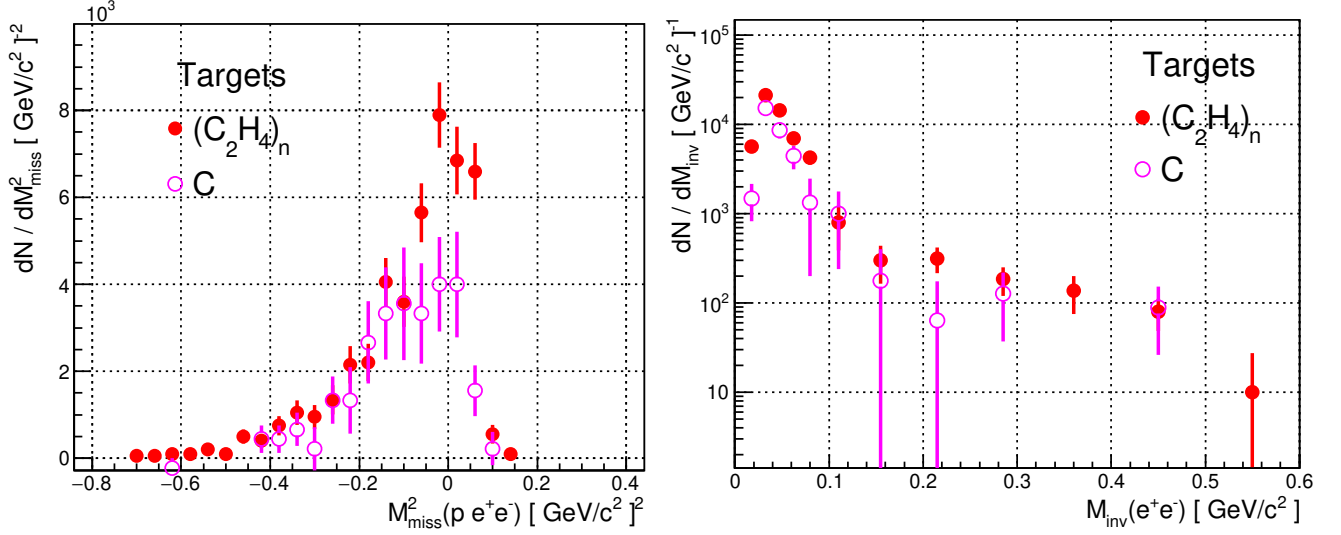


Figure 5.21.: $p e^+ e^-$ missing mass squared (left panel) and $e^+ e^-$ invariant mass (right panel) for the measurements with polyethylene (red full dots) and carbon (magenta open dots) target.

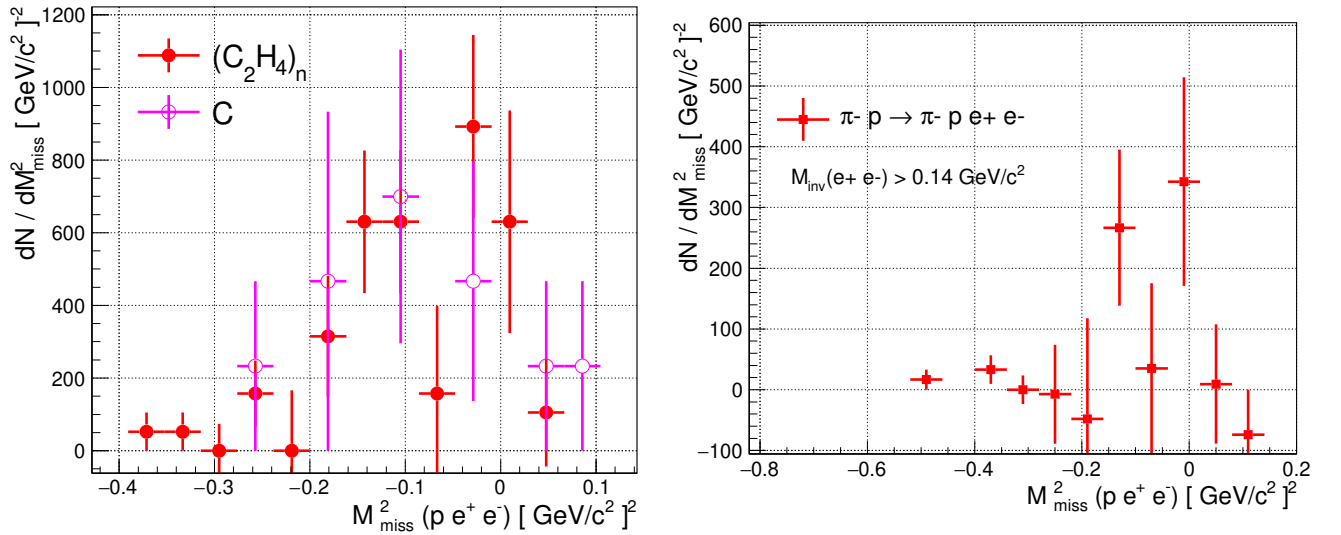


Figure 5.22.: $p e^+ e^-$ missing mass squared distribution for $(C_2H_4)_n$ (red dots) and C (magenta dots) target - left panel and $p e^+ e^-$ missing mass squared distribution after subtraction of the carbon contribution from the polyethylene target - right panel. Both are plotted for invariant mass above the π^0 mass region and are normalized to the bin width.

The missing mass squared distribution after applying the cut on the $e^+ e^-$ invariant mass: $M_{\text{inv}}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ is shown in Fig. 5.22. In the left panel, the $p e^+ e^-$ missing mass squared distribution for polyethylene and carbon target, is presented. In the right panel, the $\pi^- p$ interaction is obtained after subtraction of the carbon contribution from the polyethylene target distribution. For the bremsstrahlung

process ($\pi^- p \rightarrow p \pi^- e^+ e^-$), a peak at the position of pion mass is expected in the distribution, however, no statistically significant peak is visible.

Both $\pi^- e^+ e^-$ and $p e^+ e^-$ hypotheses show the evidence of the $p \pi^- e^+ e^-$ final state selection, however, missing mass is well identified when hypothesis contains negative pion as a measured particle ($\pi^- e^+ e^-$ hypothesis).

Table 5.3 provides statistics for the three-particle identification ($p e^+ e^-$), for both targets $(C_2H_4)_n$ and C, and for the region above $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$. The combinatorial background (CB) is included. The statistics of events is slightly higher than in the case of the $\pi^- e^+ e^-$ identification (see Table 5.3).

	$(C_2H_4)_n$		Carbon	
Type	Signal	CB	Signal	CB
BT method	907	256	538	169
BT method $M_{Inv}(e^+ e^-) > 0.140 \text{ GeV}/c^2$	70	36	49	13

Table 5.4.: Statistics of reconstructed 3-prong events ($p e^+ e^-$) in measurements with $(C_2H_4)_n$ and C targets. The signal is obtained after the combinatorial background subtraction. The first row in table corresponds to the number of events with dileptons reconstructed in the RICH by means of the backtracking (BT) algorithm and in the case of the second row the cut on invariant mass $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ was used in addition to the BT algorithm.

5.7.3 Two-prong e^+e^- analysis

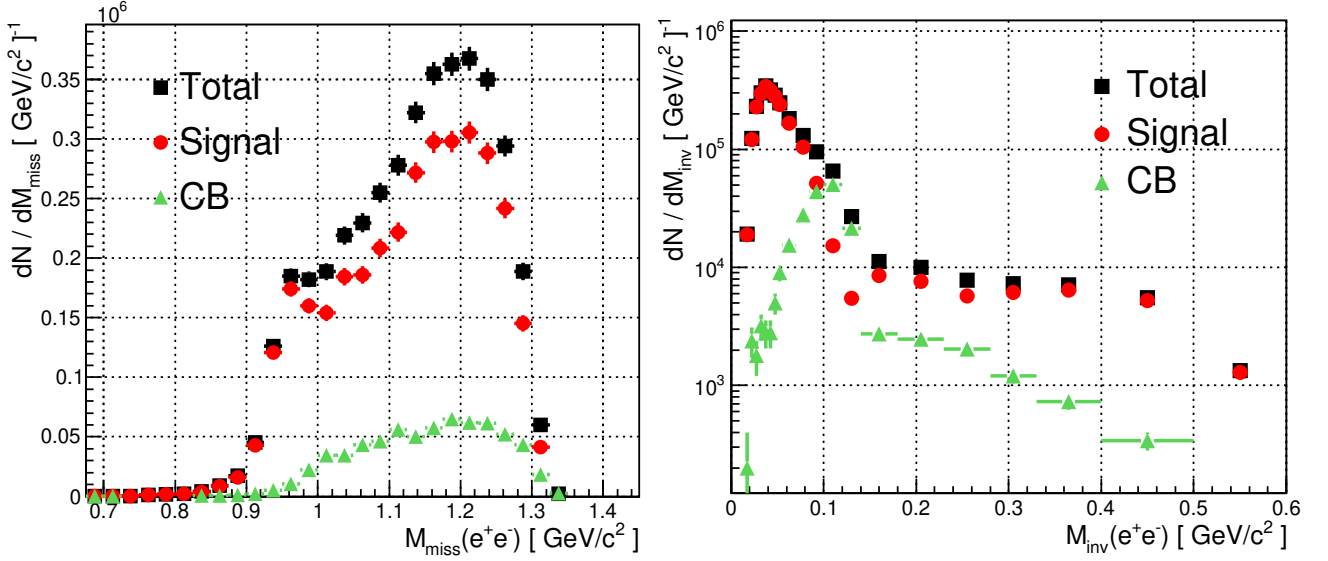


Figure 5.23.: Inclusive e^+e^- spectra: missing mass (left panel) and invariant mass (right panel) for measurement with polyethylene target. Black squares represent all e^+e^- events (total), green triangles – the combinatorial background, and red dots represent e^+e^- pairs remaining after the CB subtraction (signal).

The investigation of dielectron production was also carried out by the identification of the e^+e^- pairs in the final state which is symbolically indicated as $e^+e^- X$ channel, where X can be one or more unidentified particles. The e^+e^- missing mass spectrum for the measurements with the polyethylene target presented in Fig. 5.23, left panel shows a two peak structure with one peak at the nucleon mass and the other peak at about 1.2 GeV/c^2 . Figure 5.23 (right panel) shows the e^+e^- invariant mass distribution with a dominating contribution from π^0 Dalitz decay (see results of the Monte Carlo simulation presented in Chapter 6) in the range below the π^0 mass. At higher masses the distribution is almost flat.

The peak visible at the nucleon mass in the missing mass spectrum for the polyethylene target, is not visible in the carbon target (see Fig. 5.24 - left panel). In turns, the e^+e^- invariant mass spectra for two targets are very similar in shape as shown in Fig. 5.24 - right panel.

In the e^+e^- missing mass spectrum (Fig. 5.25, left panel), where the cut: $M_{\text{inv}}(e^+e^-) > 0.14 \text{ GeV}/c^2$ is applied, a prominent peak around a neutron mass is clearly reconstructed. The production mechanisms contributing to the $n e^+e^-$ final state are tightly related to the channels identified in the two-pion hadronic analysis, (see Ref. [80]). The dominant process is the Dalitz decay of $N(1520) \rightarrow N e^+e^-$ with an intermediate ρ meson coupling to the resonance and to the virtual photon (see Ref. [33]), which provides a good explanation for data obtained from the inclusive analysis (see Chapter 6).

The bremsstrahlung contribution can be identified only with the help of Monte Carlo simulations, as it will be discussed in the next chapter. Figure 5.25, right panel, shows that the carbon contribution in the measurement with polyethylene target is significant and due to statistics cannot be subtracted but rather included into the model descriptions.

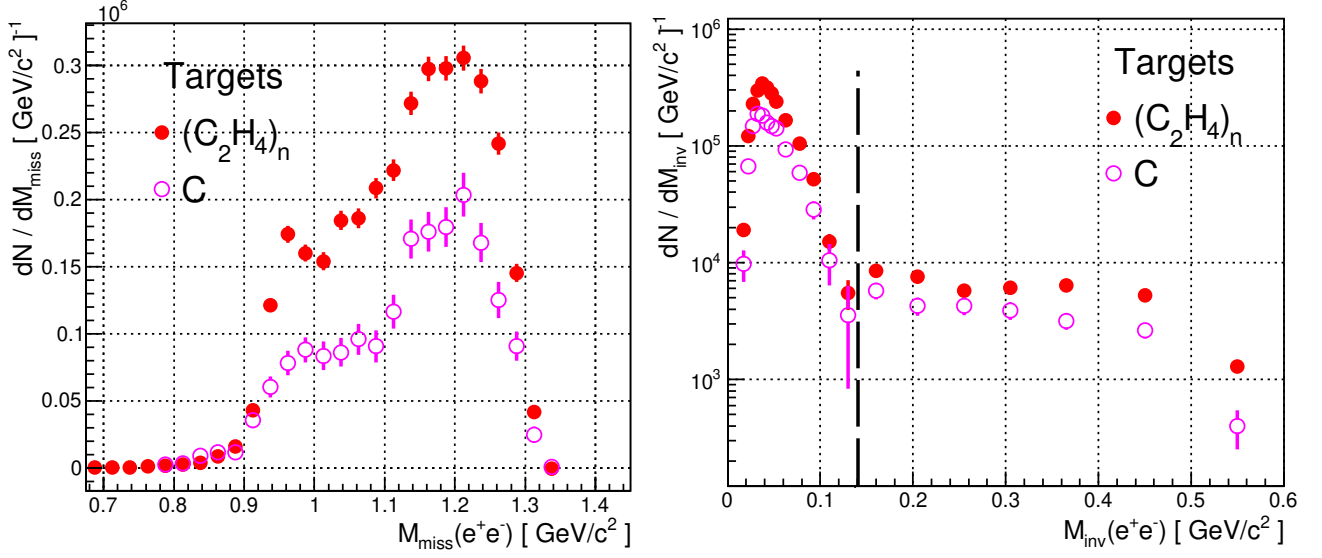


Figure 5.24.: Comparison of e^+e^- missing mass spectra (left panel) and e^+e^- invariant mass spectra (right panel) determined in measurements with polyethylene target (red full dots) and carbon target (magenta open dots). Normalization to the bin width has been applied. In the right panel, vertical black dashed line is the upper kinematical limit of π^0 Dalitz decay contribution.

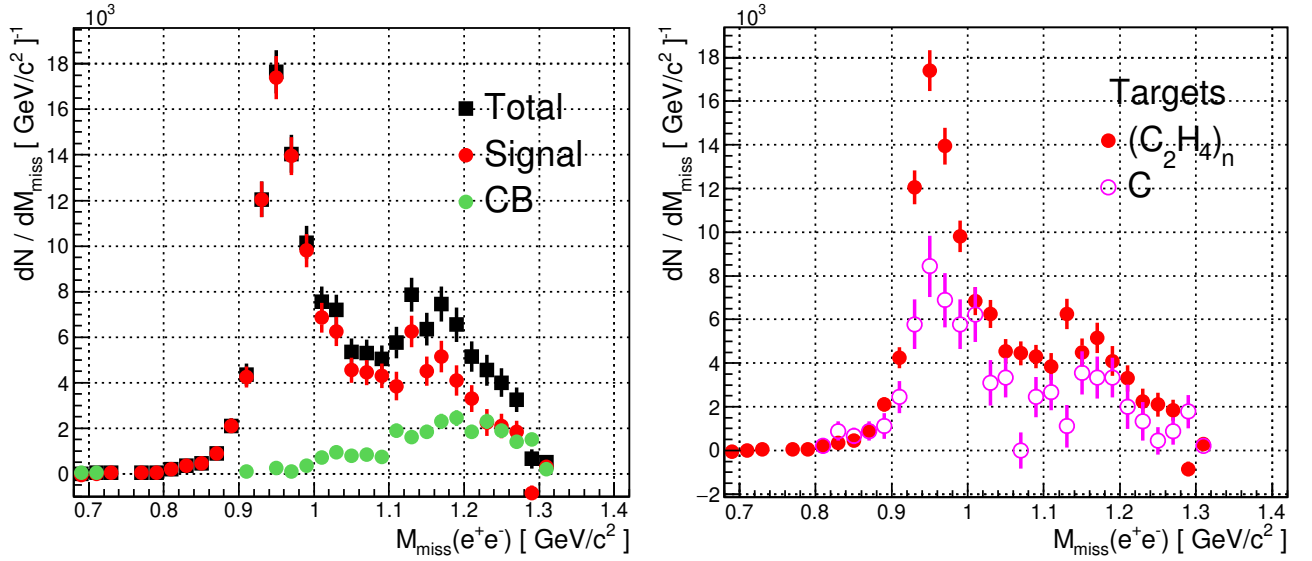


Figure 5.25.: e^+e^- missing mass distributions after the cut $M_{\text{inv}}(e^+e^-) > 0.14 \text{ GeV}/c^2$ for polyethylene target (left panel) and for comparison of polyethylene and carbon events (right panel). Left panel: all e^+e^- events (black squares, total), combinatorial background (green triangles) and signal (red full dots) are presented. Right panel: red full dots and magenta open dots represent measurements with polyethylene and carbon target, respectively. Normalization to the bin width has been applied.

Table 5.5 provides statistics for the e^+e^-X events for both targets. The total counts for polyethylene and carbon target equal to 16880 and 9566, respectively, are about 20 times higher than the corresponding statistics for the $\pi^-e^+e^-$ 3-prong events (see Table 5.3). For invariant masses higher than $M_{\text{inv}}(e^+e^-) > 0.14 \text{ GeV}/c^2$, the number of events for polyethylene and carbon target is 2416 and 1377, respectively, and is about 40 times higher than in the 3-prong analysis.

	$(C_2H_4)_n$		Carbon	
Type	Signal	CB	Signal	CB
BT method	16880	3360	9566	2053
BT $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$	2416	484	1377	222

Table 5.5.: Statistics of reconstructed 2-prong events ($e^+ e^-$) in measurements with $(C_2H_4)_n$ and C targets. The signal is obtained after the combinatorial background subtraction. The first row in table corresponds to the number of events with dileptons reconstructed in the RICH by means of the backtracking (BT) algorithm and in the case of the second row the cut on invariant mass: $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ was used in addition to the BT algorithm.

