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Selected Features of Chiral Doubling for Hadrons

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Chapter 1

General Introduction

The beginning of the XXI century started the new era in the experimental physics of charm. Several major experiments have observed unexpected patterns in the spectra of open and hidden charm, and these observations have surprised almost everyone. Let us outline some of these experiments and their spectacular results:

- In April 2003 BaBar Collaboration [1] has announced new, narrow meson $D^{\star}_{sJ}(2317)^{+}$, decaying into D^{+}_{s} and π^{0}. In May this observation was confirmed by CLEO Collaboration [2], which also noticed another narrow state, $D_{sJ}(2460)^{+}$, decaying into $D^{\star}_{s}$ and π^{0}. Both states were confirmed by Belle Collaboration [3], and finally, the CLEO observation was also confirmed by BaBar [4].

- In July 2003 Belle Collaboration measured the narrow excited states D_1, D_2 with foreseen quantum numbers $(1^{+}, 2^{+})$, and provided the first evidence for two new, broad states $D^{\star}_{0}$ $(2308 \pm 17 \pm 15 \pm 28)$ and $D^{'}_{1}$ $(2427 \pm 26 \pm 20 \pm 17)$ [5]. Both of them are approximately 400 MeV above the usual D_0, D^{*} states and seem to have opposite to them parity. Recently, observation of the D^{*}_{0} was confirmed by FOCUS Collaboration [6] and also CLEO observed second state $D^{'}_{1}$ [2]. Above states were seen in decays mode with pion, i.e. $D^{\star 0}_{0} \rightarrow D^{+}\ \pi^{-}$, $D^{\star +}_{0} \rightarrow D^{0}\ \pi^{+}$ and $D^{'\,0}_{1} \rightarrow D^{\star +}\pi^{-}$.

- SELEX Collaboration has provided preliminary data for doubly charmed baryons [7]. On top of known since 2002 state $\Xi^{+}_{cc}(3520)$ (ccd), four other cascade (conjectured as $j_{\ell} = 1/2$ states) are visible, in particular the pair of opposite chirality ccu states separated by the mass gap of the order 337 MeV.
Few months after, SELEX Collaboration also announced a new, surprisingly

narrow state $D_{sJ}^{+}(2632)$ [8], which mainly appeared in $D_{sJ}^{+} \to D_s^{+}\eta$ decays. However, till today neither of these states has been confirmed by other experiments.

- H1 experiment at DESY has announced [9] a signature for charmed pentaquark Θ_c^0 ($\bar{c}udud$) at mass 3099 MeV, *i.e.* approximately 400 MeV higher than the expected estimates known in the literature [42, 11, 12, 43].
Till today this H1 state was however not confirmed by other experiments.

- In July 2006 BaBar Collaboration [14] observed a new member of $\bar{c}s$ family decaying into D^0K^+ and $D^+K_s^0$, a narrow $\Gamma = (48 \pm 7 \pm 10)$MeV with estimated mass $(2865.6 \pm 1.5 \pm 5.0)$ MeV. In the same mass distribution they also found a broad structure X with mass of $(2688 \pm 4 \pm 3)$MeV and width $\Gamma = (112 \pm 7 \pm 36)$ MeV. Information about possible quantum numbers for both is still unavailable.

- In August 2006 Belle Collaboration [15] reported the observation of a new D_{sJ} meson with a spin-parity (1^-) and a mass of $(2715 \pm 11^{+11}_{-14})$ MeV. Its width was determined to be $\Gamma = (115 \pm 20)$ MeV. This charmed, strange state needs to be confirmed.

All above states refer to the newest observation in open charm physics. Till the end of XX century quark model claimed to explain the open charm spectra, but the new narrow D_s states with positive parity did not match any predicted pattern. Fortunately, from theoretical point of view, it is obvious that heavy-light systems belong to the realm of strictly non-perturbative physics, hence the possibility of better understanding of these states using the concepts of non-perturbative physics.

However, equally surprising experimental results appeared soon in the physics of hidden charm. The hidden charm mesons were considered to be relatively well understood using the concept of charmonia in the family of potential models, assuming that charm is heavy enough to look at mesons as hydrogen-like spectra of the bound charmed quark and antiquark orbiting around each other.

We list here some of the recent results in the hidden charm sector.
- In 2003 Belle Collaboration [20] discovered a new charm resonance marked as $X(3872)$ in B decays. This new state was quickly confirmed by other major experiments [21]. Till today this state has been observed in five major experiments (Belle, CLEO, BaBar, D0, CDF). Since it lies above $D\bar{D}$ threshold, it has got many features in common with a so-called deuson [22], i.e. a loosely bound state of $D^0\bar{D}^{*0}$ mesons. Other possible interpretation of

$X(3872)$ include a tetraquark description, i.e tightly bound charm diquark-antidiquark state [23].

- In 2004 Belle Collaboration [24] observed another state denoted by $Y(3940)$ and produced in $B^+ \to K\omega J/\psi$ decay. According to one hypothesis this new state may be viewed as a $c\bar{c}$-gluon hybrid.

- In 2005 BaBar Collaboration [25] has studied processes in $e^+e^- \to \gamma\pi^+\pi^- J/\psi$ and found a broad structure with mass near 4.26 GeV. A new state was marked as $Y(4260)$ with quantum numbers $J^{PC} = 1^{--}$ and soon confirmed by CLEO [26] and Belle [27] Collaborations. Its mass is consistent with a conventional charmonium state $\psi(4S)$ or may be interpreted as a tetraquark ($cs\bar{c}\bar{s}$) [28].

- One year after, in August 2007 Belle Collaboration [16] found the new narrow structure denoted by $Z^{\pm}$, with an accurate mass $M = (4433 \pm 4 \pm 2)$MeV and the width about $\Gamma = 45$ MeV. The state was seen in decay via $(\psi' + \pi^{\pm})$. The uniqueness of this state stems from the fact that it is the first charmonium-like meson candidate with nonzero electric charge, which excludes the minimal $c\bar{c}$ content. Many interpretation have been proposed. The Z state was viewed for example as a bound state of the diquark-antidiquark $cu\bar{c}\bar{d}$ and interpreted as a first radial excitation of the multiplet included tetraquarks $X(3872)(cu\bar{c}\bar{u})$ and $X(3876)$ $(cd\bar{c}\bar{d})$.

- In June 2008 Belle Collaboration [17] observed the two additional states of similar character (charged) with the resonance parameters: ($M_1 = 4051$ MeV, $\Gamma_1 = 82$ MeV) and ($M_2 = 4248$ MeV, $\Gamma_2 = 177$ MeV). The doubly peaked structure was seen in $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ decays. However, neither of the charged states has been observed or confirmed by BaBar experiment.

