Classical and Quantum Computation of Strongly Correlated Systems

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INTRODUCTION

Strongly correlated condensed matter systems are one of the greatest challenges of the contemporary physics [1]. Correlation effects induced by strong interactions are most prominent in one and two space dimensions (1D and 2D) where it is more difficult for particles to avoid each other than in 3D. The strong correlations result in exotic states of matter like the Luttinger liquid in 1D or the high-temperature superconductivity and topologically ordered states with anyonic excitations in 2D. The strongly correlated systems are promising candidates for quantum-technological applications like, e.g., topological quantum computers [2], but at the same time they are notoriously hard to solve by either analytical or numerical methods. Typically they have no weakly interacting regime that could be a starting point for a systematic perturbative expansion.

The general problem is somewhat less challenging in 1D with its prominent examples of exactly solvable integrable systems and powerful numerical methods like the density matrix renormalization group (DMRG) [3]. The DMRG provides essentially exact results for the ground states with a finite energy gap equivalent to a finite correlation length and a finite entropy of entanglement between two "halves" of an infinite chain. The finite entropy means that the ground state satisfies a 1D version of the *area law* for entanglement [4]. The power of the DMRG comes in part from the possibility to make the Schmidt decomposition between the left and right halves of a 1D chain. This is less useful in 2D.

In 2D the above problems return in their full complexity. Here the analytic methods often boil down to different versions of the mean-field theory that turn out to be unreliable in most interesting situations. Variational wave-functions – and variational quantum Monte Carlo algorithms – are biased towards the assumed class of wave-functions, while the unbiased quantum Monte Carlo is plagued with the notorious sign problem originating either from frustration or fermionic statistics. In this situation quantum tensor networks emerge as a promising candidate to make a breakthrough in a wide class of systems that are beyond reach of the quantum Monte Carlo [5–11]. As variational methods, they are free of the sign problem but, at the same time, the only bias introduced by a tensor network variational ansatz is that the entanglement entropy must be limited by a 2D version of the area law: the entropy of entanglement of a subsystem with the rest of the lattice is proportional to the size of the subsystem's boundary. In a wide class of gapful ground states that satisfy the area law, tensor networks can be considered as unbiased. The same is expected for thermal states of quantum Hamiltonians. By definition, the Gibbs state is the mixed state with the maximal entropy possible for a given average energy. Since this maximal entropy is actually the entropy of entanglement between the system and the rest of the universe, then – thanks to the monogamy of entanglement – there is little entanglement left between different parts of the system. This intuition is made rigorous by, e.g., an area law for mutual information satisfied by any thermal state [12].

2D tensor networks [7–9, 11] can be considered a generalization of the 1D DMRG in the sense that the variational ansatz that is optimized by the DMRG algorithm is a matrix product state (MPS), an example of a 1D tensor network. Figure 1 shows the PEPS ansatz that is

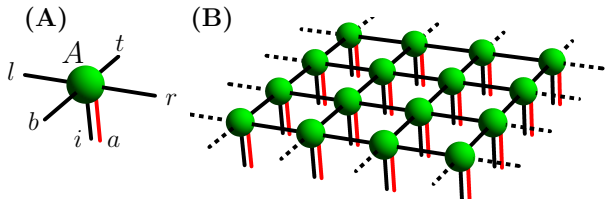

FIG. 1. This figure shows a PEPS tensor network on an infinite square lattice. Here we show only part of the whole network corresponding to 4×4 sites of the infinite lattice. The dashed lines connect this part with (not shown) rest of the network. In A, the basic building block is a tensor A with physical indices i, a numbering physical states on a single lattice site (say, spin and orbital degrees of freedom, respectively), and bond indices t, r, b, l that provide entanglement with other sites on the lattice. In B, the A-tensors are contracted through the bond indices to provide the probability amplitude for different basis states in the Hilbert space on the square lattice. In a sense, the matrix product state (MPS) in 1D can be obtained as a single column or row of this PEPS ansatz.

a 2D version of the 1D MPS. However, since – unlike in MPS – cutting a single bond in the PEPS ansatz is not equivalent to an orthogonal Schmidt decomposition, the algorithms to optimize the PEPS ansatz [7, 13] have to be founded on very different principles than the powerful DMRG algorithm. In spite of this challenge, the PEPS provided the best approximations, with the lowest energies ever achieved, to the ground states of the paradigmatic $t - J$ and Hubbard models [14] of the high-temperature superconductivity. By no means less spectacular was the solution to the long-standing magnetization-plateaux puzzle in Ref. [31].