5.8 Polyethylene to carbon ratio

In order to verify the deduced scaling factors of cross-sections for $\pi^- p$ collisions to the case of $\pi^- C$ collisions, presented in Chapter 4.3.2, we determined experimental ratio of dilepton events measured with the polyethylene and carbon target. This ratio is presented in Fig. 5.26 for the four considered hypotheses and no restriction on the e^+e^- invariant mass. Since the analyzed events originate predominately from the π^0 Dalitz decay, the carbon to proton scaling factor is $r = \sigma(C)/\sigma(p) = 3.3$ (see Chapter 4.3.2). Taking into account that the polyethylene target contains twice as many hydrogen atoms as carbon atoms, the ratio of the e^+e^- yield with polyethylene and carbon target is calculated as:

$$\frac{\sigma(PE)}{\sigma(C)} = \frac{2\sigma(p) + \sigma(C)}{\sigma(C)} = \frac{2 + r}{r} = 1.61. \quad (5.8.1)$$

The obtained value of the polyethylene to carbon ratio, indicated with red dashed line in Fig. 5.26, is in agreement with the experimental results.

The ratio of the reconstructed e^+e^- events with $(C_2H_4)_n$ and carbon target but for the e^+e^- invariant mass above the π^0 mass region is presented in Fig. 5.27 for three different hypotheses. The ratio was not calculated for the $\pi^- p e^+e^-$ hypothesis as no events were registered with the carbon target. For the exclusive e^+e^- production in the invariant mass range above the π^0 mass, we expect the dominant contribution from the bremsstrahlung process. According to the calculations presented in Chapter 4, the carbon to proton scaling factor for the $\pi^- e^+e^-$ and e^+e^- hypothesis is 4.4 and for the $p e^+e^-$ hypothesis is 3.4. Taking these factors into account, the polyethylene to carbon ratio is calculated using the formula:

$$\frac{\sigma(PE)}{\sigma(C)} = \frac{2 \sigma(p) (\epsilon^p \times A^p) + \sigma(C) (\epsilon^C \times A^C)}{\sigma(C) (\epsilon^C \times A^C)} = \frac{2 + \frac{\sigma(C)}{\sigma(p)} \frac{(\epsilon^C \times A^C)}{(\epsilon^p \times A^p)}}{\frac{\sigma(C)}{\sigma(p)} \frac{(\epsilon^C \times A^C)}{(\epsilon^p \times A^p)}}, \quad (5.8.2)$$

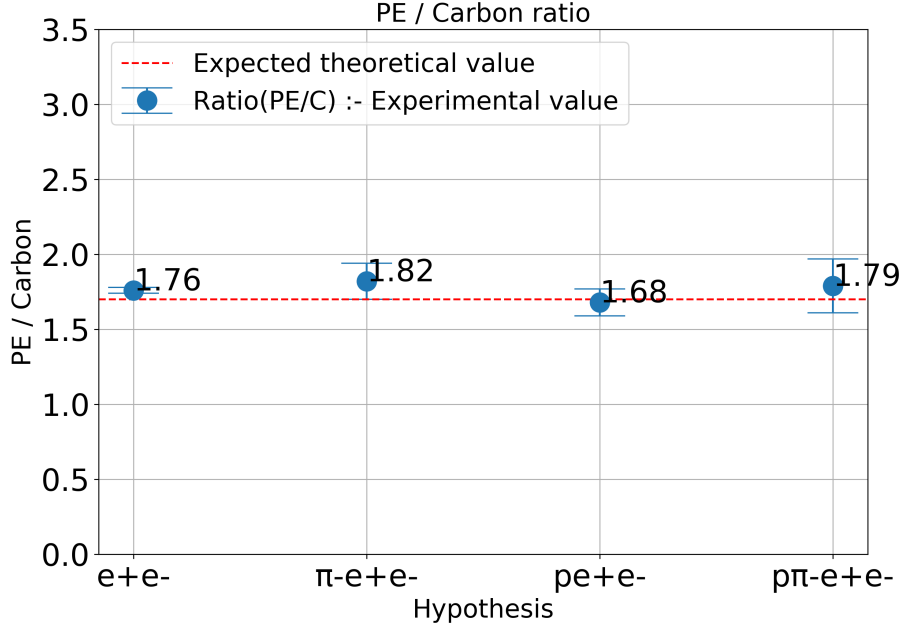


Figure 5.26.: Ratio of the reconstructed dilepton events for $(C_2H_4)_n$ and C target, for four different hypotheses (blue points) compared to result based on the proton to carbon scaling factor (red dashed line).

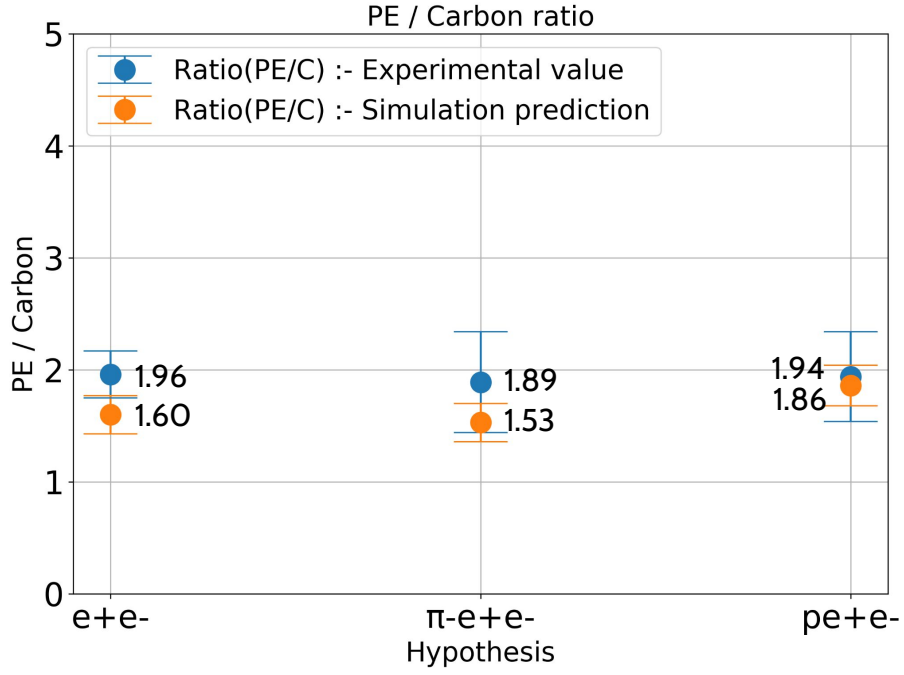


Figure 5.27.: Ratio of the reconstructed dilepton events above π^0 mass region for $(C_2H_4)_n$ and C target, for three different hypotheses. The values obtained from the experimental data and simulations are presented with blue and orange points, respectively.

where $(\epsilon^p \times A^p)$ and $(\epsilon^C \times A^C)$ are the product of detection efficiency and acceptance for the proton and carbon target, respectively. The acceptance and efficiency values were determined in the simulations of the bremsstrahlung process and are given in Table 5.6. The ratios obtained from Eq. 5.8.2 agree well with experimental values as shown in Fig. 5.27.

	Reaction channel (Acceptance $A \times$ Efficiency ϵ)		
	$\pi^- p$	$\pi^- {}^{12}\text{C}$	
Hypotheses			Ratio: $\frac{\pi^- {}^{12}\text{C}}{\pi^- p}$
$e^+ e^-$	6.89 %	5.25 %	0.76 ± 0.06
$\pi^- e^+ e^-$	0.98 %	0.83 %	0.85 ± 0.21
$p e^+ e^-$	3.91 %	3.02 %	0.68 ± 0.10

Table 5.6.: Acceptance times efficiency determined in simulations of e^+e^- bremsstrahlung events for π^- beam interacting with proton (2nd column) and carbon target (3rd column) for three studied hypothesis. The last (4th) column contains the ratio of the corresponding products together with statistical uncertainty resulting from limited numbers of simulated events.

Chapter 6

Data versus simulations

Precision and exactness are not intellectual values in themselves, and we should never try to be more precise or exact than is demanded by the problem in hand.

Karl Raimund Popper

The aim of this chapter is to interpret the obtained data by comparing them with predictions of theoretical models. The procedure takes into account the acceptance and efficiency of the HADES detection system determined by the Monte Carlo simulations. Section 6.1 presents the simulation results for the processes giving dominant contributions to the studied exclusive channels. Section 6.2 gives an interpretation of the experimental spectra for the e^+e^-X inclusive channel, and the next three sections, 6.3, 6.4 and 6.5, present interpretation of obtained spectra for the exclusive channels, $\pi^- p e^+e^-$, $\pi^- e^+e^-$ and $p e^+e^-$, respectively. The estimation of the total cross section for the bremsstrahlung process is presented in Section 6.6.

6.1 Simulations of $\pi^- p \rightarrow \pi^- p e^+e^-$ and $\pi^- p \rightarrow \pi^- p \gamma e^+e^-$

The following three intermediate states and decay processes were taken into account in the Monte Carlo simulations of the $\pi^- p \rightarrow p \pi^- e^+e^- (\gamma)$ reaction channel:

- (a) $p \pi^- \pi^0 (\gamma e^+ e^-)$,
- (b) $\pi^- \Delta^+ (p e^+ e^-)$,
- (c) Bremsstrahlung process: $\pi^- p e^+ e^-$.

The results of measurements of the two-pion production (a) performed at HADES are presented in detail in [80]. The obtained data was described by means of the partial wave analysis. The obtained parametrization of the data is used in the present work in the event generator to model $p \pi^- \pi^0$ reaction with included π^0 Dalitz decay. Also, the Δ^+ resonance contribution is obtained from partial wave analysis which is used to model the reaction $\pi^- \Delta^+ (p e^+ e^-)$ (b). The interesting channel of bremsstrahlung process (c) is modeled with the soft-photon approximation, as described in Chapter 1.5.3. In all (a)-(c) cases, the final state events were generated within the PLUTO event generator [75,76], and were multiplied by the weights calculated from the appropriate model.

Figure 6.1 presents differential cross-section as a function of e^+e^- invariant mass for π^0 Dalitz decay and Δ^+ Dalitz decay derived from the partial wave solution [100]. Also, the cross-section for bremsstrahlung process calculated by using the soft-photon approximation is presented.

One can see in Figure 6.1 that the Δ^+ Dalitz decay contribution is lower than the bremsstrahlung soft-photon approximation by about two order of magnitude. Besides, the extraction of the bremsstrahlung signal in the range of invariant mass below π^0 mass is difficult, as the obtained cross section for the π^0 channel is higher by roughly an order of magnitude than the bremsstrahlung cross section. Therefore,

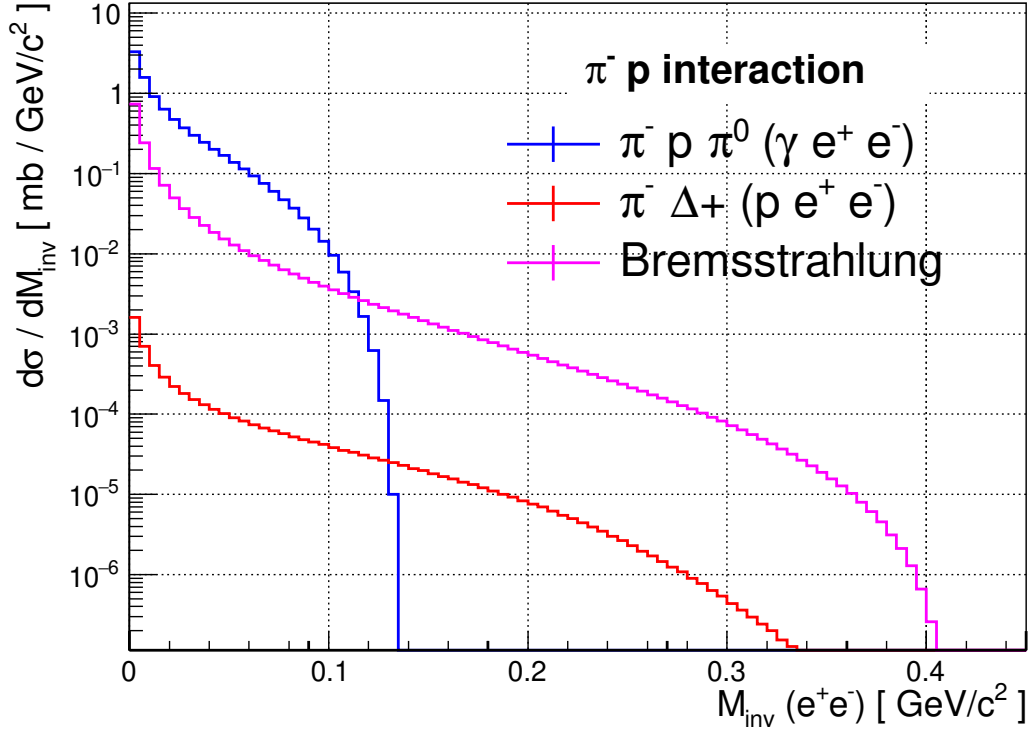


Figure 6.1.: $\pi^- p$ reaction at $p_{beam} = 0.685$ GeV/c: differential cross-section as a function of e^+e^- invariant mass for π^0 Dalitz decay (blue histogram), Δ^+ Dalitz decay (red histogram) and the soft-photon approximation of bremsstrahlung process (magenta histogram).

in order to address the $p \pi^- e^+ e^-$ final state free from the π^0 Dalitz decay contribution, one has to constraint it to the invariant mass above the π^0 mass ($M_{inv}(e^+e^-) > 0.14$ GeV/c²). However, the $M_{inv}(e^+e^-) < 0.14$ GeV/c² region and the reconstruction of π^0 Dalitz decay signal is a very important reference channel which helps to demonstrate that the reconstruction of dielectron pairs as well as hadronic tracks (p , π^- here) is under control from the point of view of the HADES spectrometer modelling. For this, the present data on the e^+e^- production in the invariant mass range below the π^0 mass is compared with the results of the partial wave analysis of the $p \pi^- \pi^0$ final state with included Dalitz decay of π^0 (see Section 6.3). Analogous procedure was successfully applied in the analysis of the Δ^+ Dalitz decay, in the reaction $p p \rightarrow p \Delta^+(p e^+ e^-)$ [86]. There was first demonstrated that the π^0 meson contribution, obtained within the partial wave analysis of the one-pion final state, can be well described also in the dielectron channel, with the π^0 Dalitz decay. The verification of various projections, including angular distributions, is of utmost importance, due to the finite spectrometer acceptance.

6.2 Inclusive $\pi^- p \rightarrow e^+ e^- X$ channel

In analysis of the $e^+e^- X$ inclusive channel, the interesting outgoing particles are e^+ and e^- , and all other particles are not reconstructed.

Figure 6.2 presents the e^+e^- missing mass spectrum obtained with $(C_2H_4)_n$ target after applying $M_{inv}(e^+e^-) > 0.14$ GeV/c² cut. The peak at the neutron mass is well explained by the results of simulations including the excitation of the N(1520) and its decays in $n e^+e^-$ channel. At higher missing mass values, the production of η meson in the process $\pi^- p \rightarrow n \eta$ with the subsequent η meson Dalitz decay (γe^+e^-) contributes. Details of the excitation and decay of the N(1520) resonance as well as the

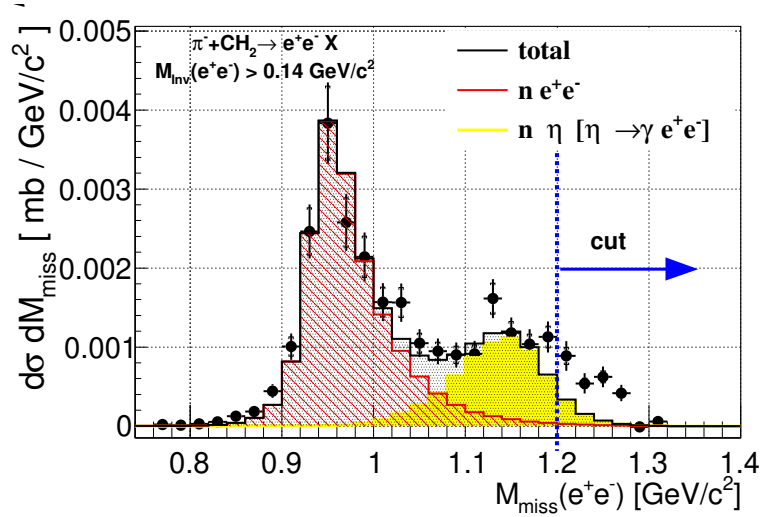


Figure 6.2.: e^+e^- missing mass (M_{miss}) spectra after applying the cut $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$. Data (black dots) are compared with full simulation results (black histogram) which contain $n e^+ e^-$ resonance channel (red histogram), and η Dalitz decay (yellow histogram). Blue dotted line defines cut $M_{miss}(e^+e^-) > 1.2 \text{ GeV}/c^2$ used for selection of bremsstrahlung events.