- Since 2007 Belle [18] reported the evidence of several other resonances named $Z(3930)$, $X(3940)$, $Y(4008)$, $X(4160)$, $Y(4350)$ (also seen be BaBar) and $Y(4660)$. The first state may be interpreted as a missing $\chi_{c2}(2P)$ state, the nature of the second is unknown, the third and fourth might be suspected to be $\psi(3S)$ and $\chi_{c0}(3P)$ states, respectively, the nature of $Y(4350)$ is unknown and the last may look like $\psi(5S)$ excitation.

In the light of these discoveries, we definitely may expect that the charm family is still growing and new other intriguing members might appear soon. At this point we would like to explain why the plethora of these states have triggered such an interest among several theorists. There are several reasons for this excitement.

QCD is the correct, but still unsolved theory of the strong interactions. The direct approach, based on massive lattice simulations of the spectra, is certainly successful, but only last year the sizes of lattice and type of physical fermions algorithms allowed to reach the physical mass of the pion, hence the very precise simulations of heavy-light hadrons are still not available. In such situation we have either to look at some "corners" of the full QCD, where certain approximations are justified and we may build a systematic expansion in sets of parameters or we have to rely on models, very often not justified rigourously enough but surprisingly successful at the phenomenological level.

One of the major insights how to organize the calculation is based on identifying proper scales and associated with them exact or approximated symmetries. We were guided by the two issues:

1. Let us start from the heavy-light mesonic system, i.e. the meson with the minimal content of the heavy quark and the light antiquark (or its complex conjugated partner). If we consider an infinitely heavy quark, we know that the heavy quark spin symmetry appears (named also as an Isgur-Wise (IW) symmetry), due to the fact that heavy spin decouples from angular momentum of the light quark j_l. The lowest partial wave of the light quark corresponds to $l = 0$, so the angular momentum equals to the spin of the light quark. Combining all light and heavy components, we get a well known pair of states $(0^-, 1^-)$, degenerated in the infinitely heavy quark limit. The observed experimentally split between these states effect scales like $1/m_Q$, where m_Q is the mass of the heavy quark, and systematic expansion in this parameter is the basics of Heavy Quark Effective Theory (HQET).

Going up in angular momentum l we find two other pairs $(1^+, 2^+)$ and $(0^+, 1^+)$. The observations of BaBar, CLEO and Belle in strange sector $(2317, 2460)$ match the spin-parity of the last pair. It was clear that the first puzzle was not the presence of these new states, but the value of their masses and decay patterns. Standard estimations based on the quark potential models (QM) were placing these states approximately 150 MeV higher then observed. Such states were therefore expected to be broad, however the observed ones were extremely narrow (with width below 10 MeV, i.e. the resolution of the experiments). The second challenge was the pattern of splitting between the opposite parity states, i.e. the mass difference between 0^+ and 0^- turned out to be identical to the mass difference between 1^+ and 1^-. The third challenge was to understand the observed decays - both hadronic and electromagnetic.

These were the three reasons which renewed an interests on open charmed hadrons spectroscopy in theoretical physics. Several possible theoretical constructions were proposed to explain the masses and possible quantum numbers. Let us briefly remind here some of those appeared in literature [30]. New charmed states were interpreted either as molecular configuration alike $D\pi$ atoms or DK molecules, their properties were studied in the framework of modified quark model (QM) and via lattice simulation. There was also another intriguing possibility, based on the concept formulated a decade [33] before the above mentioned experiments, that the new states are the result of the restrictions on the heavy-light systems imposed by the spontaneous breakdown of the chiral symmetry of the light quarks. In other words, simultaneous constraints emerging both from infinitely heavy quark sector (IW symmetry) and the massless light quark sector (chiral symmetry breakdown) impose the augmentation of the Heavy Quark Effective Theory with the crucial chiral component. This hypothesis seems to match well the observed spectra [36], but its consequences are far more dramatic. If correct, it implies that every hadron composed of heavy (H) and light (L) quarks may have an opposite chirality "doubler". This chiral doubling is the main topic studied in this work.

2. Let us explore another intriguing symmetry, appearing when the system has two infinitely heavy quarks. Let us start with the doubly heavy baryon, of the generic type HHL, where we do not speculate at this moment if heavy quarks have identical or different flavors. From the point of view of infinitely heavy color source, the two heavy quarks sitting on top on each other behave as an anti-triplet color source, since for SU(3) color ($3 \otimes 3 = 6 \oplus \bar{3}$), and the sextet is excluded due to the failure to form the color neutral (white) state with the remaining light quark in color triplet state. This means that in the infinitely heavy limit appears an additional "supersymmetry" between doubly-heavy diquark HH and heavy antiquark $\bar{H}$. This supersymmetry (we name it Savage-Wise (SW) after their discoverers) imposes the degeneration of the spectra between the doubly heavy baryons HHL and heavy-light mesons $\bar{H}L$. It also implies another dramatic effect, the appearance of the tetraquarks, since according to the same reasoning the heavy-light-light baryons HLL are expected, due to charge conjugation, to be degenerated with the $\bar{H}\bar{H}LL$ states. From this perspective it is clear now how crucial is observation by SELEX of the double heavy baryonic states and how important is an independent confirmation by other experiments. Three important theoretical problems can be immediately addressed:

– is the mass of the charm quark heavy enough to see the effects of SW symmetry?

– does the observed spectrum includes chiral doublers?

– can we learn from these data more on charm-charm interaction by extracting the information from charm-anticharm interaction in observed quarkonia?

We will try to answer these questions in this work as well.

The paper is organized as follows. In Chapter 2, we review the consequences of spontaneous breakdown of the chiral symmetry for the heavy light-systems, discussing respectively the chiral doublers for heavy-light mesons, the chiral doublers for heavy-heavy-light baryons and, for completeness, the chiral doublers for heavy-light-light baryons. There are many phenomenological ways to incorporate the spontaneous breakdown of the chiral symmetry – we decided to choose the instanton liquid picture for the QCD vacuum, due to its phenomenological success for the low-lying hadronic sector and limited number of the parameters. Brief recollection of assumptions of this model concludes Chapter 2. In Chapter 3, we consider a simple toy model incorporating the physics of instantons with heavy-light system. This model serves as a verification that the appearing scales and types of interaction are the correct ones and justifies full strength approach to heavy-light mesons based on extensive numerical simulations (Chapter 4). In Chapter 5, we address the issue of doubly heavy baryons and we speculate on possible interpretation of the SELEX data. Due to the technical complications of the problem and in order to gain some physical insight, we abandon brute force numerical simulation for the baryons and we rather rely on careful estimation of various scales based on semi-analytic calculations in various models. Chapter 6 includes discussion of the results and the summary. Two Appendices hide necessary, but very technical details of some of our calculations.