Motivated by the success of PEPS in the study of strongly correlated ground states, one of the objectives of the present project is to develop tensor networks for efficient computation of thermal states of 2D strongly correlated quantum systems. We also want to develop a new PEPS-based algorithm to search for topological order in ground states of strongly correlated systems.

The starting point for the work on thermal networks will be our original proposal [15], where a tractable 2D thermal density matrix is obtained by dimensional reduction of an intractable 3D network that is an exact representation of the thermal state on the 2D lattice. This algorithm was successfully tested in a number of benchmark applications including the high-temperature regime of the Hubbard model [15, 16]. One of the objectives of this project is to upgrade and then apply the algorithm to find finite-temperature phase diagrams that are beyond the range of applicability of more conventional methods. The Holy Grail is a complete finite-temperature phase diagram of the Hubbard model, but characterizing unknown phase diagrams of frustrated spin systems would also be quite fulfilling.

The starting point for the work on 2D topological tensor networks will be some of the general ideas formulated in the classic paper [17] that have not been implemented in practice yet. We are working on a more powerful PEPS algorithm able to find a topologically-ordered ground state. We also want to develop a new tensor network algorithm suitable for detecting and characterizing the topological order in the PEPS network representing the ground state. A combination of these two algorithms could be used in a systematic search for Hamiltonians with topologically ordered ground states and thus pave a way to physical implementation of topological quantum computers.

As mentioned above, the area law for quantum entanglement is a practical limit for the applicability of tensor networks. Some critical ground states just as well as most dynamical ones – that are able to develop *volume law* in a short time – remain beyond their reach. In these very interesting regimes of large entanglement great hopes are placed in quantum computers or their less challenging version, i.e., quantum simulators. Today around 150 quantum simulators are in operation all over the world. They vary in technology and quality, the degree of their actual "quantumness" is debatable, but they all to a greater or lesser extent are based on the idea of an adiabatic quantum simulator/computer [18–20]. The most explicit implementation of the adiabatic quantum simulator (AQS) is the famous, and somewhat controversial, D-Wave [21]. The idea of AQS is to evolve adiabatically from a simple ground state of an initial Hamiltonian

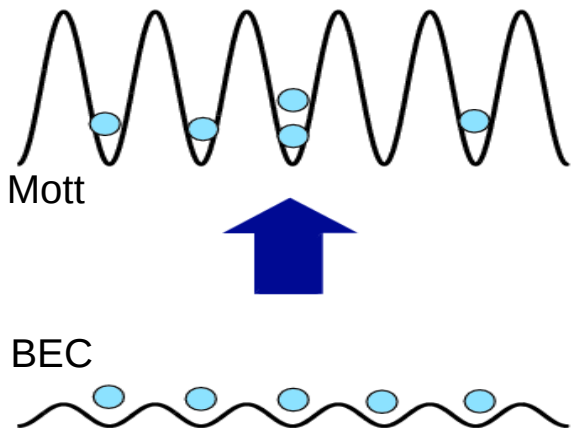

FIG. 2. A typical example of adiabatic quantum state preparation in ultracold atomic gases. Initially a Bose-Einstein condensate is prepared. This state is *simple* in the sense that it is relatively easy to prepare by evaporative cooling. Once the atoms are condensed, an optical lattice potential is turned on. Ideally, the evolution is adiabatic and the atoms finally end in the Mott insulator ground state in a deep lattice potential. This *interesting* ground state is approximately the Fock state with precisely 1 atom in each lattice well. Unfortunately, the final Mott state and the initial superfluid state are different enough to be separated by a quantum phase transition with a gap vanishing at the critical point. Consequently, the evolution is not adiabatic and the final state is excited with respect to the desired ground state of the final Hamiltonian. Here the excitations are the empty and doubly occupied wells. The number of excitations and the excitation energy depend on the rate at which the lattice potential is turned on. We quantify this rate by a time-scale τ_Q known as a "quench time" for historical reasons. The Kibble-Zurek mechanism (KZM) predicts that the excitation decays with an inverse power of the quench time.