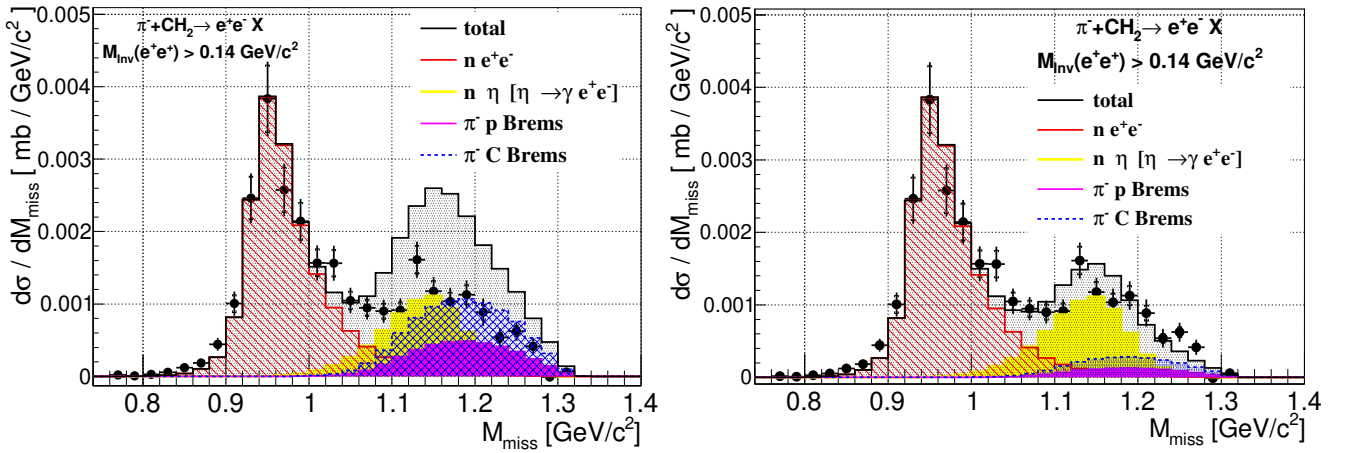


Figure 6.3.: e^+e^- missing mass (M_{miss}) spectra after applying the cut $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$. Data (black dots) are compared with full simulation results (black histogram) which contain $n e^+ e^-$ resonance channel (red histogram), η Dalitz decay (yellow histogram) and bremsstrahlung contribution ($\pi^- p$ – magenta, and $\pi^- C$ – blue histogram). Bremsstrahlung simulations are presented without scaling factor (left panel) and with the scaling factor – 4.05 (right panel).

$\eta(\gamma e^+e^-)$ Dalitz decay were taken from Ref. [33]. The data points above $1.2 \text{ GeV}/c^2$ are underestimated by the η Dalitz decay process. Events corresponding to $M_{miss}(e^+e^-) > 1.2 \text{ GeV}/c^2$ are possible candidates for bremsstrahlung process. The contribution of bremsstrahlung is described by the $\pi^- p \rightarrow \pi^- p e^+e^-$ process occurring in $\pi^- p$ and $\pi^- C$ collisions using the soft-photon approximation (see chapter 4). However, it overshoots the data significantly, (see Fig. 6.3, left panel). After scaling down the bremsstrahlung contribution by the factor 4.05, which is the same as the one applied in the case of the exclusive $\pi^- e^+e^-$ channel (see Sec. 6.4), the experimental data points are well described by the simulations (see Fig. 6.3, right panel).

Figure 6.4 presents invariant mass spectra $M_{inv}(e^+e^-)$ for the missing mass cut $M_{miss}(e^+e^-) > 1.2 \text{ GeV}/c^2$ (black dots), for the measurements with $(C_2H_4)_n$ target (left panel), C target (middle panel), and after subtraction of carbon contribution (right panel). Inclusive π^0 yield is a total sum of three dominant

Reaction	Cross-section
$\pi^- p \rightarrow n \pi^0$	9.0 mb
$\pi^- p \rightarrow n \pi^0 \pi^0$	2.0 mb
$\pi^- p \rightarrow p \pi^- \pi^0$	3.9 mb
Total	15.0 mb

Table 6.1.: Dominant π^0 production channels in the $\pi^- p$ collisions together with corresponding total cross-section values, taken from Ref. [33].

contributions of processes specified in Table 6.1. Simulations including these processes reproduce well the experimental data points in the invariant mass range below the π^0 mass. At higher invariant masses, the contribution from bremsstrahlung scaled by the factor 4.05 together with the η Dalitz decay contribution describe the data points well.

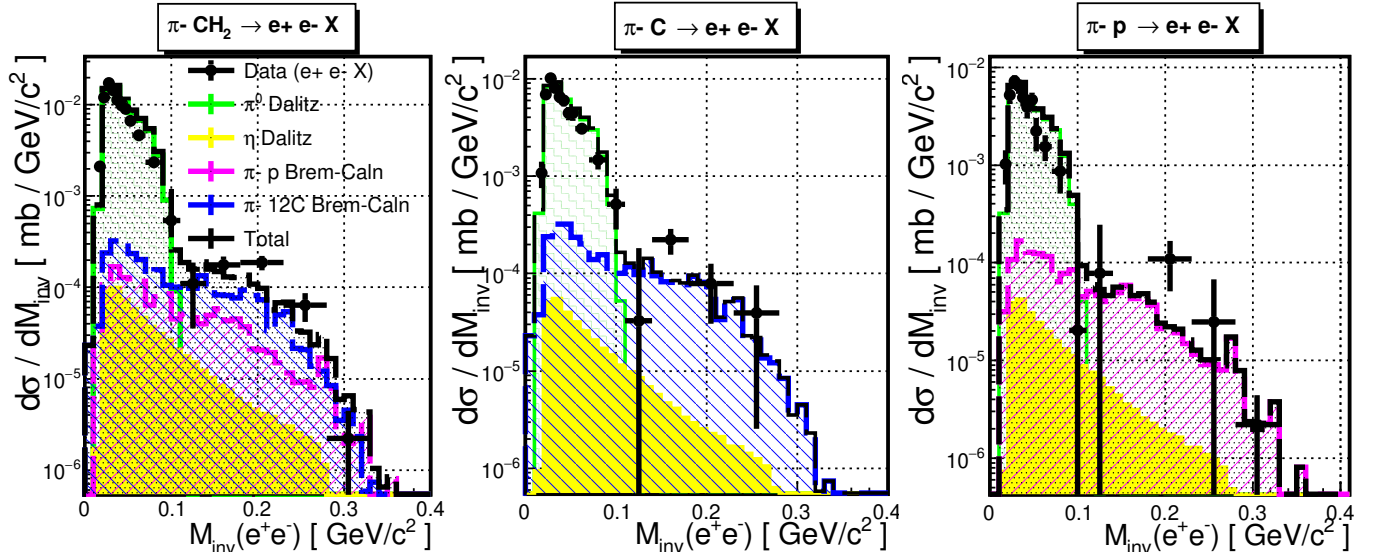


Figure 6.4.: e^+e^- invariant mass distribution with the missing mass cut $M_{miss}(e^+e^-) > 1.2 \text{ GeV}/c^2$ for $\pi^- (\text{C}_2\text{H}_4)_n$ (left panel), $\pi^- \text{C}$ (middle panel) and $\pi^- p$ (right panel) reactions. Data denoted by black points are compared with full simulation results (black histogram) which contain π^0 (green), bremsstrahlung contribution, ($\pi^- p$, magenta and $\pi^- \text{C}$, blue) and η Dalitz decay (yellow).

6.3 Exclusive $\pi^- p \pi^0(\gamma e^+e^-)$ channel

The analysis of the π^0 Dalitz decay is used to demonstrate that the reconstruction of dielectron pairs is under control from the point of view of the HADES spectrometer modelling. For this, the experimental data for the $\pi^- p \rightarrow \pi^- p \pi^0(\gamma e^+e^-)$ channel are compared to simulation results based on the partial wave analysis of the two pion production channel ($\pi^- p \rightarrow \pi^- p \pi^0$) [80] supplemented by the π^0 Dalitz decay.

6.3.1 Missing mass distributions

The $\pi^0 \rightarrow \gamma e^+ e^-$ Dalitz decay is the dominant contribution in the measured spectrum of dielectrons. It can be identified in a complete kinematical reconstruction of the final state $\pi^- p \gamma e^+ e^-$, where the four final state charged particles are measured. To extract the signal for the π^0 Dalitz decay, a cut on the two particle missing mass: $M_{miss}(\pi^- p) > 0 \text{ GeV}/c^2$ and on the invariant mass: $M_{inv}(e^+e^-) < 0.14 \text{ GeV}/c^2$ was applied. Figure 6.5 presents a correlation of two-particle missing mass $M_{miss}(\pi^- p)$ versus four-particle missing mass in the measurement with the $(C_2H_4)_n$ target, clearly indicating the area with the π^0 Dalitz decay contribution.

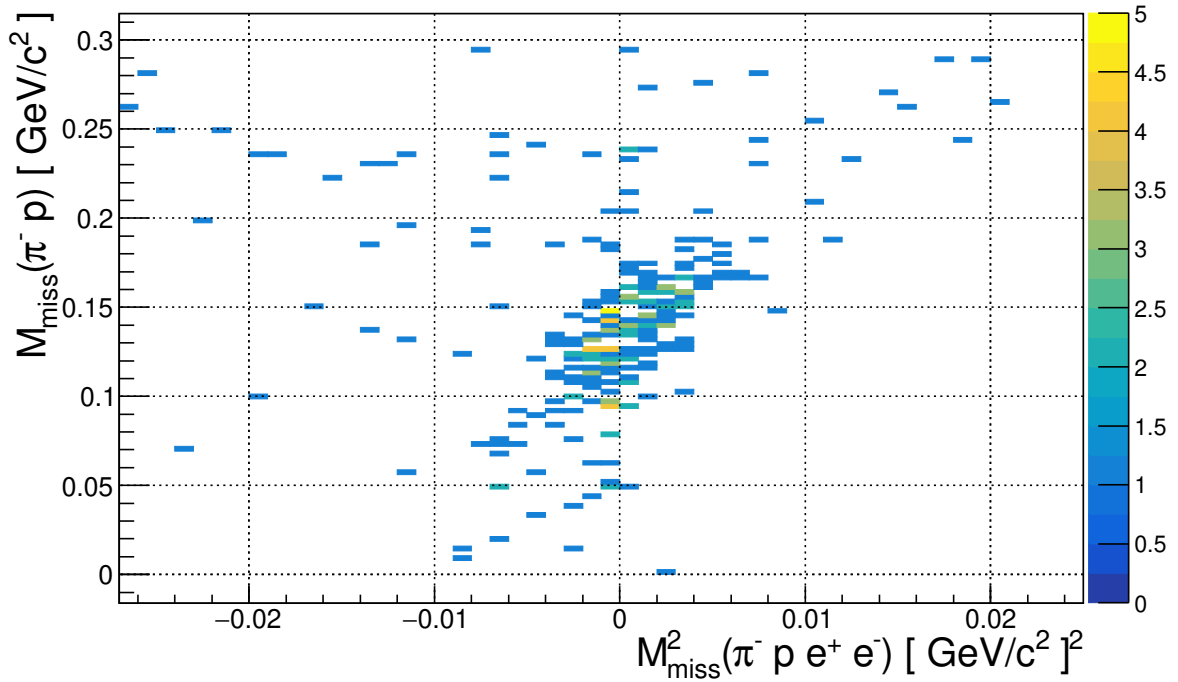


Figure 6.5.: $M_{miss}(\pi^- p)$ vs $M_{miss}^2(\pi^- p e^+ e^-)$ for $(C_2H_4)_n$ target.

Fig. 6.6 shows $M_{miss}(\pi^- p)$ and $M_{miss}^2(\pi^- p e^+ e^-)$ distributions after subtracting the carbon contribution from $(C_2H_4)_n$ target. The partial wave analysis of the two pion production channel with included π^0 Dalitz decay overshoots the data but the widths of missing mass distributions are reproduced well.

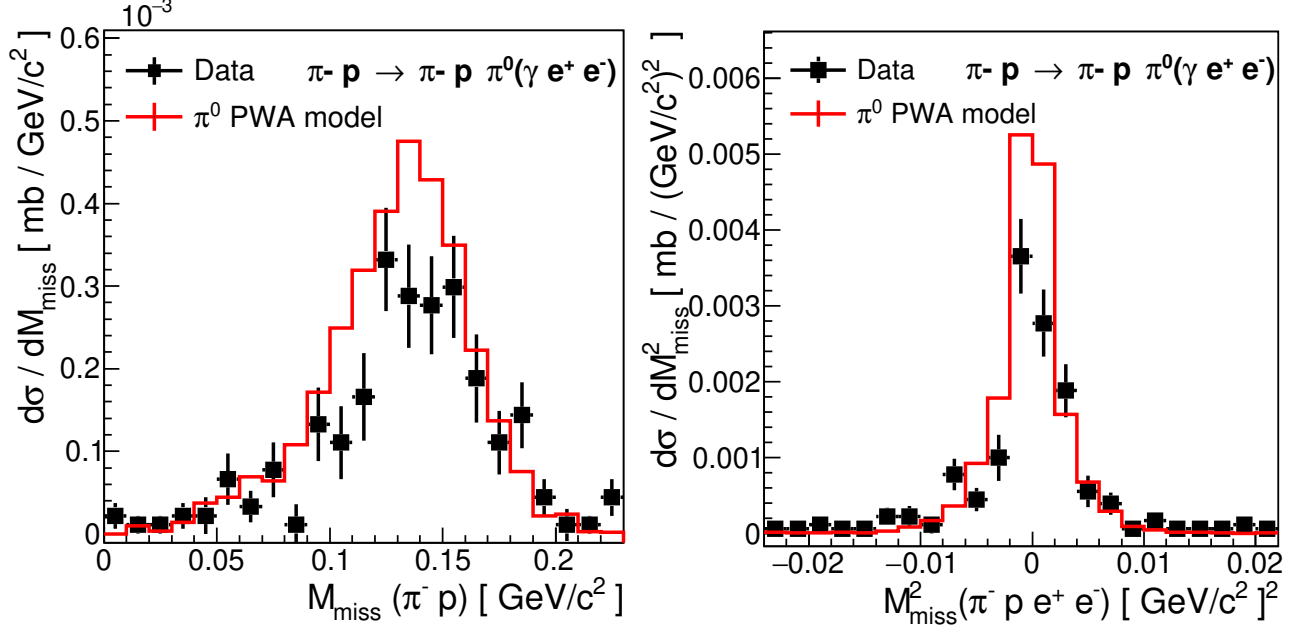


Figure 6.6.: Two-particle missing mass $M_{miss}(\pi^- p)$ (left panel) and four-particle missing mass squared $M_{miss}^2(\pi^- p e^+ e^-)$ (right panel) for the $\pi^- p \rightarrow \pi^- p \gamma e^+ e^-$ reaction channel. Data is presented with black squares and red histogram shows the simulation of π^0 Dalitz decay.

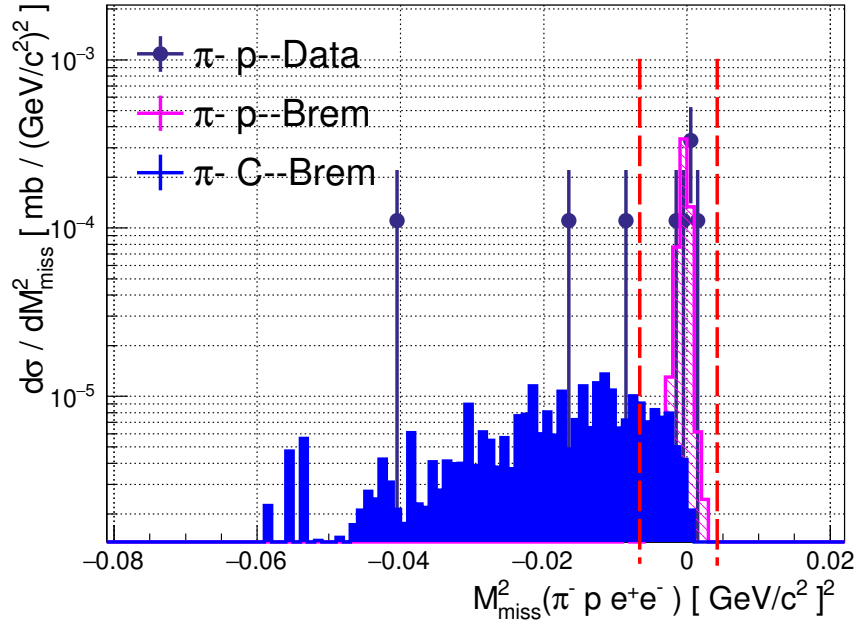


Figure 6.7.: Four-particle missing mass squared $M_{miss}^2(\pi^- p e^+ e^-)$ in measurement with $(C_2H_4)_n$ target after applying $M_{inv}(e^+ e^-) > 0.14$ GeV/c² cut. The magenta and blue histograms represent the $\pi^- p$ and $\pi^- C$ bremsstrahlung contributions, respectively. The vertical red dashed lines indicate the region selected for calculating the $\pi^- p$ bremsstrahlung cross-section.

Fig. 6.7, shows the comparison of $M_{miss}^2(\pi^- p e^+ e^-)$ distributions for proton, obtained after subtracting contribution of carbon from polyethylene data and simulation for $\pi^- p \rightarrow \pi^- p e^+ e^-$ bremsstrahlung channel. Data and simulation both are plotted above the π^0 mass region.

6.3.2 e^+e^- distributions

Figure 6.8 presents experimental distributions of the e^+e^- invariant mass ($M_{inv}(e^+e^-)$), rapidity (y), and transverse-momentum (p_T) compared with the simulations of the π^0 Dalitz decay. The experimental data points are well reproduced by the simulations.