Chapter 2

Chiral Symmetry in Heavy-Light Systems

In this Chapter we remind the idea of chiral doubling, and we discuss subsequently this phenomenon for mesonic and baryonic systems in light of the accessible experimental data. Then, we summarize the main idea of the instanton vacuum picture and their importance for describing the spontaneous breakdown of chiral symmetry in Quantum Chromodynamics.

2.1 Chiral Doublers for Heavy-Light Mesons

With respect to $\Lambda_{\rm QCD}$, the fundamental scale of the Quantum Chromodynamics, strong interactions involve three light flavors (q=u,d,s) and three heavy flavors (Q=c,b,t), (see Fig.2.1). It is instructive to consider the limits $m_q \to 0$ and $m_Q \to \infty$.

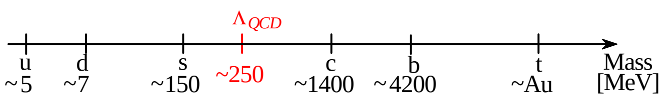

Figure 2.1: Schematic QCD mass scales.

Both limits (massless quarks and infinitely heavy mass) unravel essential symmetries of strong interactions. The light sector (massless light quark limit) is characterized by the spontaneous breakdown of the chiral symmetry (SBχS). Vacuum state is respecting only vector part of the symmetry, i.e. $SU_V(N_q) \times SU_A(N_q) \to SU_V(N_q)$, whereas axial symmetry is broken, as a

results we have massless Goldstones excitations for each broken generator. Since on top of the spontaneous breakdown of the chiral symmetry we have and explicit breaking due to current masses of up and down quark of order few MeV, pion is not massless, but still is the lightest hadronic particle. Spontaneous breakdown of the chiral symmetry is the cornerstone of the chiral perturbation theory.
The heavy sector (infinite heavy quark mass limit) exhibits heavy quark symmetry (known as a Isgur-Wise symmetry) [37]. In this limit, dynamics of the heavy quark becomes independent of its spin. As a result the masses of the pseudoscalar (0^-) and vector (1^-) mesonic states including heavy quark become degenerate, since there is no difference due to the singlet or triplet spin configuration of the system.
Heavy-light mesons are the simplest objects subjected to the simultaneous restrictions of both above-mentioned symmetries. Constraints from both symmetries enforce the form of the effective interaction of such mesons. An explicit answer from theoretical point of view was found in 1992 and 1993 [33, 34]. In brief, the novel aspect of derivation was that the interaction requires an introduction of the chiral partners ($0^+, 1^+$) (denoted in original work by G)

$$G = \frac{1+\not{v}}{2}(\gamma^\mu\gamma_5\tilde{D}^\star_\mu + \tilde{D}) \tag{2.1}$$

for well known and the "standard" H-multiplet ($0^-, 1^-$)

$$H = \frac{1+\not{v}}{2}(\gamma^\mu D^\star_\mu + i\gamma_5 D)\,. \tag{2.2}$$

Chiral partners ($0^+, 1^+$) are parity duplications for the standard multiplet ($0^-, 1^-$). Using a proper expansion introduced in [33, 34] we can write effective Lagrangian density for usual heavy-light mesons (H) and their parity partners (G). The key difference in this approach is the opposite sign of the mass contribution in chiral copies, contrary to similar term in H. Technically, the difference for chiral masses origins from the γ_5 difference in the definition of the fields H and G. In other words, it is sensitive to the parity content of the heavy-light field since $H\not{v} = -H$ and $G\not{v} = +G$, where $\not{v}$ is the Feynman-slashed velocity of the heavy quark. Since $\{\gamma_5, \gamma_\mu\} = 0$, the origin of the opposite shift is indeed chiral. Physically, it corresponds to the fact that contrary to heavy quark, light quark feels the asymmetric response of the vacuum with respect to the axial and vector properties, when correlated pair of quarks propagates between the sources either rotated or unrotated by γ_5. What is important, the mass splitting between both heavy mesons fields imply the mass relation to order $m_Q^0 N_c^0$

$$m_G - m_H = m(\tilde{D}^\star) - m(D^\star) = m(\tilde{D}) - m(D) = \mathcal{O}(\Sigma)\,, \tag{2.3}$$

where Σ denotes one loop heavy meson self-energy [33, 34, 36]. Such a generic phenomenon cannot be model dependent, so one may expect the chiral doublers for all the heavy-light hadrons. The effect should be more dramatic the heavier the heavy quark is (the better heavy quark symmetry), so similar structure is expected to appear at the level of the B mesons, involving the b quark, approximately three times more heavy comparing to the charmed one. We move now towards the experimental data. To simplify the comparison, we employ visualizations in a form of cartoons (cubes or slashed pyramides) for each flavor content. The rungs of our cubes are aligned along three directions which represent the effect of the symmetries considered. These three-dimensional cubes are organized as follows (see Fig.2.2):
- chiral symmetry breaking denoted by $SB\chi S$ (horizontal),
- Isgur-Wise symmetry breaking $1/m_Q$ (skew),
- total light angular momentum j_l (vertical).

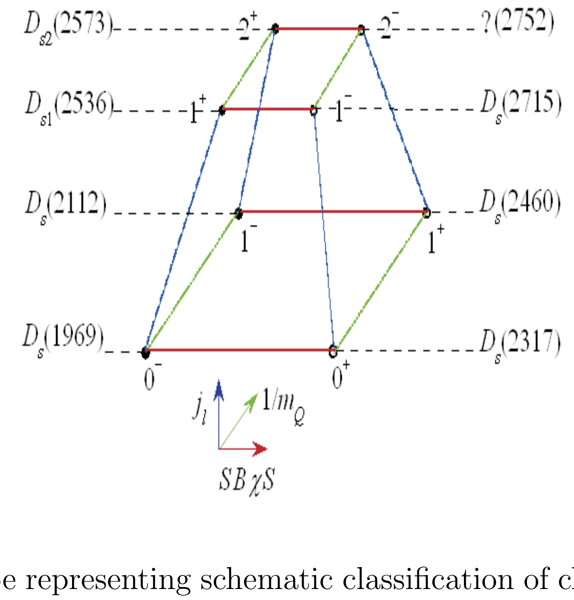

Figure 2.2: Cube representing schematic classification of chiral doublers (the right wall). Labels correspond to the case of $c\bar{s}$ mesons. Here, Belle signal $D_s(2715)$ is interpreted as an excited doubler, see text.