to an interesting ground state of a final Hamiltonian. Unfortunately, the simple and the interesting are often different enough to be separated by a quantum phase transition, see figure 2. Therefore, in order to perform its function, the AQS must be able to evolve adiabatically in a neighbourhood of a quantum critical point. This evolution is described by a quantum generalization of the Kibble-Zurek mechanism (KZM), see the review papers [22, 23]. One of the objectives of this project is better understanding of the influence of different imperfections, like disorder or dissipation and decoherence, on a very susceptible adiabatic ground state near a quantum critical point. This knowledge may help to construct more stable versions of the D-Wave and other machines [24].

OBJECTIVE I: THERMAL TENSOR NETWORKS

Significance

While the tensor network community is mainly occupied with increasingly more accurate calculation of ground states we – encouraged by their success – proposed a tensor network to calculate thermal states of strongly correlated systems [15]. Its general structure is shown in figure 3. Like all tensor networks, it is free of the fermionic sign problem. Its accuracy was demonstrated in benchmark applications to the 2D quantum Ising model, the quantum compass model, and the Hubbard model [15, 16]. Using modest numerical resources (a few desktops) we achieved accuracy comparable to the most accurate conventional unbiased numerical methods (quntum Monte Carlo, cluster DMFT). Our benchmark results in the strongly interacting regime of the Hubbard model are shown in figure 4. It is time to attack unsolved problems that are beyond the conventional means. Preliminary results of our first attempt are shown in Fig. 5. We think that, after some improvements, our thermal network may repeat the success of the ground-state networks and become the standard method of choice for finite-temperature problems that are beyond reach of the quantum Monte Carlo.

Workplan

This research will follow two main lines:

- application of the existing algorithm to obtain phase diagrams of frustrated and fermionic systems;
- conceptual developments in order to attack the most challenging problems like full characterization of the finite-T phase diagram of the Hubbard model.

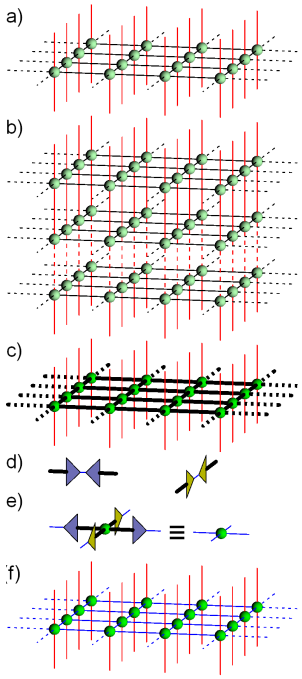

FIG. 3. This figure shows the general structure of the thermal tensor network proposed by us in Ref. [15]. The Gibbs operator $e^{-\beta H}$, where β is the inverse temperature and H the Hamiltonian, on an infinite square lattice can be written with Suzuki-Trotter decomposition as a 3D tensor network represented by the diagram in (b). This diagram is build from 2D horizontal layers (one of them shown in (a)), each of them corresponding to an operator $e^{-d\beta H}$, where $d\beta$ is a small time step in the decomposition. The dashed line convention from Fig. 1 is used here, as shown part of the network corresponds to 4×4 sites of the infinite lattice and 3 of many layers. Each ball represents a rank-6 tensor with its 6 indices represented by 6 lines sticking out of it. A line connecting two balls represents tensor contraction. Each (red) vertical line is an index numbering basis states on its lattice site. The black (contracted) indices are called the bond indices The third vertical dimension is the imaginary time β. After making all red contractions the 3D network in (b) becomes formally a 2D network in (c), but the bond dimension of its (thick black) bond indices is huge. These indices have to be truncated, see (e), by the projectors in (d). The result of this truncation is the 2D network in (f) with a small truncated bond dimension D (limiting entanglement in the truncated state) of the blue indices. Thanks to the area law for entanglement, for large enough D this truncation does not affect the accuracy of the thermal network provided that the projectors (d) are chosen optimally. To optimize the huge projectors efficiently, they are represented by another tensor network made of small isometric tensors which are optimized one-by-one.