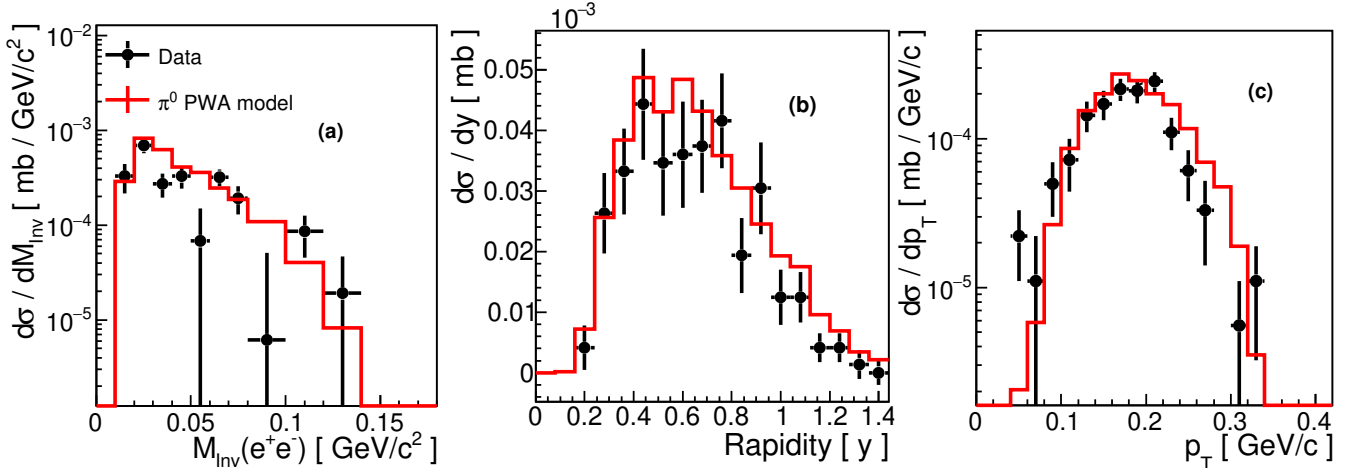


Figure 6.8.: Distribution of e^+e^- invariant mass (a), rapidity (b) and transverse momentum (c) for the $\pi^-p \rightarrow \pi^-p \gamma e^+e^-$ reaction channel, obtained after subtracting carbon contribution from $(C_2H_4)_n$ data. Data is presented with black dots and simulation results with red histogram.

6.3.3 Angular distributions

In this section, experimental angular distributions for the three final state particles: $p \pi^- \pi^0$ are compared with the simulations based on the partial wave analysis solution. This comparison is important in view of the finite acceptance and efficiency of the HADES detection system in the measurements of charged hadron tracks but also of dileptons originating from the π^0 Dalitz decay. The angular distributions are studied in three different reference frames: center-of-mass (CM) system, the Gottfried-Jackson (GJ) frame, and the helicity (H) frame.

Center-of-mass system

In the center-of-mass (CM) system the total momentum is zero and projectile and target particles have equal momenta oriented in opposite directions. Figure 6.9 presents $\cos \theta^{CM}$ distribution for the reconstructed p , π^- , and π^0 from $(C_2H_4)_n$ and C target. The π^0 momentum was determined as the missing momentum of the $p \pi^-$ system. The experimental cross-sections for carbon are much smaller as compared to the $(C_2H_4)_n$. Therefore we conclude, that the dominant contribution to the cross-section measured with the $(C_2H_4)_n$ target originates from the pion interaction with protons in $(C_2H_4)_n$. Also for this reason the angular distributions measured with $(C_2H_4)_n$ are well described by the simulation of the $\pi^-p \rightarrow \pi^-p \pi^0$ with included π^0 Dalitz decay.

Figure 6.10 presents $\cos \theta^{CM}$ distribution for the reconstructed p , π^- , and π^0 , after subtracting carbon contribution from $(C_2H_4)_n$ target. For the most forward and backward angles the data is overestimated by the simulation, however at the intermediate angles the agreement with the simulation is much better. It is remarkable that all of the studied angular distributions are approximately symmetric with respect to

90° and are strongly peaked in forward and backward direction. The underlying reaction mechanism was studied within meson exchange models including excitation of intermediate resonances in Ref. [80].

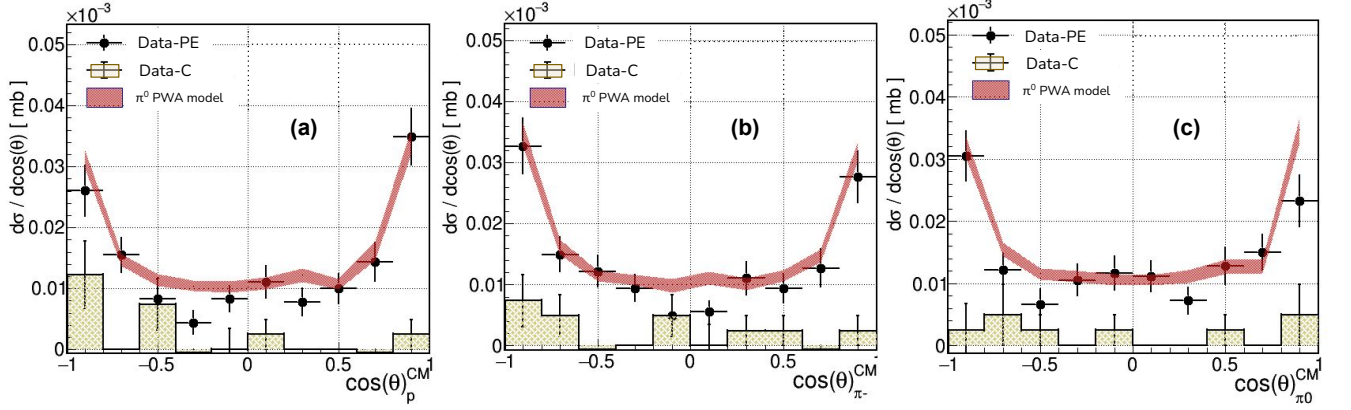


Figure 6.9.: Angular distribution for protons (a), π^- (b) and π^0 (c) in CM frame in measurements with $(C_2H_4)_n$ (black dots) and C (brown histogram bars) target. The data points are compared with the PWA simulation results for the $\pi^- p \rightarrow \pi^- p \pi^0$ channel, represented by red band.

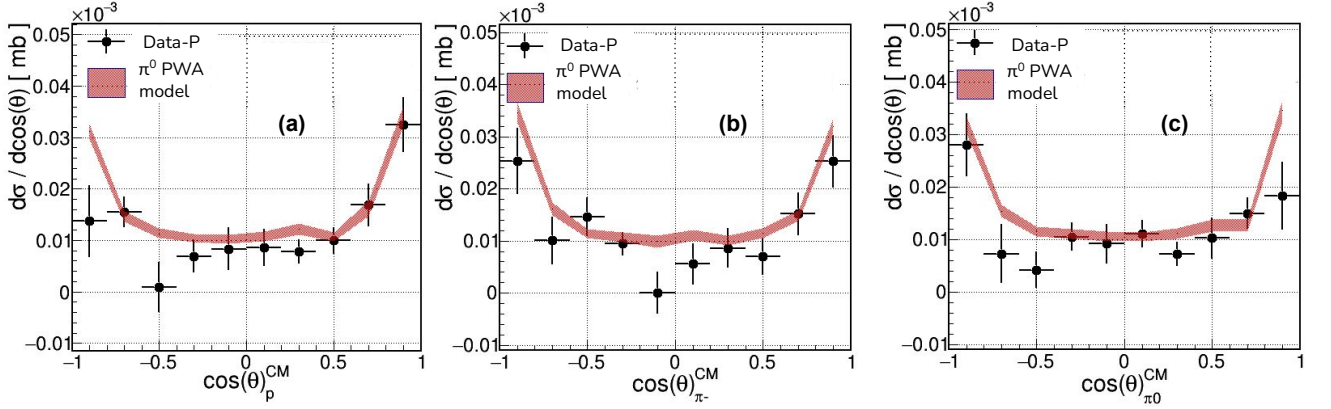


Figure 6.10.: Angular distribution for protons (a), π^- (b) and π^0 (c) in CM frame after subtracting carbon contribution from $(C_2H_4)_n$ target. The data points are compared with the PWA simulation results for the $\pi^- p \rightarrow \pi^- p \pi^0$ channel (red band).

The rest frame of two out of three particles in the final state is the Gottfried-Jackson (GJ) frame. The GJ angle is defined in this frame as angle between one of the two particles in the rest frame and the beam particle (here pion) boosted to the rest frame. In this way, the two-body scattering, e.g. $\pi^- p$ interaction, with a particle exchanged between them during the time of interaction can be studied using the associated distribution. For example, for the rest frame defined by the final state $p \pi^-$ pair, the GJ angle between the incoming pion projectile and the proton in the rest frame is represented by $\theta_{\pi^- p}^p$. For the three particle final state, three combinations of the two-particle rest frames are possible and for each rest frame two different GJ angles can be determined. Together, this gives six different GJ angles.

Figure 6.11 presents distributions of six GJ angles for the $\pi^- p \rightarrow \pi^- p \pi^0$. Each row corresponds to

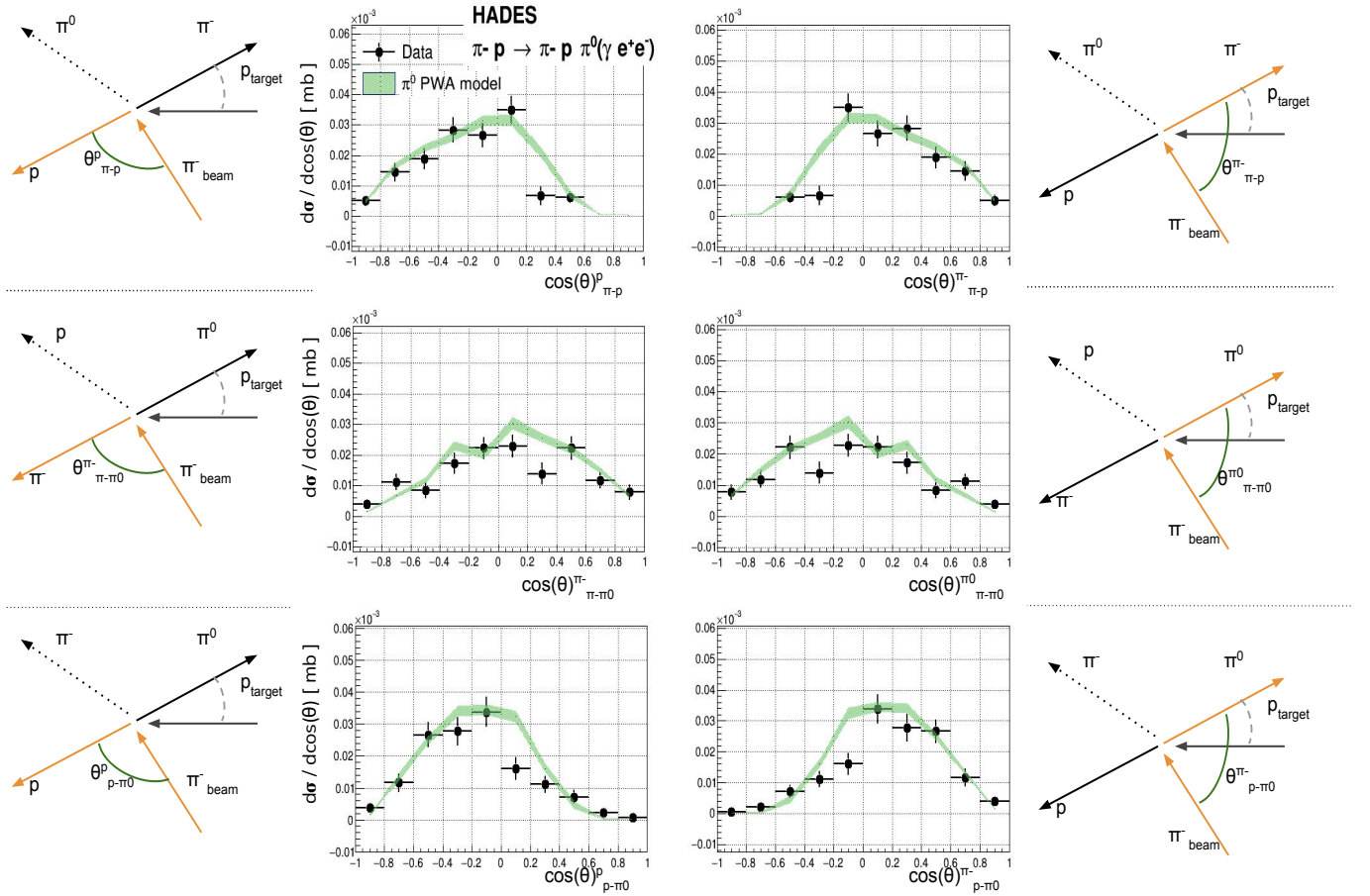


Figure 6.11.: Differential cross-sections for $\pi^- p \rightarrow \pi^- p \pi^0$ ($\gamma e^+ e^-$) channel in function of the GJ angles in the $p \pi^- \pi^0$ state. The GJ angles are defined in the text. Data points (black dots) are compared with PWA solution represented by green band, where thickness of the band defines statistical error [88].

one rest frame and two columns represent two respective angles in that frame. Two distributions in each row – the left and right one – are mirror reflections of each other, since the sum of the corresponding GJ angles equals 180° . Angular distributions in the GJ frame are well suited for the study of the production mechanism proceeding via two particle intermediate state. According to the PWA of the $\pi^- p \rightarrow \pi^- p \pi^0$ reaction [80], the dominant contribution to the $\pi^- \pi^0$ pair production comes from the excitation of the intermediate $\Delta(1232)$ resonance decaying into a pion-nucleon and from the production of the intermediate ρ meson decaying into two pions. The presented angular distributions of the data agree well with the simulation based on the PWA solution.

Helicity frame

The helicity (H) frame is also the rest frame of two out of the three particles in the final state but the helicity angle is defined between one of the rest frame particles and the third particle in the final state, boosted in this rest frame. For example, for the rest frame defined by the final state $p \pi^-$ pair, the helicity angle between the outgoing π^0 and the π^- in the rest frame is represented by $\theta_{\pi^0 \pi^-}^{\pi^- p}$. For the three particle final state, three different two particle rest frames are possible and for each rest frame two different helicity angles can be determined. Together, this gives six different helicity angles. Figure 6.12 presents distributions of the six helicity angles for the $\pi^- p \pi^0$ final state. Each row corresponds to one rest frame and two columns represent to two respective angles in that frame.

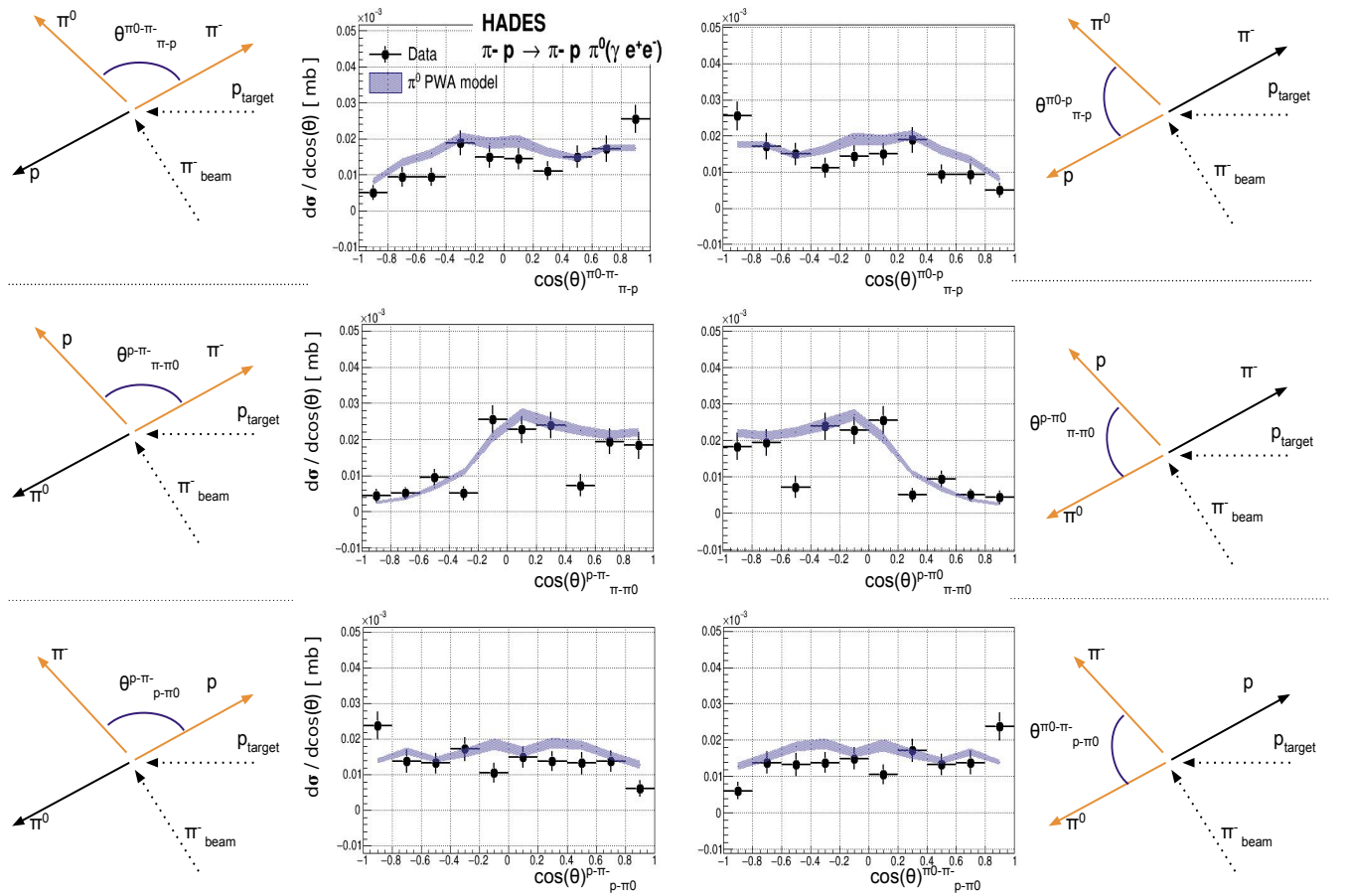


Figure 6.12.: Differential cross-sections for $\pi^- p \rightarrow \pi^- p \pi^0$ ($\gamma e^+ e^-$) channel in function of the H angles in the $p \pi^- \pi^0$ state. The H angles are defined in the text. Data points (black dots) are compared with PWA solution represented by blue band, where thickness of the band defines statistical error [88].