We focus on a cartoon for strange charmed mesons, i.e. D_s-cube. Lower left rung represents known pseudoscalar (0^-) $D_s(1969)$ and vector (1^-) $D_s^\star(2112)$, with $j_l = 1/2$ light angular momentum. The splitting between them (143 MeV) is an $1/m_c$ effect and is expected to vanish in infinitely heavy charm quark limit, i.e. both particles would have form the H multiplet. The upper left rung corresponds to $j_l = 3/2$ representation, i.e. 1^+ and 2^+ excited multiplet. Here, $D_{s1}(2536)$ and $D^\star_{sJ}(2573)$ are the candidates, separated by (smaller for excited states, here only 37 MeV) $1/m_c$ origin mass splitting. Concluding, our left wall of the D_s-cube represents well known states, before BaBar and Belle discoveries. The novel aspect of the chiral doublers scenario is the appearance of the right wall. First, we expect two chiral partners for D_s and $D_s^\star$, representing right lower rung. Recently discovered $D^\star_{sJ}(2317)$ and $D_{sJ}(2460)$ are the candidates for the $(0^+, 1^+)$ scalar-axial G multiplet. The averaged splitting for $(0^+, 0^-)$ and the averaged splitting for $(1^+, 1^-)$ are (349.2 ± 0.8) MeV and (346.8 ± 1.1) MeV, respectively, i.e. almost identical, as predicted a decade ago [33, 34]. The mass difference within the G multiplet, i.e. between the the new BaBar and CLEO states, is identical to the splitting between the $(1^-, 0^-)$ pair. Moving to upper values of the light angular momentum (j_l direction on Fig.2) we would also expect the chiral partners for the excited $j_l = 3/2$ multiplet, i.e. new chiral pair $(1^-, 2^-)$ [38]. Alternatively, this pair could be also viewed as the $j_l = 3/2$ excitation of the BaBar-CLEO $(0^+, 1^+)$ multiplet. The states within this new multiplet would be separated by similar $1/m_c$ split, like the split between D_{s2} and D_{s1}, i.e. by 37 MeV. However, the question is how large is the chiral split for the excited states? Is it also equal to 350 MeV alike the chiral split for the $j_l = 1/2$ plaquette or is different? The possible answer was proposed by [38]. In this work the chiral shift for excited states was approximately half of the value of the shift for $j_l = 1/2$ multiplet (175 MeV). The fact that excited states are less sensitive to the effects of the QCD vacuum is not totally unexpected, see e.g. [39]. Of course, the precise value of the chiral shift for the excited doubler can be provided only by an experiment. It is tempting to speculate that the recent signal reported by Belle [15] (2715) is a (1^-) doubler of D_{s1}. Therefore, the chiral shift for excited strange charmed mesons would be of order of 179 MeV. If indeed this is the case, a natural expectation in the chiral doubler scenario is the presence of the chiral doubler for D_{s2} state as well, i.e. one would expect new, 2^- state within few MeV around 2752 MeV, possibly in $D_s^\star \eta$ channel, to follow the pattern of the decay of other doublers. It is important to notice that the spectacular, very narrow width of charmed mesonic chiral doublers is a result of particular "conspiracy" of scales. The most natural decay pattern of the doubler to his opposite chirality lower partner by the emission of the Goldstone boson is blocked: strange Goldstones (kaons) are too heavy comparing to the value of chiral gap, and pions are isotriplets, so the decay would violate the isospin, since both charm and strange quarks are isosinglets.

Similar cartoon is expected for non-strange sector. Let us mention that, here the natural candidates for lower ($j_l = 1/2$) right rung of the D-cube are the new states reported by Belle, i.e. $D_0^\star$ $(2308 \pm 17 \pm 15 \pm 28)$ and D_1' $(2427 \pm 26 \pm 20 \pm 17)$ [5]. They can be viewed as chiral partners of well known pair $D(1865)$ and $D^\star(2010)$ respectively. In this case the chiral shift seems to be equal or even larger then for the strange ones which is not in contradiction with certain models of spontaneous breakdown of the chiral symmetry [36]. The precise value of the chiral shift is still an open problem, due to the experimental errors and systematic difference between the FOCUS [6] and Belle [5] signals. Since D mesons are isodoublets, there are no restrictions due to the emission of pions between the doublers, hence the width of non-strange doublers is much broader comparing to their strange counterparts.

Similar chiral doublers are also expected for heavier members of mesons family, i.e. B and B_s. Additionally, the chiral doubling should be more pronounced in this case, since the $1/m_Q$ corrections are three times smaller. This feature implies that one of our axes (skew) is three times shorter. More details and predictions of chiral splitting for B and B_s one can find in [36].

2.2 Chiral Doublers for Heavy-Light-Light Baryons and Exotic States

Chiral doublers should appear also for more complicated states, including baryons and hypothetic exotic states. To avoid the introduction of new degrees of freedom, in [40] we discussed the possibility of an extension of the chiral doublers scenario for heavy-light-light baryons, including the exotic states like heavy pentaquarks, using the concept of effective mesonic Lagrangians. In other words, we parallel the original construction of Skyrme, who identified nucleon as a soliton of the effective Lagrangian for pions (non-linear sigma model). Similarly, we view baryons as solitons of the effective mesonic Lagrangian including both chiral copies (H and G) of heavy-light mesons, a point addressed already in [33]. We are working in large N_c limit, which justifies the soliton (Skyrmion) picture, and large heavy quark mass limit, since we have exploited the constraints from Isgur-Wise symmetry of the heavy quark. This approach could be viewed as a starting point for including $1/m_Q$ corrections from the finite mass of the heavy quark, explicit breaking of chiral symmetry, etc., alike the presented scheme does it for the mesons. Charmed hyperons emerge as bound states of D and $D^\star$ in the presence of the $SU(2)$ Skyrme background. First the pseudoscalar-vector heavy meson pair is being bound in the background of the static soliton, generating the $O(N_c^0)$ binding. Vibrational modes are the "fast degrees" of the freedom. The adiabatical rotation of the bound system by quantization of collective coordinates of the $SU(2)$ Skyrmions alike proposed by Witten [44] corresponds then to "slow degrees" of freedom. It is well known, that in this case the rotation is not the free one. Fast degrees of freedom in Born-Oppenheimer approximation generate the effective "gauge" potential, of a Berry phase [45] type. In the case of degenerate pesudoscalar and vector mesons (IW limit) the phases coming from D meson and $D^\star$ meson are equal, but opposite. Their cancelation corresponds to the realization of the Isgur-Wise symmetry at the baryonic level, therefore degeneration of spin 1/2 and 3/2 multiplets. In the following [40] we have chosen the same philosophy but in contrary to the other similar works in the literature [41, 42, 43] we considered the full heavy-light effective Lagrangian with both chiral copies [33, 34] and we included the crucial effects of the chiral shift. Therefore, the Lagrangian density reads

$$\mathcal{L} \;=\; \mathcal{L}_{Skyrme} + \mathcal{L}_H + \mathcal{L}_G + \mathcal{L}_{HG}\,. \tag{2.4}$$