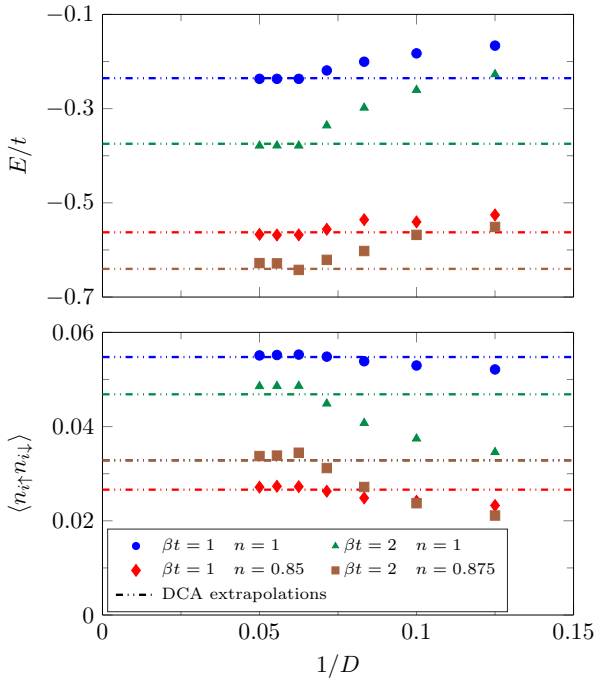

FIG. 4. This figure, originating from Ref. [16], shows high-temperature benchmark results in the Hubbard model: $H = -\sum_{\langle i,j\rangle\sigma} t\left(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma}\right) + \sum_i U\left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right)$, where $c^\dagger_{i\sigma}$ and $c_{i\sigma}$ create and annihilate, respectively, an electron with spin $\sigma = \uparrow, \downarrow$ on site i, $n_{i\sigma} = c^\dagger_{i\sigma}c_{i\sigma}$ is the number operator, $U > 0$ is on-site repulsion strength, and $\langle i, j\rangle$ denotes summation over nearest neighbour (NN) pairs with hopping energy t. The points in the figure show typical benchmarks for this model: total energy E per site and double occupancy $\langle n_{i\uparrow}n_{i\downarrow}\rangle$ as a function of the bond dimension $8 \leq D \leq 20$ obtained in strongly correlated half-filled and underdoped regime - $U/t = 8$ with our tensor network shown in Fig. 3 (details in Ref. [16]). Circles, triangles, squares and diamonds mark different inverse temperatures β and electron densities n (per lattice site). For comparison, lines show corresponding results from cluster DMFT extrapolated with the cluster size sent to infinity [25, 26], which is considered to be the best unbiased numerical method in this regime [25].

Encouraged by our benchmark tests, we believe that our present algorithm is ready for a systematic study of most frustrated spin systems with nearest neighbour interactions and some paradigmatic fermionic and spin-orbital models like spinless interacting fermions on honeycomb lattice [27]). Including additional next-nearest-