Two distributions in each row – the left and right one – are mirror reflections of each other, since the sum of the corresponding H angles equals 180° . The angular distribution in a helicity reference frame is strongly related to the excitation of resonances decaying into two out of the three final state particles. For example, the two final state pions originating from the decay of intermediate ρ meson have a high relative momentum due to their large invariant mass, and in the pion-nucleon helicity frame the respective

enhancement is expected at large opening angle between the two pions (or small opening angle between the proton and pion). Indeed, such enhancement is visible in the $\theta_{p\pi}^{\pi^-\pi^0}$ distribution corrected for the HADES acceptance, presented in Ref. [80]. Summarizing, all presented mass, rapidity, transverse-momentum and angular distributions were compared with the PWA model and are well described by this model.

6.4 Exclusive $\pi^-e^+e^-$ channel

In the exclusive $\pi^-e^+e^-$ analysis, proton is an unmeasured particle in the final state. The missing proton in this scenario can come not only from the bremsstrahlung channel but also from the Δ^+ Dalitz decay channel ($\Delta^+ \rightarrow p e^+e^-$), due to the same final state. The Δ^+ π^- contribution was determined by the PWA solution and was modelled in the Monte Carlo simulation.

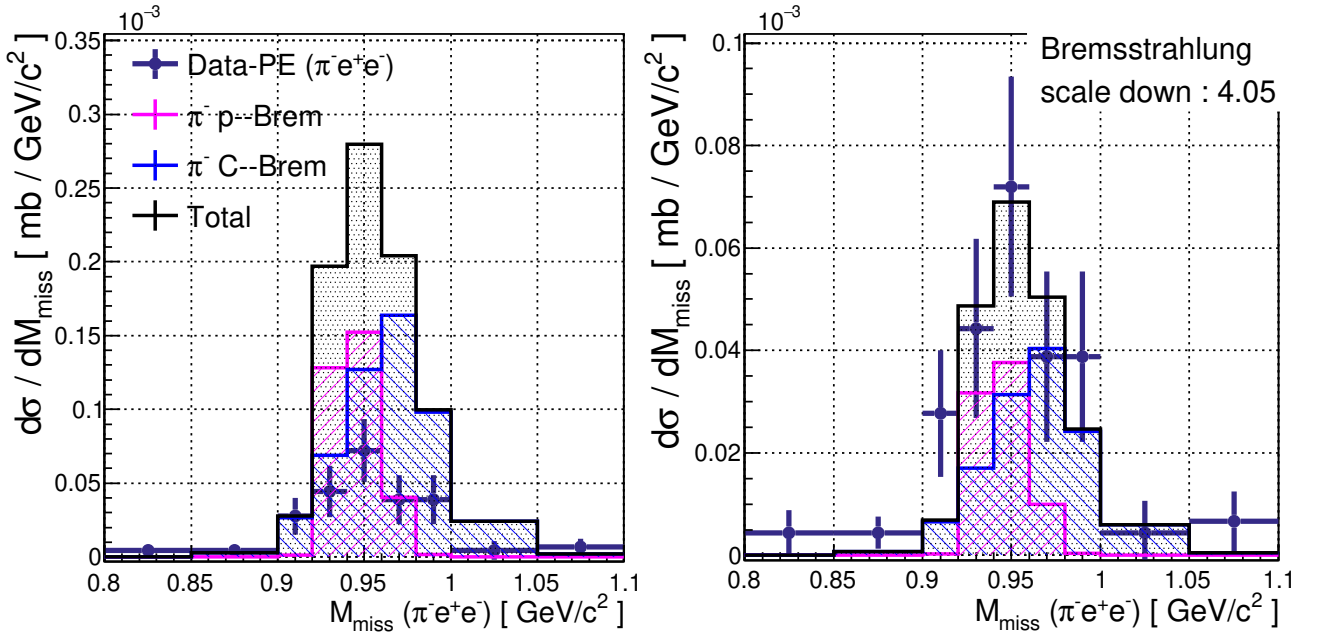


Figure 6.13.: $\pi^- e^+ e^-$ missing mass distribution in measurement with $(C_2H_4)_n$ target after applying $M_{inv}(e^+e^-) > 0.14$ GeV/c² cut. The magenta and blue histograms represent the π^-p and π^-C bremsstrahlung contributions, respectively. The sum of these two contributions are shown as black histogram. In the right panel, the simulated contributions are scaled down by a factor 4.05 compared to the left panel.

To suppress the background originating from the π^0 Dalitz decay, the e^+e^- invariant mass values above the π^0 mass are selected. Figure 6.13 presents the $\pi^- e^+ e^-$ missing mass (M_{miss}) spectrum for the measurement on $(C_2H_4)_n$ target, with the prominent peak at the proton mass. Histograms represent contributions from the Monte Carlo simulations of the bremsstrahlung process for π^-p collisions (magenta histogram) and π^-C collisions (blue histogram). The Δ Dalitz decay contribution is about ≈ 70 times smaller than the bremsstrahlung contribution and, therefore, not shown in the figure. The bremsstrahlung prediction overestimates the data (see Fig. 6.13, left panel). In order to explain the experimental data, a downscale factor for the bremsstrahlung simulation is introduced. This scaling factor is determined by minimizing the χ^2 value calculated between the data points and the total bremsstrahlung simulation, presented in Fig. 6.13 (right panel). The obtained value of the downscale factor amounts to 4.05. After downscaling, the total bremsstrahlung simulation provides good description of data as shown in right panel of Fig. 6.13.

The data points in Fig. 6.13 are fitted using the Gaussian distribution and the obtained mean and standard deviation value is $0.951 \pm 0.007 \text{ GeV}/c^2$ and $0.033 \pm 0.005 \text{ GeV}/c^2$, respectively. Similarly, fitting the Gaussian distribution to the simulation (black histogram), results in the mean value $0.952 \pm 0.001 \text{ GeV}/c^2$ and standard deviation $0.027 \pm 0.0005 \text{ GeV}/c^2$. The experimental and simulated distributions are consistent in terms of the peak position and the width.

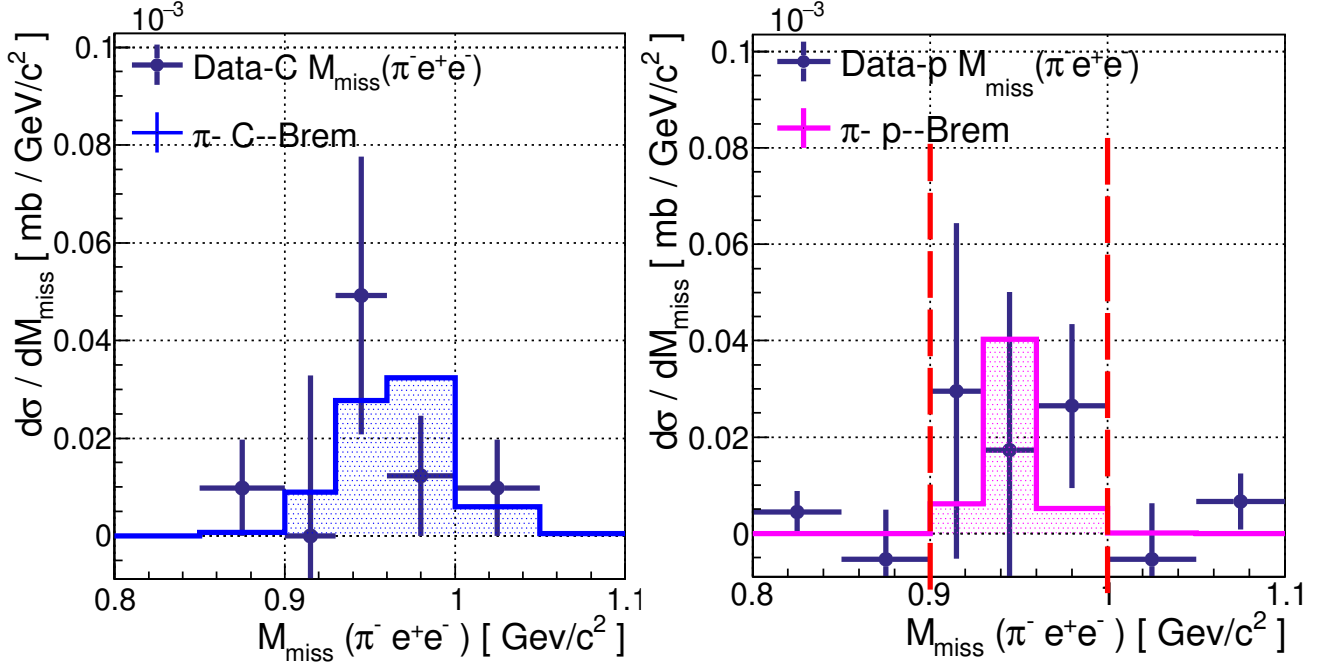


Figure 6.14.: $\pi^- e^+ e^-$ missing mass distribution in measurement with C target (left panel) and in measurement with $(C_2H_4)_n$ after subtracting the carbon contribution (right panel). The magenta and blue histograms represent the $\pi^- p$ and $\pi^- C$ bremsstrahlung simulations, respectively. The simulations are presented with included scaling down factor – 4.05. The vertical red dashed lines indicate the region selected for calculating the $\pi^- p$ bremsstrahlung cross-section.

The missing mass spectra in measurement with the C target and with the $(C_2H_4)_n$ target after subtracting the carbon contribution are presented in Fig. 6.14. Data points for the carbon target are biased by the large statistical uncertainties. Therefore, the obtained spectra for polyethylene target after subtraction the carbon contribution contain also high uncertainties and no statistically significant signal indicating for the $\pi^- p$ bremsstrahlung is visible.

The e^+e^- invariant mass distribution for the $(C_2H_4)_n$ target, for the C target, and for the $(C_2H_4)_n$ target after subtracting the carbon contribution, are shown in Fig. 6.15. Black data points are compared with the sum of simulations (black histogram) including $\pi^- p$ bremsstrahlung (magenta hatched histogram), $\pi^- C$ bremsstrahlung (blue hatched histogram), π^0 Dalitz decay (green hatched histogram) and Δ Dalitz decay (yellow hatched histogram). The simulations of the bremsstrahlung are pre-scaled by the factor of 4.05. For $(C_2H_4)_n$ and C targets, the simulations overestimate the data in the invariant mass region below π^0 mass almost by a factor of 2. The possible explanation for this discrepancy is that the simulation of π^0 production on carbon is based on very simple model using a scaling factor of $Z^{2/3}$ as described in the Chapter 4.3.2. This model does not take into account, e.g. the secondary interactions of particles produced in the π^- collisions with proton in carbon nucleus with the remaining nucleons in the

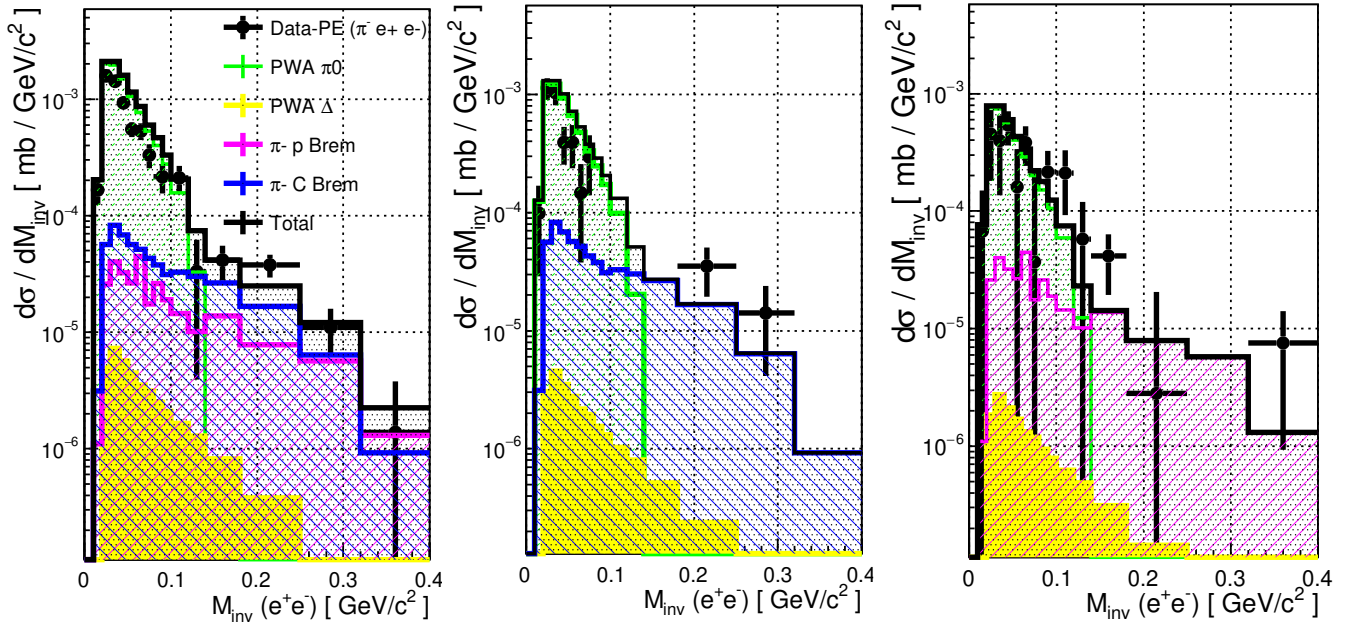


Figure 6.15.: e^+e^- invariant mass distribution for π^- (C_2H_4) $_n$ (left panel), π^- C (middle panel) and π^- p (right panel). Data points (black dots) are compared with the sum of simulations (black histogram) including π^- p bremsstrahlung (magenta hatched histogram), π^- C bremsstrahlung (blue hatched histogram), π^0 Dalitz decay (green hatched histogram) and Δ Dalitz decay (yellow hatched histogram).

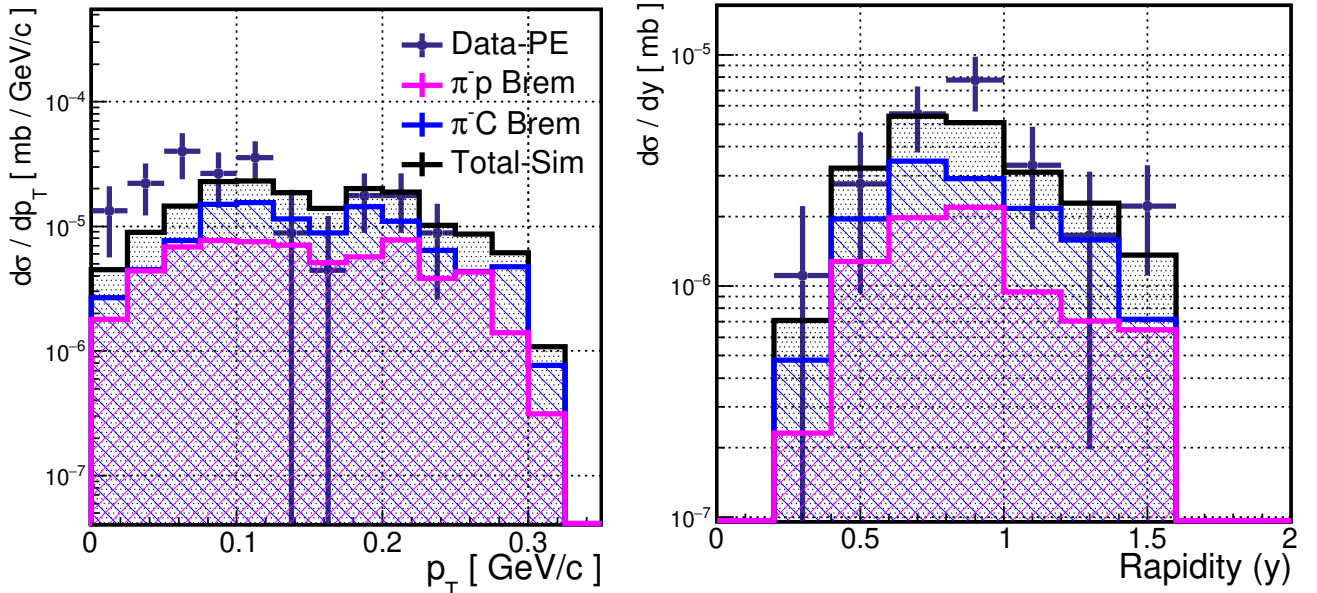


Figure 6.16.: The transverse momentum (left panel) and rapidity (right panel) distribution for the missing proton for (C_2H_4) $_n$ target. Data points (dark blue dots) are compared with the sum of simulations (black histogram) including π^- p bremsstrahlung (magenta hatched histogram) and π^- C bremsstrahlung (blue hatched histogram).

nucleus.

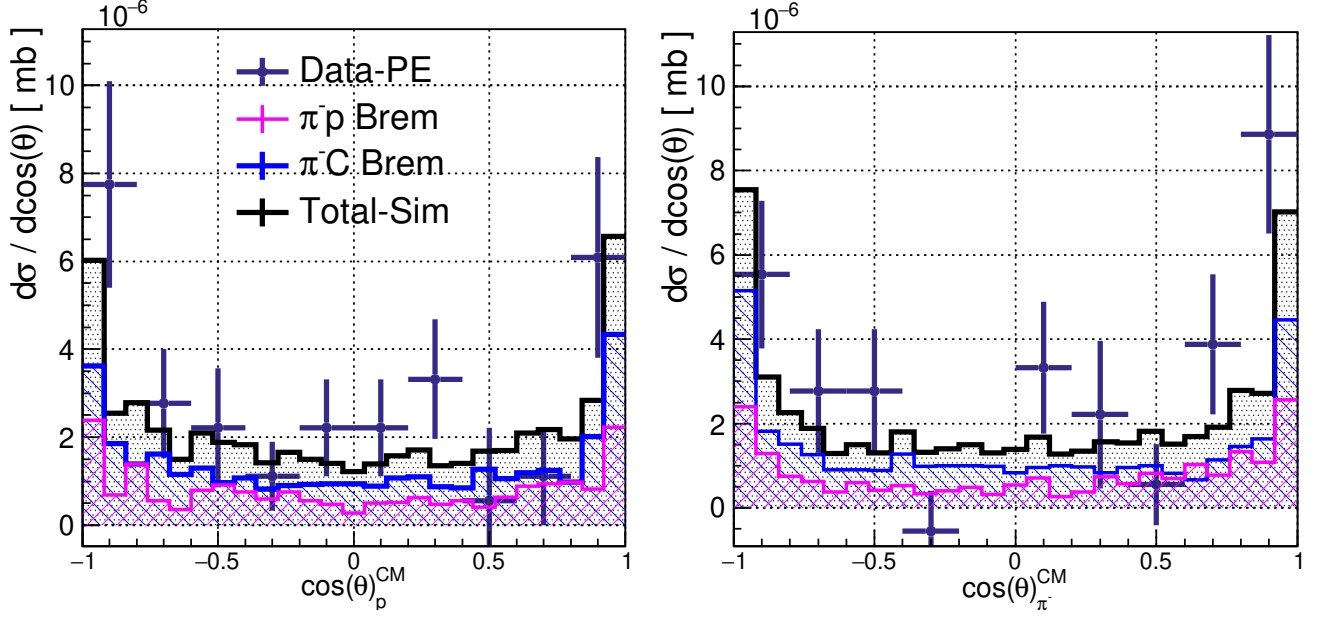


Figure 6.17.: Distribution of cosine of the proton polar angle: $\cos(\theta_p^{CM})$ (left panel) and of the pion polar angle: $\cos(\theta_{\pi^-}^{CM})$ (right panel) in center-of-mass frame, for $(C_2H_4)_n$ target. Data points (dark blue dots) are compared with the sum of simulations (black line) including $\pi^- p$ bremsstrahlung (magenta hatched histogram) and $\pi^- C$ bremsstrahlung (blue hatched histogram).