Here, $\mathcal{L}_{Skyrme}$ is the nonlinear Lagrangian density (in case of only two light flavors - up and down) which carries a winding number identified as a baryon number $B = 1$

$$\mathcal{L}_{Skyrme} = \frac{f_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2} Tr[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2\,, \tag{2.5}$$

where U is the $SU(2)$ matrix, transforming as $U \to AUB^{-1}$ under $SU(2) \times SU(2)$ chiral rotations and $f_\pi = 93$ MeV is the pion decay constant. The first term in this Lagrangian is the leading nonlinear σ model Lagrangian. The second term, which contains the dimensionless parameter e was introduced by Skyrme to stabilize the soliton. Other parts labeled by H and G refer to the heavy meson fields [33, 34].

In this case, we find four different scenarios:

- Soliton of the light sector with baryon number 1 binds the H-multiplet - the resulting bound states exhibits the quantum numbers of the charmed baryons with standard $1/2^+$ parity.
- Soliton of the light sector with baryon number 1 binds the G-multiplet - the resulting bound states exhibits the quantum numbers of the charmed baryons with opposite $(1/2^-)$ parity.
- Soliton of the light sector with baryon number 1 binds the anti-flavored $\bar{H}$-multiplet - the resulting bound states exhibits the quantum numbers of the charmed baryon with minimal content of five quarks with standard $1/2^+$

parity, i.e. charmed pentaquark.
- Soliton of the light sector with baryon number 1 binds the anti-flavored $\bar{G}$-multiplet - the resulting bound states exhibits the quantum numbers of the charmed baryon with minimal content of five quarks with opposite $1/2^-$ parity, i.e. the chiral partner of the pentaquark.
We do not have any contribution from the interaction term L_{HG}, since that due to the properties of the heavy spin symmetry, one can trade $\gamma^\mu A_\mu$ into $v^\mu A_\mu$. This implies, that in the rest frame static Skyrmion background decouples the G and H Lagrangians. This decoupling allows immediately to write down the generic mass formula for opposite parity partner of the isoscalar baryon and for opposite parity partner of the isoscalar pentaquark (denoted here by tilde)

$$\begin{aligned} \tilde{M} &= M_{sol} + m_{\tilde{D}} - 3/2 g_G F'(0) + 3/(8I_1) , \\ \tilde{M}_5 &= M_{sol} + m_{\tilde{D}} - 1/2 g_G F'(0) + 3/(8I_1) , \end{aligned} \tag{2.6}$$

in analogy to identical formulae for the known sector for H, with D mesons and g_H axial couplings, respectively

$$\begin{aligned} M &= M_{sol} + m_D - 3/2 g_H F'(0) + 3/(8I_1) , \\ M_5 &= M_{sol} + m_D - 1/2 g_H F'(0) + 3/(8I_1) . \end{aligned} \tag{2.7}$$

In all above expressions , second terms m_D and $m_{\tilde{D}}$ denote the averaged mass of heavy-mesons. The ordering of mass terms is as follows: the first term corresponds to classical mass od the soliton (of order N_c), the before-last one measures the (model-dependent via the shape of the soliton profile $F(r)$) binding with respect to the mass of the meson (independent on the number of colors) and the last term measures the $1/N_c$ split due to the moment of inertia I_1 of the soliton. It is of primary importance that both Hamiltonians for H and G sectors have the same functional form of lowest eigenvalue: M_5 for H and $\tilde{M}_5$ for G. Hence both parity partners emerge as H and G bound states in the $SU(2)$ solitonic background. The mass difference comes in the first approximation solely from the difference of the coupling constants $g_G - g_H$ and meson mass difference $m_{\tilde{D}} - m_D$ where $m_{\tilde{D}} = (3M_{\tilde{D}^\star} + M_{\tilde{D}})/4$ is the averaged over heavy-spin mass of the $(1^+, 0^+)$ mesons. Constant g_G is the axial coupling constant in the opposite parity channel, responsible for pionic decays of the 1^+ axial states into 0^+ scalars. Using recent Belle data [3], *i.e.* 0^+ candidate $D_0^\star$ $(2308 \pm 17 \pm 15 \pm 28)$ and 1^+ candidate $D_1^{'}$ $(2427 \pm 26 \pm 20 \pm 17)$, we get $M_{\tilde{D}} = 2397$ MeV, unfortunately with still large errors. One can easily combine the formulae for four, above mentioned, generic scenarios. Fist, we notice, that the mass splitting between the usual baryons of opposite parity leads to

$$\Delta_B \;=\; \Delta_M + 3/2F'(0)g_H\delta g\,, \tag{2.8}$$

where $\Delta_M = M_{\tilde{D}} - M_D$ is the mass shift between the opposite parity heavy-light mesons and $\delta_g = 1 - g_G/g_H$ measures the difference between the axial couplings for both copies. Similar reasoning leads to the formula for the parity splitting between the opposite parity pentaquarks:

$$\Delta_P = \Delta_M + 1/2F'(0)g_H\delta g\ . \tag{2.9}$$

Combining both formulae we get

$$\Delta_P = \frac{\Delta_B + 2\Delta_M}{3}\ . \tag{2.10}$$

Let us turn now towards the available data. Using the shift of the opposite parity heavy charmed mesons from very recent Belle [5] data we arrive at Δ_M = 425 MeV unfortunately with still large errors. As a result, the mass shift between the lowest Λ_c states of opposite parities, $\Lambda_c(1/2^+,\ 2285)$ and $\Lambda_c(1/2^-,\ 2593)$ is approximately $\Delta_B = 310$ MeV. Similarly for isotriplet $(3/8I_1 \to 11/8I_1)$, taking masses $\Xi_c(1/2^+,\ 2470)$ and $\Xi_c(1/2^-,\ 2790)$ we have $\Delta_B = 320$ MeV. These two numbers allow us to estimate $\Delta_P = (350 \pm 60)$ MeV, *i.e.* we get the mass of the chiral doubler of the isoscalar pentaquark as high as (3052 ± 60) MeV. Let us contrast these predictions to others in the literature (see Table). We note that our intension was to demonstrate the