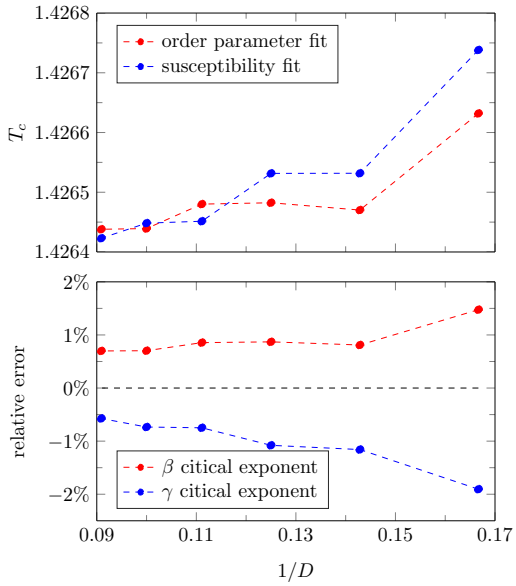

FIG. 5. Preliminary results for a phase transition in the quantum e_g model on an infinite square lattice: $H = -\sum_{\langle ij\rangle||a} \tau_i^a \tau_j^a - \sum_{\langle ij\rangle||b} \tau_i^b \tau_j^b$, where $\sum_{\langle ij\rangle||a}(\sum_{\langle ij\rangle||b})$ denotes a sum over nearest-neighbour bonds along the a-axis (b-axis), $\tau^a = \frac{\sqrt{3}}{2}\sigma^z + \frac{1}{2}\sigma^x$, $\tau^b = -\frac{\sqrt{3}}{2}\sigma^z + \frac{1}{2}\sigma^x$ and σ^x, σ^z are Pauli matrices representing orbital degrees of freedom. A second order phase transition is expected to a phase with non-zero orbital polarization $\langle\tau^z\rangle$ (one of the orbitals is occupied more frequently than the other). Estimates of its critical temperature T_c and critical exponents β, γ are obtained for different values of the bond dimension $6 \leq D \leq 11$. The critical temperature T_c is converged in D up to 4 significant digits and the estimates of the critical exponents agree with the expected 2D Ising universality class ($\beta = 1/8$, $\gamma = 7/4$) within 1%. They approach the exact ones with increasing D. The phase diagram of this model is inaccessible to quantum Monte Carlo because of the sign problem.

neighbour interactions is straightforward and can further increase its range of applicability. To begin with, we plan to apply the algorithm to the spin-orbital e_g-model [10, 28], the Kitaev-Heisenberg model [29], the spin-1 Heisenberg model on the hexagonal lattice [30], and the Shastry-Sunderland model [31]. Preliminary results for the e_g model are shown in Fig. 5. Ground states of these models were already simulated by tensor networks and some major breakthroughs were made like the solution of the long-standing magnetization-plateaus puzzle [31]. A systematic study of their finite-T phase diagrams will provide further data to be compared with experiments.

At temperatures far below the benchmark in figure 4, the Hubbard model still remains a hard problem. Rather than to push the present algorithm towards lower temperatures by a brute force, we want to rethink some of its basic principles, make improvements, and then to attack this notoriously difficult problem again with an upgraded version.

Methodology

The application of the existing algorithm will require optimizing the thermal network for different values of Hamiltonian parameters and temperatures in order to scan the phase diagram. In analogy to the state of the art PEPS ground state simulations [14, 31], large unit cells will be used to allow for states with many different patterns of translational symmetry breaking.

The conceptual improvements may open a path to efficient simulation of the Hubbard model. They include

- generalization of the orthogonal projectors used in the current ansatz, see Fig. 3d, to the most general non-orthogonal ones [32] that were successfully used to improve efficiency of tensor networks in a different context [14];

- redefining the figure of merit for projector optimization to target directly the desired observables;

- generalization of the finite entanglement scalings (FES) – proposed originally [33] for quantum phase transition in 1D systems – to our method in order to improve the accuracy of critical temperatures and exponents.

The finite entanglement scalings are equivalent to the finite size scalings known in the quantum Monte Carlo [34] where they are essential for its unmatched precision and reliability. Combining FES with our method can be a crucial step

towards obtaining reliability and precision comparable with the best unbiased quantum Monte Carlo. It may bring the Monte Carlos's precision and reliability to systems with a sign problem that are not tractable by the Monte Carlo itself.

Collaboration

We hope to benefit from a collaboration with Prof. Philippe Corboz from Amsterdam who is a leader in applications of the PEPS network to ground states of strongly correlated systems.

OBJECTIVE II: TOPOLOGICAL TENSOR NETWORKS

Significance

The quantum topological order is a non-local phenomenon evading the Landau classification based on local order parameters [37]. It supports anyonic excitations that can be employed in topological quantum computation. Thanks to the non-locality the computation is going to be robust against local decoherence. There is an urgent demand for strongly correlated systems with topologically ordered ground states. In order to meet this demand an algorithm was developed where a quasi-1D tensor network is used to study topological order in the ground state of a 2D strongly correlated system [17], see figure 6b. However, a mismatch between the structure of the 1D network and the 2D lattice limits its applicability to ground states with very small correlation lengths. In this project, encouraged by the recent breakthroughs in minimization of PEPS energy [13], we will follow the general strategy of the classic paper [17] but with the 2D PEPS in place of the 1D network. Since the structure of the 2D PEPS can match that of the 2D lattice, see figure 6a, in principle there are no upper limits to the correlation lengths that can be considered. If successful, our project will provide numerical tools for a systematic search for topological order in strongly correlated systems.