For the study of the bremsstrahlung process, various distributions of experimental differential cross-sections for the $(C_2H_4)_n$ target are compared to the SPA simulations. They are presented in Fig. 6.16 as a function of the proton transverse momentum p_T (left panel) and the proton rapidity y (right panel). Proton is meant here to be reconstructed as the $\pi^- e^+ e^-$ missing mass. In Fig. 6.17, cross-sections as a function of the cosine of the proton (left panel) and π^- (right panel) polar angle in the center-of-mass frame is presented. Figures 6.16 and 6.17 are prepared after applying the invariant mass cut: $M_{inv}(e^+e^-) > 0.14 \text{ GeV}/c^2$. In the figures, the Monte Carlo simulation of the bremsstrahlung was scaled down by the factor 4.05, as discussed above in this chapter. The presented experimental distributions are well described by the simulations, except the low p_T region.

6.5 Exclusive $p e^+ e^-$ channel

Another hypothesis which allows for the reconstruction of the four particles in the final state, is three particle hypothesis: $p e^+ e^-$, where the missing pion is identified using the missing mass technique. The analysis is complementary to $\pi^- e^+ e^-$ where the missing particle was a proton, however, as it is shown in Chapter 5, the pion acceptance surpasses the proton acceptance. Protons are mostly produced at small polar angles with respect to the beam, where the HADES spectrometer lacks the acceptance at $\theta < 18^\circ$. Distributions of the e^+e^- invariant mass $M_{inv}(e^+e^-)$ are shown in Fig. 6.18. The distributions represent π^- interaction with different targets, i.e. $(C_2H_4)_n$ (left panel), C (middle panel) and p (right panel). For the $\pi^- p$ process, the data points are reasonably well described by the total simulation including π^0 Dalitz decay and $\pi^- p$ bremsstrahlung process. The simulations also include Δ Dalitz decay process but this contribution is negligibly small. The total simulation for the π^- interacting with $(C_2H_4)_n$ and C target overestimates the data points in the invariant mass range from about 0.1 to 0.3 GeV/c^2 .

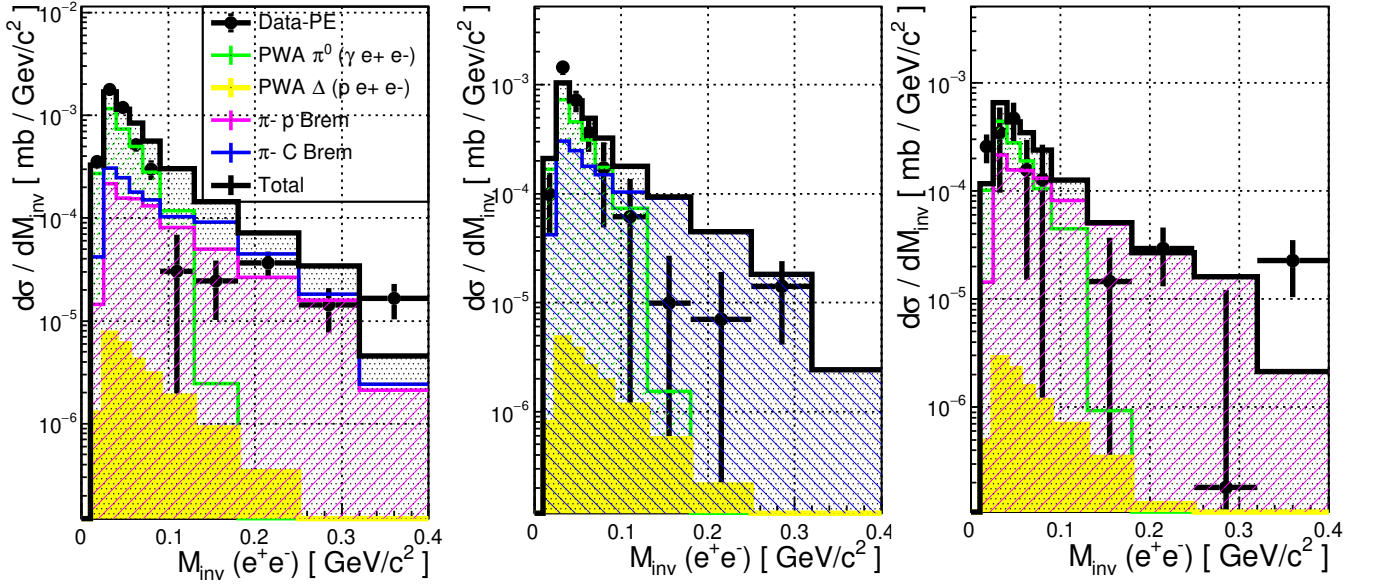


Figure 6.18.: e^+e^- invariant mass distribution for $\pi^- (C_2H_4)_n$ (left panel), $\pi^- C$ (middle panel) and $\pi^- p$ (right panel), for the $p e^+ e^-$ three-particle hypothesis. Data points (black dots) are compared with the SPA bremsstrahlung for $\pi^- p$ (magenta hatched histogram) and $\pi^- C$ (blue hatched histogram) reactions. The π^0 Dalitz decay (green hatched histogram) and Δ Dalitz decay (yellow hatched histogram) derived from PWA solution are also presented.

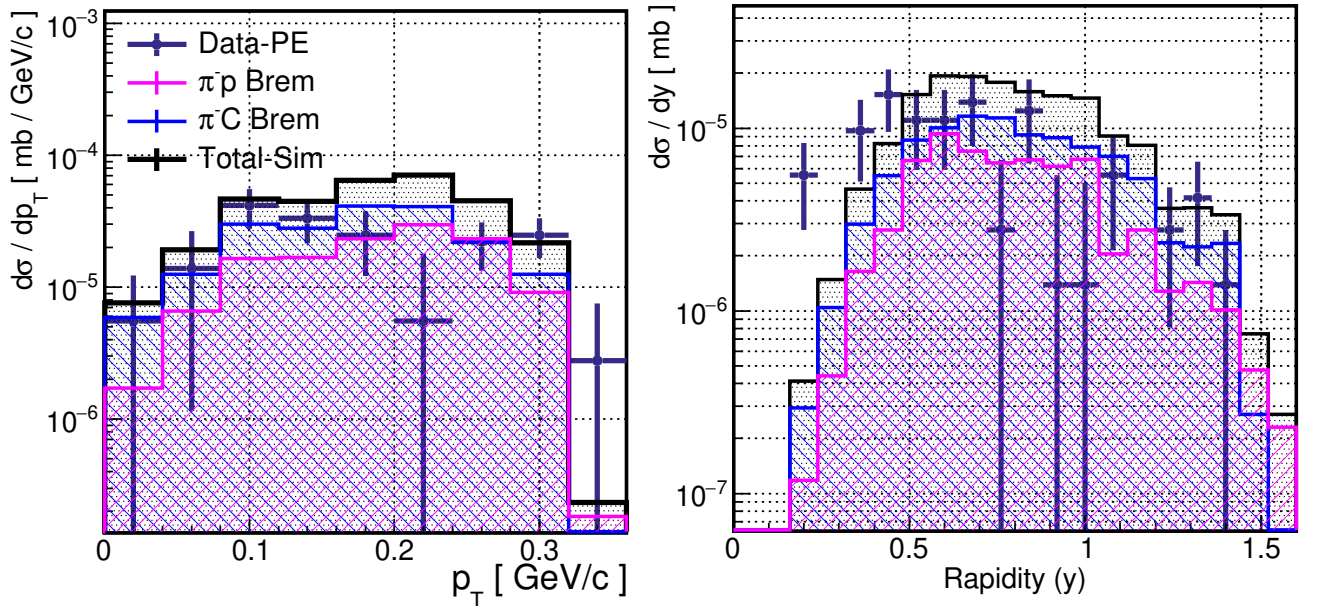


Figure 6.19.: The transverse momentum (left panel) and rapidity (right panel) distribution for the missing pion for $(C_2H_4)_n$ target. Data points (dark blue dots) are compared with the sum of simulations (black line) including $\pi^- p$ bremsstrahlung (magenta hatched histogram) and $\pi^- C$ bremsstrahlung (blue hatched histogram).

For the study of the bremsstrahlung process, various distributions of experimental differential cross-sections for the $(C_2H_4)_n$ target are compared to the SPA simulations. They are presented in Fig. 6.19 as a function of the pion transverse momentum p_T (left panel) and the pion rapidity y (right panel). Pion is meant here

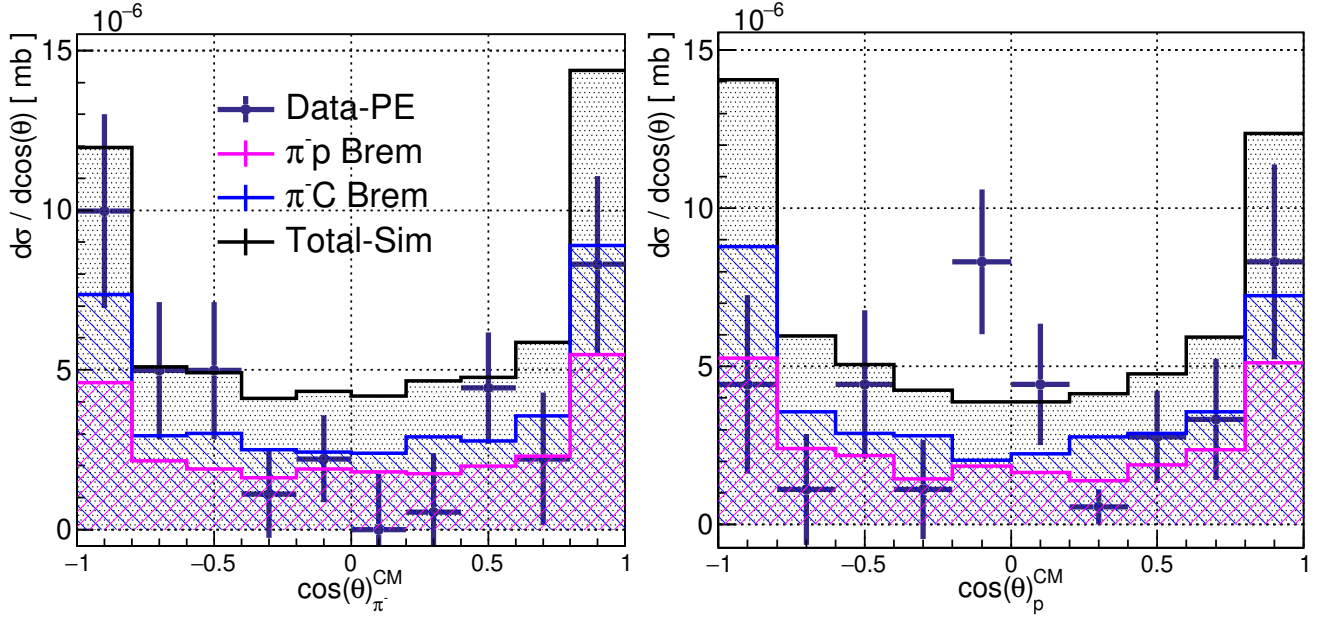


Figure 6.20.: Distribution of cosine of the pion polar angle: $\cos(\theta_{\pi^-}^{CM})$ (left panel) and of the proton polar angle: $\cos(\theta_p^{CM})$ (right panel) in center-of-mass frame, for $(C_2H_4)_n$ target. Data points (dark blue dots) are compared with the sum of simulations (black histogram) including $\pi^- p$ bremsstrahlung (magenta hatched histogram) and $\pi^- C$ bremsstrahlung (blue hatched histogram).

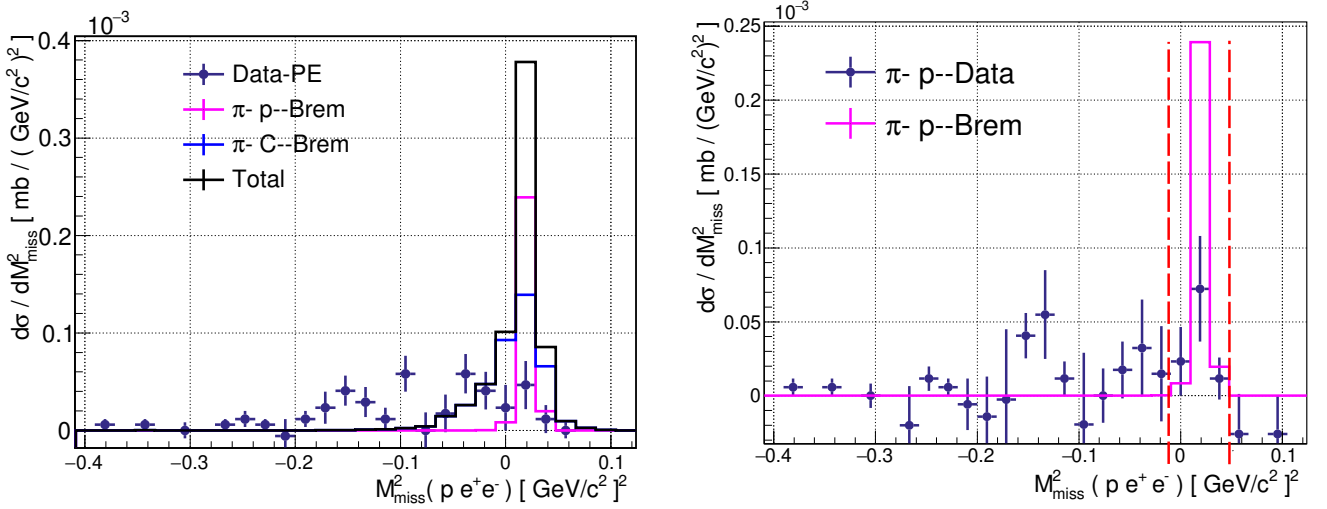


Figure 6.21.: The missing mass squared $M_{miss}^2(p e^+ e^-)$ distribution for the $(C_2H_4)_n$ target (left panel) and after subtracting carbon contribution from $(C_2H_4)_n$ data (right panel) for $e^+ e^-$ invariant mass above the π^0 mass range. Data points (dark blue dots) are compared with the sum of simulations (black histogram) including $\pi^- p$ bremsstrahlung (magenta histogram) and $\pi^- C$ bremsstrahlung (blue histogram). The vertical red dashed lines indicate the region selected for calculating the $\pi^- p$ bremsstrahlung cross-section.

to be reconstructed as the $p e^+ e^-$ missing particle. In Fig.6.20, differential cross-sections as a function of the cosine of the pion polar angle and the proton polar angle in the center-of-mass frame are presented. Figure 6.19 and 6.20 are prepared after applying the invariant mass cut: $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$. In the figures, the Monte Carlo simulation of the bremsstrahlung overestimates the presented experimental

distributions in some ranges of the kinematical variables.

The missing mass squared appears to need additional smearing with respect to Fermi-momentum (p_F) (Fig. 6.21, left panel), where the impact of the Fermi-momentum is already predicted for pion interaction with the bound protons in carbon atom. Data (dark blue dots) are compared to a black histogram that represents the sum of the π^-p (magenta) and π^-C (blue) bremsstrahlung processes. The peak predicted by the simulations at the missing mass squared corresponding to the pion mass exceeds by far the experimental data points. In the right panel of the same figure, data is plotted after subtracting carbon contribution from $(C_2H_4)_n$ target and compared with the proton bremsstrahlung simulation. Also in the case of proton target the simulation exceeds the data points at the missing mass value corresponding to the pion mass.

6.6 Extraction of cross-sections

This section is dedicated to the estimation of the total cross-section for the π^0 production in the $\pi^- p \rightarrow p \pi^- \pi^0$ reaction as well as for the bremsstrahlung process on the $(\text{C}_2\text{H}_4)_n$ target. As described in Chapter 5.3, the measured yields are transformed to cross-sections by applying relative normalization to the elastic pion-proton scattering data taken from the literature. This corresponds to division of the measured yield N by the integrated luminosity L , which was calculated as the product of the number of beam pions and the areal density of protons in the polyethylene target. In order to correct these cross-sections for the limited detector acceptance A and efficiency ϵ they are divided by the acceptance and efficiency values. Thus the total cross-section is calculated as:

$$\sigma_{TOT} = \frac{N}{L} \frac{1}{A \epsilon}. \quad (6.6.1)$$

The detector acceptance A is determined in simulations as a ratio of the sum of weights of events with the tracks passing through the active part of the HADES spectrometer to the sum of weights of all events generated in the full solid angle (see Chapter 4.4). The detection efficiency ϵ is determined as a ratio of the sum of weights of the reconstructed events to the sum of weights of all events generated within the HADES acceptance.