Table 2.1: Predicted masses of charmed pentaquark Θ_c^0 $(udud\bar{c})$:

Model	Mass [MeV]	Ref.
constituent quark model (FS)	2902	[46]
diquark model	2710	[11]
diquark-triquark model	2985 ± 50	[12]
chiral soliton model	2704	[42, 43]
chiral doublers scenario	2700 ; 3052 ± 60	[40]
lattice calculation	2977	[47]

order of magnitude for parity splitting for heavy baryons and pentaquarks (in large m_Q and large N_c limits). One is therefore tempted to interpret the recent H1 state [9] as a parity partner $\tilde{\Theta}_c$ of the yet undiscovered isosinglet pentaquark Θ_c of opposite parity and $M_5 \approx 2700$ MeV. Similar reasoning applies to other isospin channels, strange charmed pentaquarks and to extensions for b quarks. Despite BaBar and CLEO data yield with the impressive accuracy the chiral mesonic shift to be equal to 350 MeV, no charmed strange baryon data for both parities do exist by now, so one cannot make similar estimation for strange charmed pentaquarks. We would also like to stress, that H1 signal for pentaquark was never confirmed by other experiments. On the positive side, we would like to stress, that any hypothetic exotic configuration, even weakly bounded, observed and confirmed at the charm level will have dramatic consequences for the beauty sector. The reason is that color interactions are flavor blind, and the positive kinetic term comparing bottom and charm quarks is three times more suppressed, so the probability of forming exotic states increases with the mass of the heavy flavor.

2.3 Chiral Doublers for Doubly Heavy Baryons

Till the end of XX century the issue of doubly heavy baryons belong to fascinating, but purely academic domain of theoretical physics, due to lack of any experimental data. In 2003 SELEX Collaboration [7] reported a discovery of the doubly charmed baryons belonging to the Ξ_{cc} family. All of those five visible states and possible quantum numbers are presented on Fig.2.3. The SELEX Collaboration has argued that the orbital angular momentum

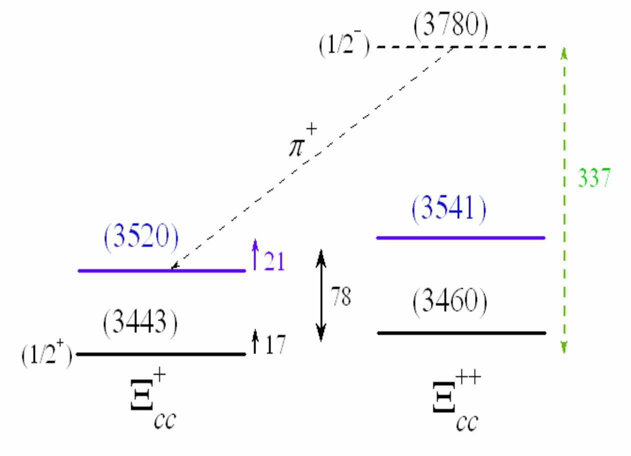

Figure 2.3: Spectrum of doubly heavy baryons (Ξ_{cc}) that have been observed by the SELEX Collaboration.

of the ground states is $L = 0$ (for both ccu and ccd) which implies positive parity. The two of the excited states, i.e. (3520) and (3541) are consistent with $L > 0$ (either negative or positive parity), however, the way of observed decays from orbital excited state $\Xi_{cc}(1/2^-)$ (3780) through the pion emission suggest that $\Xi_{cc}^+(3520)$ could have negative parity. Till today, all these five states remain a big challenge for theoretical interpretation. There also remains a challenge in experimental physics – the SELEX discovery was not confirmed by other experiments. Moreover, it is surprising that the new states were observed via weak decay products, and not by more expected, faster electromagnetic decays. Surprisingly, there is one aspect of these data which is very interesting for us from the point of view of the chiral doublers. When we look at the difference between the opposite parity states $(\frac{1}{2}^- - \frac{1}{2}^+)$, we get the mass shift of the order 337 MeV. This value is very close to the observed split between the hypothetic charmed hyperons of opposite chirality discussed in the previous section, i.e. the split of 310 MeV for charmed Λ isosinglets and 320 MeV for charmed Ξ. It is therefore tempting to speculate, that the two of the observed states represent the pair of HHL chiral doublers. Whereas the splits of order of 20 MeV between different charged states may be attributed to the difference between the masses of up and down quarks, the 78 MeV split between the $\Xi_{cc}^+(3520)$ and $\Xi_{cc}^+(3443)$ is a real challenge. Motivated by analysis of heavy-light mesons, we may ask if this split is not related to Savage-Wise symmetry mentioned in the introduction. One of the consequences of this symmetry is the relation [51] between the split between the masses of doubly heavy antibaryons with corresponding spins 3/2 and 1/2 (we denote it as Δ_{SW}) and the split between the masses between heavy-light mesons with corresponding spins 1 and 0 (we denote is as ΔIW[1]:

$$\Delta_{SW} \;=\; \frac{3}{4}\Delta_{IW}\,. \tag{2.11}$$

Since the $O(1/m_c)$ split between the $D*$ and D reads 145 MeV, the predicted split for doubly heavy baryons reads 109 MeV, *assuming* the Savage-Wise symmetry holds already at the level of the charm quark. This number has to be confronted with the SELEX split of 78 MeV – is the Savage-Wise symmetry already an approximate symmetry at the level of the quark with the mass of 1.45 GeV? If this is the case, one could expect similar approximate symmetries between heavy hyperons and tetraquarks, as depicted schematically on Fig 2.4.

The issue if the 30% discrepancy between the SELEX data and the predicted spin split for doubly heavy baryons is a consequence of the new approximate symmetry at the hadronic level or the numerical accident, is heavily

[1]We would like to thank Nora Brambilla and Tom Mehen for informing us prior to publications that the corrected factor is 3/4 and not 3/2 as in original Savage-Wise paper.