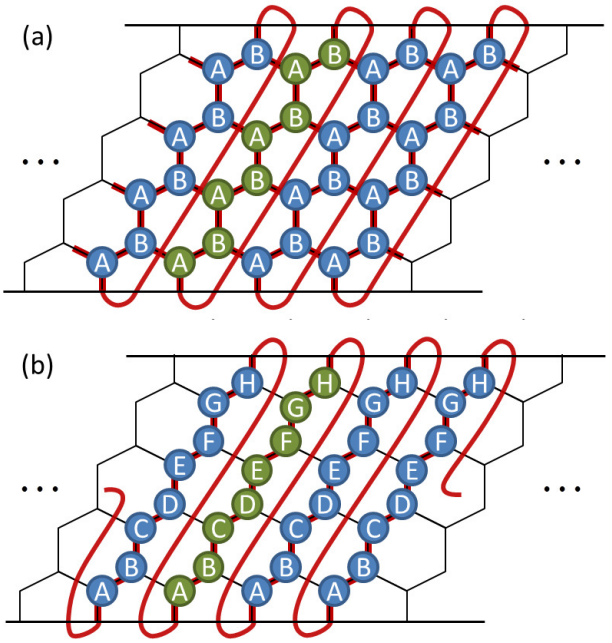

FIG. 6. This figure comes from the pioneering work [17]. It shows how to describe a topologically-ordered ground state on an infinite cylinder with periodic boundary conditions in the vertical direction. Panel (a) shows a natural covering of the cylinder with a 2D PEPS tensor network. The red lines, connecting the sites of the hexagonal lattice, represent the structure of this tensor network that is also hexagonal. Panel (b) shows a less natural covering with a quasi-1D network used in DMRG calculations. The red lines represent its quasi-1D structure. Thus far method (b) was mainly developed (but see [35]) because it could benefit from the well established DMRG algorithms. However, due to the incompatibility of the 1D tensor network with the 2D lattice, its applicability was limited to very small correlation lengths. Recent breakthroughs in the PEPS algorithms [13] encourage us to renew the idea (a), where the correlation length can in principle be arbitrarily large.

Workplan

We want to create a comprehensive numerical tool made of two key elements:

- an algorithm to minimize the energy of a 2D PEPS tensor network representing the ground state of a topologically ordered system on an infinite 2D lattice;
- an algorithm to detect and characterize the topological order in the PEPS ap-

proximating the ground state suspected of topological ordering.

These two elements can be developed either in parallel or one after another.

Methodology

In order to obtain sufficiently accurate representation of ground states with the PEPS tensor network, we want to develop further the new variational methods to minimize its energy [13]. We will go beyond the nearest neighbour couplings in a Hamiltonian and introduce at least the next-nearest-neighbours. Furthermore, in the iterative optimization procedure we will update clusters of a few lattice sites rather than individual tensors on individual lattice sites. This will make the minimization more robust against being trapped in local energy minima.

In order to detect and characterize topological order, a PEPS representing the unique ground state on an infinite lattice can be wrapped on a cylinder or a torus, as shown in figure 6a, where it becomes a linear combination of degenerate ground states. These ground states are locally indistinguishable but topologically different. Matrix product states are going to be employed to represent projectors on the different ground states. When projected out, overlaps between ground states, subject to modular transformations, will reveal their topological universality class.

Collaboration

We plan a collaboration with Dr Łukasz Cincio – currently an Oppenheimer Fellow in the Los Alamos National Laboratory and formerly a 5-year postdoc in the Perimeter Institute – who is a leader in application of DMRG to topological order [17].