6.6.1 Estimation of systematic uncertainties

Source of uncertainty	Error estimate [%]
Normalization	4
Hadron identification	2
e^+e^- identification	10
Combinatorial background subtraction	5
Total	12

Table 6.2.: Systematic uncertainties for the dilepton analysis of π^- beam experiment. The total uncertainty is calculated by adding the individual contributions in quadrature.

The main contributions to systematic error of the measured cross-sections are collected in Table 6.2. The error of the normalization to the elastic scattering is 4% (see Chapter 5.6.3). The uncertainty due to applying the p and π^- identification cuts (see Chapter 5.6.3) is estimated to be equal to 2%. The uncertainty of the dilepton identification (see Chapter 5.6.3) dominated by backtracking algorithm due efficiency of RICH detector is equal to 10% (Ref. [33]). The systematic error of the combinatorial background (CB) subtraction is estimated by taking the difference between the CB calculated using the geometric and arithmetic mean which is divided by the respective signal. This error is of about 3% – 5%.

6.6.2 Determination of $\pi^- p \rightarrow \pi^- p \pi^0$ cross-section

Reaction	$\pi^- p \pi^0$
HADES acceptance cross-section	$(2.34 \pm 0.26_{stat} \pm 0.28_{sys}) \times 10^{-3} \text{ mb}$
Acceptance (A)	1.66 %
Efficiency (ϵ)	4.45 %
4π total cross-section	$3.20 \pm 0.35_{stat} \pm 0.38_{sys} \text{ mb}$

Table 6.3.: Extracted cross section of the π^0 production for the $\pi^- p \pi^0$ final state: 1st and last row present the value within the HADES acceptance, and the value extrapolated to the full solid angle, respectively, while the second and third row give the acceptance and efficiency used for the extrapolation.

In order to determine the $\pi^- p \pi^0$ reaction cross-section, the number of events from the $\pi^- p$ missing mass spectrum (see Fig. 5.13) is divided by the integrated luminosity. Additionally, to account for the Dalitz decay for π^0 meson registered in the experiment, the obtained cross-section is divided by the branching ratio for the π^0 Dalitz decay ($\gamma e^+ e^-$, $\text{BR} = 0.0117 \pm 0.0003$). The resulting $\pi^- p \pi^0$ reaction cross-section in the HADES acceptance is $(2.34 \pm 0.26_{stat} \pm 0.28_{sys}) \times 10^{-3} \text{ mb}$. The indicated systematic error contains contributions given in Table 6.2 with the exception of the uncertainty of the e^+e^- identification. This uncertainty is taken into account while correcting the cross-section for the limited detection efficiency. The $\pi^- p \pi^0$ total cross-section for the full solid angle is calculated by taking into account the limited detection efficiency and the acceptance of HADES obtained in the simulations based on the PWA and given in Table 6.3. It amounts to $3.20 \pm 0.35_{stat} \pm 0.38_{sys} \text{ mb}$ and agrees with the value $3.54 \pm 0.01_{stat} \pm (+0.23, -0.21)_{sys} \text{ mb}$, obtained in the recent HADES publication on two pion channel analysis [80]. The agreement between the present result for the total cross-section obtained in the analysis of the dilepton channel ($\pi^- p \gamma e^+ e^-$) and the cited result for the hadronic final state ($\pi^0 p \pi^-$) indicates that the reconstruction of dileptons in the present analysis is well under control.

6.6.3 Bremsstrahlung cross-section with polyethylene target

In this thesis, the first attempt to evaluate the total cross section for the virtual bremsstrahlung emission in the collision of π^- beam with polyethylene target is made. For the cross-section evaluation we take exclusive 3-prong ($\pi^- e^+ e^-$) and inclusive 2-prong ($e^+ e^-$) hypotheses in consideration. The 4-prong hypothesis ($\pi^- p e^+ e^-$) is not used for the estimation because the corresponding signal in the four particle missing mass spectrum (see Fig. 5.16) is not statistically significant. In turns, the 3-prong ($p e^+ e^-$) hypothesis is not used because the three particle missing mass spectrum in measurement with polyethylene target (see Fig. 6.21) is not understood in terms of the performed simulations.

In the case of $\pi^- e^+ e^-$ hypothesis, the bremsstrahlung cross-section within the HADES acceptance, for $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$, was calculated as integral of entries in the $\pi^- e^+ e^-$ missing mass spectrum shown in Fig. 6.13, right panel. The obtained value of the cross-section is $(5.87 \pm 0.94_{stat} \pm 0.70_{sys}) \times 10^{-6} \text{ mb}$. After correcting this result for the HADES acceptance and detection efficiency, the 4π bremsstrahlung cross-section amounts to $(6.76 \pm 1.11_{stat} \pm 0.81_{sys}) \times 10^{-4} \text{ mb}$. This value is further extrapolated to the entire range of $e^+ e^-$ invariant mass by taking into account the acceptance and efficiency corrections calculated based on the SPA model. Resulting total cross-section for the virtual photon bremsstrahlung in collisions of the $0.685 \text{ GeV}/c$ π^- beam with $(C_2H_4)_n$ target is $(2.58 \pm 0.44_{stat} \pm 0.31_{sys}) \times 10^{-2} \text{ mb}$. The obtained values of the cross-section along with the detector efficiency and acceptance are collected in Table 6.4. To estimate the total cross-section for the $\pi^- p$ bremsstrahlung process, the above result for the $(C_2H_4)_n$ target was divided by the factor 3.2, resulting from Eq. 4.3.8 in Chapter 4. The resulting value of the total cross section for the $\pi^- p$ bremsstrahlung is $(8.1 \pm 1.4_{stat} \pm 0.97_{sys}) \times 10^{-3} \text{ mb}$. To the best of our knowledge, this is the first cross section data for the virtual bremsstrahlung emission in the collisions of charged pions with nucleons. The obtained value of the total cross-section is about 4 times smaller than the prediction of the soft-photon approximation (SPA) model, which is $28 \times 10^{-3} \text{ mb}$. The observed discrepancy between the current cross section and the prediction of the SPA model can be understood in the light of theoretical estimations presented by Eggers *et al.* in Ref. [101]. The authors show, that the assumption of the SPA model, that the electromagnetic and strong processes factorize, is not fulfilled even for small dilepton invariant masses in the range from 10 to 300 MeV. Based on a derived formula for the dilepton cross section in pion-pion collisions, which does not rely on such factorization, they show, that the SPA overestimates the dilepton cross sections by factors 2-8.

Reaction	$\pi^- p e^+ e^-$ ($\pi^- e^+ e^- X$ hypothesis)	
HADES acceptance $M_{inv} > 0.14 \text{ GeV}/c^2$	$(5.87 \pm 0.94_{stat} \pm 0.70_{sys}) \times 10^{-6} \text{ mb}$	
Conditions	$M_{inv} > 0.14 \text{ GeV}/c^2$	Full M_{inv} range
Acceptance (A)	2.71 %	0.75 %
Efficiency (ϵ)	32.03 %	3.03 %
4π total cross-section	$(6.76 \pm 1.11_{stat} \pm 0.81_{sys}) \times 10^{-4} \text{ mb}$	$(2.58 \pm 0.44_{stat} \pm 0.31_{sys}) \times 10^{-2} \text{ mb}$

Table 6.4.: Estimated bremsstrahlung cross-section for polyethylene target within HADES acceptance and $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ (2nd row), extrapolated to full solid angle (last row, middle column) and further extrapolated to the entire range of $M_{inv}(e^+ e^-)$ (last row, right column). Acceptance (A) and efficiency (ϵ) values from the Monte Carlo simulations are also presented.

In the case of $e^+ e^- X$ inclusive hypothesis, the bremsstrahlung cross-section within the HADES acceptance and for $M_{miss} > 1.2 \text{ GeV}/c^2$ was calculated as an integral of experimental entries in the $e^+ e^-$ missing mass spectrum shown in Fig. 6.2, right panel, reduced by the corresponding contribution from the simulations

of the η Dalitz decay. The resulting value of the cross-section is $(1.82 \pm 0.22_{stat} \pm 0.18_{sys}) \times 10^{-5}$ mb. This value is extrapolated to the entire range of e^+e^- invariant mass by taking into account the acceptance and efficiency corrections calculated based on the SPA model. The resulting total 4π bremsstrahlung cross-section in collisions of the π^- beam with $(C_2H_4)_n$ target is $(3.43 \pm 0.42_{stat} \pm 0.34_{sys}) \times 10^{-2}$ mb. The obtained values of cross-section along with detector efficiency and acceptance are collected in Table 6.5. The results for the bremsstrahlung cross-section in $\pi^-(C_2H_4)_n$ collisions presented above for the exclusive ($\pi^-e^+e^-X$) analysis and the inclusive (e^+e^-X) analysis are consistent within the indicated experimental uncertainties.

Inclusive hypothesis	e^+e^-X
σ [mb] within HADES acceptance $M_{inv} > 0.14 \text{ GeV}/c^2$ and $M_{miss} > 1.2 \text{ GeV}/c^2$	$(1.82 \pm 0.22_{stat} \pm 0.18_{sys}) \times 10^{-5} \text{ mb}$
$A \times \epsilon$	0.053 %
σ [mb] 4π acceptance full M_{inv} and M_{miss} range	$(3.43 \pm 0.42_{stat} \pm 0.34_{sys}) \times 10^{-2} \text{ mb}$

Table 6.5.: Inclusive bremsstrahlung cross-section for polyethylene target within HADES acceptance, $M_{miss} > 1.2 \text{ GeV}/c^2$ and $M_{inv} > 0.14 \text{ GeV}/c^2$ (2nd row), product of acceptance (A) and efficiency (ϵ) (third row) and the cross-section corrected for the acceptance and efficiency, extrapolated to the entire range of $M_{inv}(e^+e^-)$ (last row).

6.6.4 Estimation $\pi^- p \rightarrow \pi^- p e^+ e^-$ bremsstrahlung cross-section

In this thesis, the total cross section for the $\pi^- p$ virtual bremsstrahlung is also determined in an alternative way by subtracting contribution of carbon in polyethylene spectra. The bremsstrahlung cross-section within the HADES acceptance for $M_{inv}(e^+ e^-) > 0.14 \text{ GeV}/c^2$ was calculated independently for the three exclusive channels: $\pi^- e^+ e^- X$, $p e^+ e^- X$ and $\pi^- p e^+ e^-$ using the missing mass spectra presented in Figs. 6.14, 6.21, and 6.7, respectively. For each spectrum, the cross-section was calculated as integral of entries in the missing mass range containing the simulated bremsstrahlung signal. The limits of integration are indicated with vertical red lines in the mentioned figures. The obtained cross sections together with statistical uncertainties are given in the second row of Table 6.6. These cross-section values were then corrected for the acceptance and efficiency of the detection system given, respectively, in the third and fourth rows of Table 6.6. The resulting bremsstrahlung cross sections in the full angular range, but still for the $e^+ e^-$ invariant mass greater than $0.14 \text{ GeV}/c^2$, are given in the fifth row of Table 6.6. Finally, these cross-sections were extrapolated to the full range of the $e^+ e^-$ invariant mass using the corresponding correction for the acceptance and efficiency obtained from the simulations (based on the SPA model). The values of the product of acceptance and efficiency are given in the sixth row of Table 6.6. The resulting values of the total cross section for the bremsstrahlung process are given in the last (the seventh) row of Table 6.6. The weighted mean of these values, with the weights equal to the reciprocals squares of the corresponding uncertainties, is taken as the final result for the total bremsstrahlung cross-section. Its value and the corresponding statistical and systematic uncertainty is $(4.68 \pm 1.44_{stat} \pm 0.56_{sys}) \times 10^{-3} \text{ mb}$.

In the case of the inclusive $e^+ e^- X$ hypothesis, the bremsstrahlung cross-section within the HADES acceptance and $M_{inv} > 0.14 \text{ GeV}/c^2$ was calculated as integral of entries in the $e^+ e^-$ missing mass spectrum (Fig. 6.4, right panel) over the missing mass values larger than $1.2 \text{ GeV}/c^2$. Obtained value was then reduced by the contribution from the η Dalitz decay obtained in the simulations. The result, given in the second row of Table 6.7, is extrapolated to the entire range of $e^+ e^-$ invariant mass and by taking into account the acceptance and efficiency corrections calculated based on the SPA model. Resulting total bremsstrahlung cross-section in $\pi^- p$ collisions is $(1.28 \pm 0.84_{stat} \pm 0.13_{sys}) \times 10^{-2} \text{ mb}$. The obtained value of cross-section along with detector efficiency and acceptance are collected in Table. 6.7. This value has a large statistical uncertainty. It is consistent with the mean value of the cross-sections obtained from the exclusive analyses.

The total cross-sections for the virtual bremsstrahlung emission in collisions of pion beam with polyethylene as well as with proton target were obtained by the extrapolation to the full solid angle of the corresponding cross-sections measured within the HADES acceptance. For the extrapolation, the simulations based of the SPA model were used. The systematic uncertainty of this extrapolation is not known since the uncertainties of the theoretical predictions of the SPA model are unknown. Therefore, for comparison of the experimental cross-sections with predictions of various theoretical models of the bremsstrahlung emission, one should rather take the cross sections determined within the HADES acceptance and the predictions of theoretical models convoluted with the HADES acceptance.

Exclusive hypothesis	$\pi^- e^+ e^- X$	$p e^+ e^- X$	$\pi^- p e^+ e^-$
σ [mb] within HADES acceptance and $M_{inv} > 0.14$ GeV/c ²	$(2.19 \pm 1.69) \times 10^{-6}$	$(2.61 \pm 1.20) \times 10^{-6}$	$(6.64 \pm 2.71) \times 10^{-7}$
Acceptance (A)	9.29 %	9.29 %	1.19 %
Efficiency (ϵ)	27.93 %	26.03 %	23.97 %
σ [mb] 4π acceptance and $M_{inv} > 0.14$ GeV/c ²	$(2.88 \pm 2.22) \times 10^{-4}$	$(9.80 \pm 4.96) \times 10^{-5}$	$(2.32 \pm 0.95) \times 10^{-4}$
$A \times \epsilon$ full M_{inv} mass	2.28×10^{-2} %	7.25×10^{-2} %	8.57×10^{-3} %
σ [mb] full M_{inv} range	$(9.61 \pm 7.42) \times 10^{-3}$	$(3.60 \pm 1.65) \times 10^{-3}$	$(7.75 \pm 3.16) \times 10^{-3}$

Table 6.6.: Bremsstrahlung cross-section for proton target within HADES acceptance and $M_{inv}(e^+e^-) > 0.14$ GeV/c² (2nd row), corrected for detector acceptance and efficiency (5th row) and extrapolated to the entire range of $M_{inv}(e^+e^-)$ (last row). Acceptance (A) and efficiency (ϵ) values from Monte Carlo simulations are also presented.

Inclusive hypothesis	e^+e^-X
σ [mb] within HADES acceptance $M_{inv} > 0.14 \text{ GeV}/c^2$ and $M_{miss} > 1.2 \text{ GeV}/c^2$	$(7.40 \pm 4.90) \times 10^{-6}$
$A \times \epsilon$	$5.83 \times 10^{-2} \%$
σ [mb] 4π acceptance full M_{inv} and M_{miss} range	$(1.28 \pm 0.84) \times 10^{-2}$

Table 6.7.: Inclusive bremsstrahlung cross-section for proton target within HADES acceptance, $M_{miss} > 1.2 \text{ GeV}/c^2$ and $M_{inv} > 0.14 \text{ GeV}/c^2$ (2nd row), product of acceptance (A) and efficiency (ϵ) (third row) and the cross-section corrected for the acceptance and efficiency, extrapolated to the entire range of $M_{inv}(e^+e^-)$ (last row).

Chapter 7

Summary and conclusions

"Science never solves a problem without creating ten more."

George Bernard Shaw

This work presents the analysis of data collected with the HADES spectrometer for the dilepton production in π^-p collisions at a pion beam momentum of 0.685 GeV/c. The measurements were performed with a polyethylene target and with a carbon target. The latter was used to determine the contribution to the dilepton production from the beam scattering on carbon in polyethylene target. The main aim of the work was to determine the contribution to the production of dileptons from the virtual bremsstrahlung emission in the elementary reaction $\pi^-p \rightarrow \pi^-p e^+e^-$. We searched for the signal from this process through the analysis of three exclusive channels: $\pi^-p e^+e^-$, $\pi^-e^+e^-X$, $p e^+e^-X$, and one inclusive channel: e^+e^-X . To estimate contributions to the production of dileptons from various possible processes, we performed Monte Carlo simulations based on theoretical models of these processes.

In the investigated analyses channels, the distributions of the e^+e^- invariant mass are dominated by the π^0 Dalitz decay. The production of π^0 mesons in the $\pi^-p \rightarrow \pi^0\pi^-p$ channel, also registered in the present experiment, was the subject of a separate analysis. Obtained differential cross-section distributions were described using partial wave analysis [80]. We analyzed the $\pi^-p \rightarrow \pi^0\pi^-p (\gamma e^+e^-)$ process including the π^0 Dalitz decay to demonstrate that the reconstruction of dielectron pairs is under control from the point of view of the HADES spectrometer modelling. The differential cross-sections were determined as function of various kinematical variables such as invariant mass, transverse momentum, rapidity or scattering angles of e^+e^- pair in various reference frames. The experimental distributions of the cross-sections are reasonably well reproduced by the simulation based on the partial-wave analysis.

One of the main conclusions of the present work is that in the collisions of the pion beam with polyethylene and carbon target, a significant contribution to the dilepton production comes from the bremsstrahlung emission process. In the e^+e^- invariant mass range below the π^0 mass, the π^0 Dalitz decay dominates the dilepton production. However, in the exclusive $\pi^-e^+e^-$ and $p e^+e^-$ channels, the distributions of the e^+e^- invariant mass in the range above the π^0 mass are dominated by the contribution from the bremsstrahlung emission. This contribution is roughly two times larger for the π^-C collisions than for the π^-p collisions. According to the performed simulations, the contributions from the Δ resonance Dalitz decay is about two orders of magnitude smaller than that from the bremsstrahlung. In the inclusive e^+e^-X analysis, significant contributions to the dilepton production come also from the η Dalitz decay and the $N(1520)$ Dalitz decay. In this case, the bremsstrahlung emission is a dominating process for high e^+e^- missing masses, in the range above around 1.2 GeV/c².