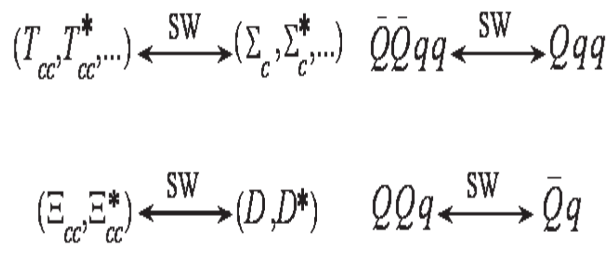

Figure 2.4: Schematic relations for doubly heavy baryons Ξ_{QQ} and tetraquarks T_{QQ} due to SW symmetry. Here, D and D^* refers to the standard $s = (0, 1)$ D-mesons, Σ_c and Σ_c^* are isospin-1, spin-1/2 and spin-3/2, respectively. The doubly charmed tetraquarks are labeled by T_{cc}, T_{cc}^* with spin 0 and 1. Excitation in light degrees of freedom may be also analyzed.

debated. A recent discussion covers framework of NRQCD [48], the relativistic and nonrelativistic quark model [49] or extended chiral perturbation theory [50]. Very interesting, more general speculations on the issue of the presence of Savage-Wise symmetry and the nature of SELEX data were discussed recently in [52]. In the similar spirit, we will investigate and try to understand properties of the doubly heavy baryons using the insights from instanton model and we will confront the results with other calculations based on potential models.

2.4 QCD Instantons and Chiral Symmetry

The fundamental mechanism responsible for the spontaneous breakdown of the chiral symmetry is still unknown, so the phenomenon is usually described at the level of some effective theories or models, Nambu-Iona-Lasinio model being the most known and popular. It is however expected and to large extend confirmed by lattice calculation, that the microscopic underlying picture is somehow related to chiral disorder, triggered by localized lumps of gluonic field. Typical snapshots of lattice gauge configurations reveal very often non-trivial topological content of the freezed configurations. These observations justify to some extend instanton picture of the QCD vacuum. In this picture, these lumps of gluonic fields are represented by certain classical, stable and topologically non-trivial configurations called instantons. These objects since their discovery in 1975 by Belavin et al. [53] have been enjoying significant attention in many theoretical studies. Their topological content is strictly related to non-Abelian character of the gluonic fields,

$$Q = \frac{g^2}{32\pi} \int d^4x G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \,, \tag{2.12}$$

where

$$\widetilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta} \,, \qquad \epsilon_{1234} = 1 \,. \tag{2.13}$$

It is easy to show that instanton carrying the topological charge n, where n is a natural number has the minimal action $S = 8n\pi/g^2$, where coupling constant in denominator points at strictly non-perturbative origin of this configuration. This property follows from the self-duality conditions $G = \widetilde{G}$. Similar arguments hold also for anti-instanton solutions except that the gauge fields obey relation $G = -\widetilde{G}$, carrying the same finite action as the instantons but opposite charge $Q = -n$. As can be seen, the fulfillment of of the conditions $G = \pm\widetilde{G}$ automatically leads to satisfaction of equations of motion $D_\mu G^a_{\mu\nu} = D_\mu \widetilde{G}^a_{\mu\nu} = 0$, confirming that the configuration is an exact solution of Yang-Mills theory. Let us mention that the explicit form of the instanton solutions depends on the gauge choice, and for general n, is known only parametrically as a system of coupled algebraic equations. We stick therefore to the lowest non-trivial topological configurations, since their solution is available in the simple analytic form. In the literature one may find several kinds of solutions corresponding to the different types of gauge, i.e. regular, singular or axial. We restrict ourselves to the singular gauge which will be used in all our further calculations due to its convenient behavior for large distances. The final expression for the instanton configuration in the singular gauge, with its center at point z_I and size ρ, has the form

$$A^a_\mu(x - z_I) = U_I \bar{\eta}^a_{\mu\nu} \tau^a U^\dagger_I \frac{(x - z_I)_\nu \rho^2}{(x - z_I)^2((x - z_I)^2 + \rho^2)} \,. \tag{2.14}$$

Here $\bar{\eta}^a_{\mu\nu}$ is the 't Hooft symbols (see Appendix (8.2)), and U_I represents the color orientation matrices of $SU(N_c)$. Note that each instanton with $Q = 1$ is characterized by $4N_c$ collective coordinates.

Behavior of fermions in the presence of instanton is of particular experience. Since a single instanton leads in a presence of massless fermions to the emergence of an effective vertex in the form of flavor determinant, instantons naturally solve the $U(1)$ problem [54] (the η' mass problem). As a consequence of Atiyah-Singer theorem, Dirac equation in the background of instanton (anti-instanton) possesses exact right-handed (left-handed) fermionic zero modes. In the dilute mixture of instantons and anti-instantons the overlaps between these modes provides the mechanism for dressing the constituent quarks, realizing the chiral disorder. To avoid unnecessary repetitions, we relegate the details to one of the well-known results [55].

We would like to stress two important shortcomings coming from such picture of the QCD vacuum. First, despite original expectations [56], instantons do not explain the nature of the confinement. Till today, despite many attempts, we still lack fundamental understanding which type of gluonic configurations is responsible for this phenomenon. On the other side, lattice studies show that the restoration of the chiral symmetry and deconfinement phase transition happens at the same critical temperature, which strongly suggests some crucial link between two phenomena. We will see some consequences of this failure of instantons when analyzing heavy-light systems. Second, since Yang-Mills theory has nonlinear equations of motion, superposition rule does not hold for the gas of instantons and anti-instanton, which forces the practitioners to choose some Ansatz for the measure for the ensemble of instantons and anti-instantons. Most successful is so-called instanon-liquid model, proposed originally in 1982 by Shuryak [71] and later investigated extensively by Diakonov and Petrov [64]. In these works, they have shown that the form on the Ansatz stabilizes the average size of instantons in the vacuum, protecting them from swelling (so-called infrared problem). The model is basically characterized by two scales:

- the density of the well separated (distance about 1 fm) instantons is of the order $n = fm^{-4}$, which allows to treat quantum corrections as the product of factorized 't Hooft small oscillations,
- all instantons have the averaged size about $\rho = \rho_0 = \frac{1}{3} fm$.

Using above parameters one can not only reproduce the phenomenological values of the quark and gluon condensates, but also one can get a very successful picture for several correlation functions in light-flavor sector of the QCD, as for example shown in [72], where propagating through the instanton vacuum quarks were probing the physical distances up to 1.5 fm. In Chapter 4 we will exploit similar strategy for the heavy-light mesonic systems. However, before we will engage ourselves into extensive numerical simulations of the propagations of heavy-light systems through the instanton liquid, we would like to get some insight how the instanton may provide the seed of chiral doubling phenomenon. To achieve this goal, we study in the next Chapter simple, but illuminating toy-model.