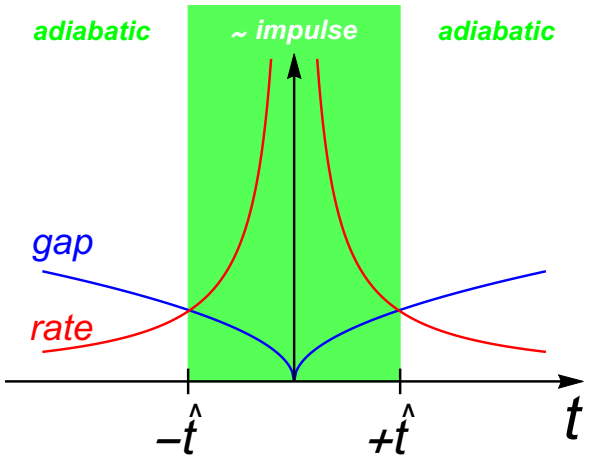

FIG. 7. When a quantum Hamiltonian is slowly driven across a quantum critical point, its energy gap closes at the critical point and, at the same time, the relative transition rate diverges to infinity. As a result both at the time $t = 0$ of crossing the critical point and in its neighbourhood from $-\hat{t}$ to $+\hat{t}$, the evolution cannot be adiabatic. The system must get excited and – according to the quantum Kibble-Zurek mechanism [22] – the excitation energy should decay with a power of the transition time. The characteristic exponent depends on the universality class of the quantum phase transition [22].

OBJECTIVE III: ADIABATICITY OF QUANTUM SIMULATORS

Significance

Adiabatic quantum simulators (AQS) are expected to become an indispensable tool for quantum computation of strongly correlated systems. They enter the game where tensor networks on classical computers must give up, i.e., when the quantum states of interest become entangled more than the limit set by the area law. The most promising implementations of AQS are atom/ion traps [19, 36] or superconducting structures like the D-Wave [21]. An AQS is supposed to evolve adiabatically – following a time-dependent Hamiltonian – from a simple initial ground state to an interesting final ground state, but the simple and the interesting are often different enough to be separated by a quantum phase transition. A slow evolution near a quantum critical point is described by a quantum version of the Kibble-Zurek mech-

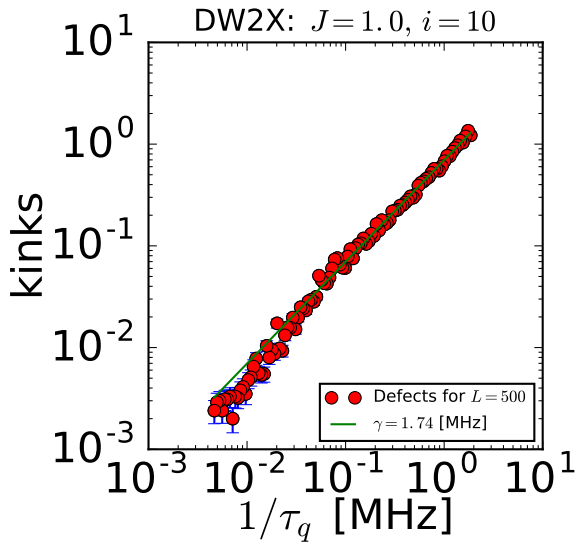

FIG. 8. This figure shows preliminary experimental results from quantum simulations of the quantum Kibble-Zurek mechanism (KZM) in the transverse-field quantum Ising chain with the D-Wave 2X adiabatic quantum simulator (Los Alamos). An average number of kinks is shown in the final ferromagnetic chain of 500 spins/qubits after a linear quench of the transverse field g^x with a quench time τ_q. The number of excitations decays with a power of τ_q, as expected from KZM, but the exponent is -1 instead of the $-1/2$ predicted in the classic paper [45]. Either weak disorder or decoherence and dissipation are candidate culprits.

anism (KZM), see the review papers [22, 23] and figure 7. This theory is already supported by a number of experiments [38]. Its most refined version – the KZ scaling hypothesis [39] – was confirmed by a recent beautiful experiment [40]. However, improving precision of experiments is revealing potential influence of various imperfections, like disorder [41] or coupling to external environment causing decoherence and dissipation [42], on a very susceptible quantum state of a system attempting an adiabatic evolution near a quantum critical point. These effects will not be negligible in any AQS and in particular they are not negligible in the D-Wave 2X machine, see our preliminary experimental results in figure fig:dwaveKZM. The effects of disorder were considered in early theoretical papers [43] but their experimental relevance was appreciated only recently [41]. More systematic understanding of these phenomena and incorporating them into the general framework of the KZM may help to construct more accurate adiabatic quantum simulators.