For the identification of the bremsstrahlung emission in the π^-p collisions leading to the $\pi^-p e^+e^-$ final state, the missing mass technique was used. To get rid of the background coming from the π^0 Dalitz decay, only events with the e^+e^- invariant mass above the π^0 mass were selected. In the missing mass spectrum of the $\pi^-e^+e^-$ system measured for the polyethylene target, a clear peak at the position of the proton mass

is observed. It is well described by the simulation as a sum of the contributions coming from the quasi-free collisions of pions with protons inside carbon nucleus and in collision with proton in hydrogen atom. On the other hand, the missing mass spectrum for the $p e^+e^-$ contains a signal which is much lower than the simulation result. Besides, the experimental spectrum contains a tail appearing at negative values of the missing mass squared, which is not described by the simulations. We suppose that these effects may be due the rescattering and absorption of protons in the carbon nucleus, which were not included in the simulation. However, these effects are not observed in the case of the $\pi^- e^+e^-$ missing mass spectra which is only understandable if the rescattering of pions is much weaker than that of protons. Verification of this hypothesis requires further theoretical research.

We used a clean signal for the bremsstrahlung emission process, observed in the exclusive $\pi^- e^+e^- X$ analysis, to determine the cross-section for this process in the collisions of the π^- beam with polyethylene target. Obtained value of the cross-section determined within the HADES acceptance and for the e^+e^- invariant mass larger than 0.14 GeV/c² is $(5.87 \pm 0.94_{stat} \pm 0.70_{sys}) \times 10^{-6}$ mb. This value, extrapolated to the full solid angle and the entire range of the e^+e^- invariant mass using simulations based on the SPA model, gives the total cross-section of $(2.58 \pm 0.44_{stat} \pm 0.31_{sys}) \times 10^{-2}$ mb.

The indicated uncertainties do not take into account the systematic error related to the extrapolation of the measured cross-section to the full solid angle. This extrapolation depends on the applied theoretical model describing the bremsstrahlung emission process. Therefore, for the verification of theoretical predictions, one should rather use the experimental values of the cross-section obtained within the HADES acceptance and convolute the theoretical calculations with this acceptance.

To estimate the total cross-section for the bremsstrahlung process on a proton target, the above result for the polyethylene target was divided by the factor 3.2, resulting from a simple model of scaling the pion-proton bremsstrahlung cross section to the pion-carbon cross section. Obtained value of the total cross section for the $\pi^- p$ bremsstrahlung is $(8.1 \pm 1.4_{stat} \pm 0.97_{sys}) \times 10^{-3}$ mb.

We also determined the $\pi^- p$ bremsstrahlung cross section in a different way, namely by subtracting the carbon contributions in the missing mass spectra for the three exclusive channels measured with the polyethylene target. Obtained values of the cross-section, extrapolated to the full solid angle and the entire range of invariant mass, are consistent with each other and their mean value is $(4.68 \pm 1.44_{stat} \pm 0.56_{sys}) \times 10^{-3}$ mb. Within the range of given uncertainties, this value is consistent with the result obtained by scaling the cross section for polyethylene target. To the best of our knowledge, these are the first cross section data for the virtual bremsstrahlung emission in the collisions of charged pions with nucleons.

The obtained experimental data for the virtual bremsstrahlung emission are based on low statistics of events and do not allow for a precise study of various distributions of differential cross section. Therefore, for the study of the $\pi^- p$ bremsstrahlung it is important to perform measurements with higher statistics. In such measurements, the use of a liquid hydrogen target instead of a polyethylene target should be considered to avoid the background from the collisions with carbon atoms contained in polyethylene. From the point of view of the theoretical description of the virtual bremsstrahlung emission, it is interesting to calculate the total cross section for this process, avoiding the approximations used in the SPA model, and to compare it with the experimental values obtained in the present work.

"However far modern science and technics have fallen short of their inherent possibilities, they have taught mankind at least one lesson: Nothing is impossible."

LEWIS MUMFORD

Appendix A

Transforming expression for bremsstrahlung cross-section from four-fold to threefold differential form

The expression for differential cross-section for bremsstrahlung process in the soft-photon approximation derived in [50] and discussed in Chapter 1.5.3 is:

$$\frac{d^4\sigma_{e+e-}}{dyd^2p_t dM} = \frac{\alpha^2}{6\pi^3} \frac{\widehat{\sigma(s)}}{ME^2} \frac{R_2(s_2)}{R_2(s)}, \quad (\text{A.0.1})$$

where

$$R_2(s) = \sqrt{1 - \frac{(m_1 + m_2)^2}{s}}, \quad s_2 = s + M^2 - 2E\sqrt{s}, \quad (\text{A.0.2})$$

and

$$\widehat{\sigma(s)} = \frac{s - (m_1 + m_2)^2}{2m_1^2} \sigma(s). \quad (\text{A.0.3})$$

The kinematic variables in above equations are explained in Chapter 1.5.3.

Using the identity $dy = \frac{dp_z}{E}$ from [43] (PDG, Kinematics) the left hand side of Eq. A.0.1 can be written as:

$$\frac{d^4\sigma_{e+e-}}{dyd^2p_t dM} = E \frac{d^4\sigma_{e+e-}}{dp_z d^2p_t dM} = E \frac{d^4\sigma}{d^3p dM}. \quad (\text{A.0.4})$$

Using another identity from PDG [43]:

$$\frac{d^3p}{E} = d\phi \, p_t dp_t \, dy, \quad (\text{A.0.5})$$

the right hand side of Eq. A.0.4 is:

$$E \frac{d^4\sigma}{d^3p dM} = \frac{d^4\sigma}{d\phi p_t dp_t dy dM}, \quad (\text{A.0.6})$$

and the Eq. A.0.1 can be written as:

$$\frac{d^4\sigma}{d\phi p_t dp_t dy dM} = \frac{\alpha^2}{6\pi^3} \frac{\widehat{\sigma(s)}}{ME^2} \frac{R_2(s_2)}{R_2(s)}. \quad (\text{A.0.7})$$

After integrating this equation over the full azimuthal angle (ϕ) and using the identity $dp_t^2 = 2p_t dp_t$, we get finally:

$$\boxed{\frac{d^3\sigma}{dydp_t^2dM} = \frac{\alpha^2}{6\pi^2} \frac{\widehat{\sigma(s)}}{ME^2} \frac{R_2(s_2)}{R_2(s)}}. \quad (\text{A.0.8})$$

"Thus, it is suggested that among all proposed theories there must be some basic variables which will bring them all together"

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Bibliography

- [1] J. Adamczewski-Mush et al., Nature Physics 15, 1040 (2019).
- [2] T. Galatyuk et al., Eur. Phys. J. A52, 131 (2016).
- [3] G. Roche et al., Phys. Rev. Lett. 61, 1069 (1988).
- [4] A. Yegneswaran et al., Nucl. Instr. and Meth. A 290, 61 (1990).
- [5] R. Porter et al., Phys. Rev. Lett. 79, 1229 (1997).
- [6] G. Roche et al., Phys. Lett. B229, 228 (1989).
- [7] E. Bratkovskaya et al., Nucl. Phys. A634, 168 (1998).
- [8] G. Agakichiev et al., Phys. Lett. B663, 43 (2008).
- [9] G. Agakichiev et al., Phys. Lett. B690, 118 (2010).
- [10] G. Agakichiev et al., Phys. Rev. C84, 014902 (2011).
- [11] B. Kardan, Nucl. Phys. A967, 812 (2017).
- [12] M. Lorenz, Nucl. Phys. A967, 27 (2017).
- [13] P. Salabura, EPJ Web Conf. 137, 09008 (2017).
- [14] W. K. Wilson et al., Phys. Lett. B316, 245 (1993).
- [15] H. Nifenecker and J. P. Bondorf, Nucl. Phys. A442, 478 (1985).
- [16] J. G. Messchendorp et al., Phys. Rev. C61, 064007 (2000).
- [17] A.Yu. Korchin and O. Scholten, Nucl. Phys. A581, 493 (1995).
- [18] G.H. Martinus, Ph.D. thesis, University of Groningen (1998).
- [19] K. Lapidus et al., arXiv:0904.1128 [nucl-th] (2017).
- [20] T. Galatyuk et al., Int. J. Mod. Phys. A24, 599 (2009).
- [21] Adamczewski-Musch, et al. Eur. Phys. J. A56, 259 (2020). <https://doi.org/10.1140/epja/s10050-020-00237-2>
- [22] R. Shyam and U. Mosel, Pramana - J. Phys. 75, 185 (2010).
- [23] R. Shyam and U. Mosel, Phys. Rev. C82, 062201 (2010).
- [24] R. Shyam and U. Mosel, Phys. Rev. C79, 035203 (2009).
- [25] J. J. Sakurai, Ann. Phys. 11, 1 (1960).
- [26] M. Bashkanov and H. Clement, Eur. Phys. J. A50, 107 (2014).
- [27] G. Aznauryan and V. D. Burkert, Few Body Syst. 59, 98 (2018).
- [28] B. Aubert et al., Phys. Rev. D73, 012005 (2006).
- [29] J. J. Sakurai, Phys. Rev. Lett. 22, 981 (1969).
- [30] P. Adlarson et al., Phys. Rev. C95, 035208 (2017).
- [31] P. Salabura et al., EPJ Web Conf. 241, 01013 (2020).
- [32] G. Ramalho and M. T. Pena, Phys. Rev. D85, 113014 (2012).
- [33] Federico Scozzi, Ph.D. Thesis, Darmstadt Technical University (2020). DOI:10.25534/tuprints-00009713
- [34] N. P. Samios, Phys. Rev. 121, 275 (1961).
- [35] S. Devons et al., Phys. Rev. 184, 1356 (1969).
- [36] H. Fonvieille et al., Phys. Lett B233, 60 (1989).
- [37] C. M. Hoffman et al., Phys. Rev. D28, 660 (1983).
- [38] B. M. K. Nefkens et al., Phys. Rev. D18, 3911 (1978).
- [39] M. K. Liou and W. T. Nutt, Phys. Rev. D16, 2176 (1977).
- [40] S. M. Playfer et al., J. Phys. G13, 297 (1987).
- [41] C. A. Meyer et al., Phys. Rev. D38, 574 (1988).

-
- [42] L. A. Kondratyuk and L. A. Ponomarev, Sov. J. Nucl. Phys. 7, 82 (1968).
- [43] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 The Review of Particle Physics (2021). <https://pdg.lbl.gov/>
- [44] F. E. Low, Phys. Rev. 110, 974 (1958).
- [45] M. K. Liou and C. K. Liu, Phys. Rev. D 26, 1635 (1982).
- [46] R. Rückl, Phys. Lett. 64B, 39 (1976).
- [47] J. Zhang, R. Tabti, C. Gale, and K. Haglin, Int. J. Mod. Phys. E6, 475 (1997).
- [48] C. Gale and J. Kapusta, Phys. Rev. C35, 2107 (1987).
- [49] M. Schafer et al., Phys. Lett. B221, 1 (1989).
- [50] Gy. Wolf et al., Nucl. Phys. A517, 615 (1990). [https://doi.org/10.1016/0375-9474\(90\)90222-8](https://doi.org/10.1016/0375-9474(90)90222-8)
- [51] J. Markert et al., J. Phys. G. Nucl. Part. Phys. 34, S1041 (2007).
- [52] <http://www-hades.gsi.de>
- [53] J. Adamczewski-Musch et al., Eur. Phys. J. A53, 188 (2017).
- [54] P. Salabura, Nuclear Physics News, Vol. 25, No. 2 (2015), <http://www.nupecc.org/npn/npn252.pdf>.
- [55] J. Diaz et al., Nucl. Instr. and Meth. A478, 511 (2002).
- [56] G. Agakichiev et al., Eur. Phys. J. A41, 243 (2009).
- [57] K.E. Spear and J.P. Dismuke, Synthetic Diamond: Electro-chemical Society. Wiley, New York (1994).
- [58] J. Pietraszko, T. Galatyuk, V. Grilj, W. Koenig, S. Spataro, and M. Träger, Nucl. Instr. and Meth. A763, 1 (2014).
- [59] Alexander Marc Schmäh, PhD thesis, Technische Universität, Darmstadt, Germany (2008). https://tuprints.ulb.tu-darmstadt.de/992/2/A_Schmah_PHD_ULB.pdf
- [60] Michael Wiebusch, PhD thesis, Johann Wolfgang Goethe Universität, Frankfurt, Germany (2019). <http://publikationen.ub.uni-frankfurt.de/frontdoor/index/index/docId/50844>
- [61] C. Agodi et al., IEEE Trans. Nucl. Sci, vol.45, no.3, 665 (1998).
- [62] Szymon Harabasz, PhD thesis, Doctor of Physics at the Jagiellonian University in Kraków and the degree of dr.rer.nat. at the Technische Universität Darmstadt (2017).
- [63] D. Beller et al., Nucl. Instr. Meth. A602, 687 (2009).
- [64] A. Blanco et al., Nucl. Instr. and Meth. A661, S114 (2012).
- [65] G. Kornakov et al., Jour. Instr, 9, C11015 (2014). <https://iopscience.iop.org/article/10.1088/1748-0221/9/11/C11015>
- [66] A. Blanco et al., JINST 8, P01004 (2013), <https://iopscience.iop.org/article/10.1088/1748-0221/8/01/P01004/pdf>
- [67] A. Balanda et al., Nucl. Instr. and Meth. A531, 445 (2004).
- [68] O. Svoboda et al., JINST 9, C05002 (2014).
- [69] Jan Michel, PhD thesis, Johann Wolfgang Goethe-Universität, Frankfurt, Germany (2012).
- [70] Manuel Sánchez García, PhD thesis, University of Santiago-de-Compostela, Spain (2003).
- [71] P. Salabura et al., Nucl. Phys. B - Proc. Supp. 701 (1995). [https://doi.org/10.1016/S0920-5632\(95\)80106-5](https://doi.org/10.1016/S0920-5632(95)80106-5).
- [72] HADES, Ph.D. thesis repository, <https://www-hades.gsi.de/?q=node/126>
- [73] P. Sellheim, J. Phys. Conf. Ser. 599, 012027 (2015). 10.1088/1742-6596/599/1/012027
- [74] Patrick Sellheim, PhD thesis, Johann Wolfgang Goethe Universität, Frankfurt, Germany (2017). <http://publikationen.ub.uni-frankfurt.de/frontdoor/index/index/docId/44417>
- [75] M. A. Kagarlis, Pluto++. GSI Report, 2000-2003, unpublished.
-

-
- [76] I. Fröhlich et al., PoS ACAT2007, 076 (2007). <https://arxiv.org/abs/0708.2382>
- [77] J. Weil, H. van Hees and U. Mosel, Eur. Phys. J. A48, 111 (2012). <https://doi.org/10.1140/epja/i2012-12111-9>
- [78] CNS Data Analysis Center, SAID Home Page <http://gwdac.phys.gwu.edu>(2018)
- [79] A. D. Brody et al., Phys. Rev. D3, 2619 (1971).
- [80] J. Adamczewski-Musch et al., Phys. Rev. C102, 024001 (2020). <https://journals.aps.org/prc/abstract/10.1103/PhysRevC.102.024001>
- [81] A. Anisovich et al., Eur. Phys. J. A24, 111 (2005). <https://link.springer.com/article/10.1140/epja/i2004-10125-6>
- [82] W. Przygoda et al., EPJ Web of Conferences 81, 02014 (2014) <https://doi.org/10.1051/epjconf/20148102014>
- [83] <https://pwa.hiskp.uni-bonn.de/>;
- [84] M. Shrestha and D. Manley, Phys. Rev. C86, 055203 (2012).
- [85] L. P. Kaptari and B. Kampfer, Nucl. Phys. A764, 338 (2006).
- [86] R. Shyam and U. Mosel, Phys. Rev. C82, 062201 (2010).
- [87] M. Bashkanov and H. Clement, Eur. Phys. J. A50, 107 (2014). <https://doi.org/10.1140/epja/i2014-14107-9>.
- [88] W. Przygoda, habilitation thesis, Jagiellonian University, Kraków, (2018).
- [89] HYDRA - HADES analysis package, <http://www-hades.gsi.de/>.
- [90] ROOT- An Object Oriented Data Analysis Framework; <http://root.cern.ch/>
- [91] HGEANT. HADES Simulation Package; <http://www-hades.gsi.de/>
- [92] GEANT, Detector Description and Simulation Tool, online user guide, <https://cds.cern.ch/record/1073159/files/cer-002728534.pdf>
- [93] <https://www.geant.org/>
- [94] I. Frohlich et al., PoS ACAT 076 (2007) arXiv:0708.2382[nucl-ex]
- [95] W. Przygoda, JPS Conf. Proc. 10, 010013 (2016).
- [96] S. Constantinescu, S. Dita, and D. Jouan, Report PNO-DER-96-01 (1996). <https://www.osti.gov/etdeweb/servlets/purl/580056>
- [97] M. Gazdzicki and M. I. Gorenstein, arXiv:hep-ph/0003319 [hep-ph]. <https://arxiv.org/abs/hep-ph/0003319>.
- [98] M. Abreu et al., Eur. Phys. J. C14, 443 (2000). <https://doi.org/10.1007/s100520000373>
- [99] A. Adare et al., Phys. Rev. C81, 034911 (2010). <http://link.aps.org/doi/10.1103/PhysRevC.81.034911>.
- [100] J. Adamczewski-Musch et al., Phys. Rev. C95, 065205 (2017).
- [101] H.C. Eggers, C. Gale, R. Tabti and K. Haglin, arXiv preprint hep-ph/9604372 (1996). <https://inspirehep.net/literature/417857>