Workplan

This project will require going back to the roots and re-examining some of the most basic notions underlying the KZ physics. To this end the quantum Ising chain is usually an indispensable testing ground [43, 45]. Its Hamiltonian reads

$$H = -\sum_{l=1}^{L-1} J_l \sigma_l^z \sigma_{l+1}^z - \sum_{l=1}^{L} (g_l^x \sigma_l^x + g_l^z \sigma_l^z). \quad (1)$$

Here J_l are coupling strengths, g_l^x are transverse fields, and g_l^z are longitudinal fields. Both in theory [43, 45] and in the D-Wave machine the adiabatic evolution is driven by a ramp of a uniform transverse field g^x from a large initial value down to $g^x = 0$. Were it an adiabatic evolution, the state would follow the ground state from the initial state of transverse-polarized spins/qubits to the final ferromagnetic state. In the pure Ising chain, without any longitudinal field g_l^z and with the couplings and transverse fields independent of l, $\tau_q^{-1/2}$ excitations/kinks are expected to be found in the final ferromagnetic state [45], where τ_q is a transition/quench time.

The plan is to consider first the influence of uniform and random longitudinal fields g_l^z on the final number of excitations. Even when turned-off, such fields cannot be completely eliminated in the D-Wave AQS, and even when very small their influence must be significant given the divergent linear susceptibility at the critical point. The next step is to couple the chain to Markovian environments whose influence can be modelled by Lindblad superoperators in the master equation. Finally, non-Markovian environments can also be considered.

Methodology

In the absence of longitudinal fields, the Ising chain is solvable by the Jordan-Wigner and Bogoliubov transformations. Dynamics of the Ising

chain coupled to external environment can often be described in terms of quadratic fermionic correlators that decouple from higher-order correlators. A quantum state during a slow evolution across a critical point satisfies the area law for entanglement [44] and thus can in principle be simulated with tensor networks. Finally, we will have access to the newest D-Wave 2X in Los Alamos to experiment with quantum simulations.

Collaboration

A long standing collaboration with Prof. Wojciech Zurek from the Los Alamos National Laboratory will be sustained. A new collaboration with Dr Bartłomiej Gardas – currently a 3-year FUGA postdoc in our group and formerly a 2-year postdoc in Los Alamos – will be enhanced.

RESEARCH AND TRAINING POTENTIAL

The project leader (PL) is an author of one of the pioneering papers on the dynamics of quantum phase transitions [45] (237 citations) and an invited review article on the same subject [22] (243 citations). He also contributed to some of the first applications of quantum tensor networks in two spatial dimensions [9, 10, 46]. PL was an advisor of 5 PhD students, four of them continue their careers in science doing postdocs in Australia, Austria, Canada, Finland, Netherlands, Italy, and the USA, in places like the famous Perimeter Institute and the legendary Los Alamos National Laboratory. Their hosts include international stars like Philippe Corboz (Amsterdam), Lew Pitaevskii and Sandro Stringari (Trento), Guifre Vidal (Perimeter), Frank Verstraete (Vienna), and Wojciech Zurek (Los Alamos). One of PL's present students is a laureate of the prestigious Diamond Grant for young researchers from the Polish Ministry of Science and Higher Education. If funded, this project will provide a high quality environment for training further students in international research collaboration. It is PL's principle to face young people with serious scientific problems from the very beginning of their careers.

SUMMARY

The proposed research is timely. The quantum tensor networks matured enough to attack important realistic models of condensed matter physics. At the same time, the adiabatic quantum simulators gained precision where even small disturbances begin to have qualitative consequences. The whole research area is promising lots of exciting and unexpected new developments in the near future. We stay in touch with the leading experts and expect to obtain results publishable in the leading journals. Finally, the project will give its young participants an opportunity of exciting research in an international environment already at the very beginning of their scientific careers.

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