Chapter 3

Toy-model for the Heavy-Light Hadrons

3.1 Motivation and Expectations

In this work we are studying the consequences of the assumption, that experimental spectra observed for opposite parity light-heavy baryons, observed first in the case of mesons by BaBar [1], Belle [3] and CLEO [2], originate from the chiral shift. We mentioned before that instantons play an unquestionable role in hadronic physics, first, explaining the $U_A(1)$ puzzle for the light mesons, second, providing the mechanism for spontaneous breakdown of the chiral symmetry. It is therefore tempting to see what is the effect of instantons on heavy-light systems. Their intimate connection with spontaneous breakdown of the chiral symmetry implies, that they should as well provide the mechanism of the chiral shift separating the opposite parity heavy-light mesons. To check if this is true we [57] propose a simple model, based on the response of the heavy-light systems to the instantons in the case of one light and one heavy flavor. Certainly, it is a toy model, since we know that the single light flavor corresponds to the $U_A(1)$ anomaly, and realistic case includes at least two light flavors. On the other side, it is tempting to see, if already the $U_A(1)$ axial anomaly alone can provide the mechanism for separating the states of opposite parity and can generate chiral shifts. After positive verification of the above statement we investigate if the instanton effects can cause also the splitting between different spin states, i.e. what is their role at the level of $1/m_Q$ corrections. Finally, we extend our toy model to the baryons, to check if indeed similar mechanism can works in their case as well.

3.2 One-Heavy and One-Light

Our approach to the heavy-light system using instantons based on the concepts [68, 59] parallels the similar construction for light-light systems [63, 64, 65]. This is a justified reasoning, if indeed the instantons play the fundamental role in understanding the effects of chiral symmetry. To see how instantons can trigger a splitting between the opposite parity heavy-light particles (in the case of one light and one heavy flavor) we exploit the instanton induced interaction worked out in [59] using the correlation function formalism and $1/N_c$ book-keeping arguments. In of the following Chapters (5) we show how to obtain these kind of effective interactions in much more involved case of doubly heavy baryons, and two realistic light flavors. Now, we recall only the final version of the effective Lagrangian in case of the heavy-light mesons

$$\mathcal{L}_{qQ} = -\left(\frac{\Delta M_Q \Delta M_q}{2nN_c}\right)\left(\bar{Q}Q\bar{q}q + \frac{1}{4}\bar{Q}\lambda^a Q\bar{q}\lambda^a q\right), \tag{3.1}$$

where $\Delta M_Q \approx 86\,\mathrm{MeV}$ is the heavy quark mass shift generated by a presence of instantons [59, 63] and $n = n_*/2N_c = (fm)^{-4}/2N_c$ is the instanton density in the vacuum. This induced interaction breaks explicitly axial U(1) symmetry, but preserves heavy-quark symmetry (or IW symmetry, i.e. invariance under heavy spin flip). We already note that one-gluon exchange involves

$$\mathcal{L}_{qQ} = -\frac{g^2}{2\Lambda_c^2}\bar{Q}\gamma_\mu\lambda^a Q\bar{q}\gamma^\mu\lambda^a q\,, \tag{3.2}$$

(with some cut-off Λ_c) which is clearly $U_A(1)$ preserving. To proceed further, we rearange the heavy mesons effective Lagrangian given by eq.(3.1) using first one of the Fierz color identities discussed in details in the Appendix. As a next step, after unraveling the colors, we use the standard Fierz relations for spin. Details of our notation and the explicit transformations used for all further calculations are relegated to the Appendix A (7.1). Taking into account all identities of color and spin, a final result can be obtain as follows

$$\begin{aligned}\mathcal{L}_{qQ} = \quad & +\left(\frac{\Delta M_Q \Delta M_q}{2nN_c}\right)\left(\frac{1}{4}\frac{1}{3} + \frac{1}{4}\frac{1}{3}\right) \\ & \times(\bar{Q}q\bar{q}Q - \bar{Q}i\gamma_5 q\bar{q}i\gamma_5 Q \\ & +\bar{Q}\gamma^\mu q\bar{q}\gamma_\mu Q + \bar{Q}\gamma_5\gamma^\mu q\bar{q}\gamma_\mu\gamma_5 Q \\ & +\frac{1}{2}\bar{Q}\sigma^{\mu\nu}q\bar{q}\sigma_{\mu\nu}Q)\,,\end{aligned} \tag{3.3}$$

where 1/4 is taken from Fierzing the spin, and 1/3 from Fierzing the color in each of the two contributions in equation (3.1). Here, we want to remind that we have already taken into account the effect of Fermi statistics of the quark fields in the final expression (3.3).

Before proceeding further, let us comment how one can rewrite the above Lagrangian in terms of physical, dressed fields. To show this, let us remind that pertinent $\bar{q}Q$ combinations carry a proper spin-parity assignments 0^+, 0^-, 1^- and 1^+. Therefore, keeping in mind the expressions for the standard heavy meson multiplets $H = (0^-, 1^-)$ and their parity partners $G = (0^+, 1^+)$ presented in the Introduction we may rewrite our Lagrangian in terms of these fields, using the so-called bosonization techniques [35]. In this way, at the expense of introducing new (bosonic) fields, we manage to convert quartic quark interaction into interaction of quark-antiquark with the relevant meson. Formal integration over quarks fields leads therefore to fermion determinant, depending on background mesonic fields and heavy and light parts of the quark propagators. Derivative expansion of such action where taking into account projectors onto heavy and light quarks sectors have to be carefully implemented leads the generic effective chiral Lagrangian for heavy-light mesons [33], incorporating the effects of chiral doublers. Instead of following this formal path, we choose simpler arguments to see immediately the role of the instantons. Now, looking at the our Lagrangian density (3.3) we conclude that the instanton induced interaction is attractive in the $(0^-, 1^-)$ channels and repulsive in the $(0^+, 1^+)$ channels. The attraction is equal in magnitude to the repulsion and proportional to the light constituent mass Σ, a situation reminiscent of [33] for two and higher flavors. We readily note, that the use of the constituent one-gluon exchange (3.2) yields a Fierzing that differs from (3.3) in one notable way: all terms in (3.3) carry positive weight. The result is a repulsion in both channels of equal magnitude. The instantons split the even/odd parity heavy-light multiplets, while one gluon exchange does not. This is visible in the correlators at short distance already for one heavy and one light flavor irrespective of chiral symmetry breaking.

Using our effective action given by (3.3) we estimate the contribution to the heavy hadron spectrum and, what is more interesting, we also obtain the contribution to the mass difference between chiral partners. One of the possible ways to do that relies on the use of the variational approach. Following arguments from [68] the contribution may be treated as a perturbation, and for wave functions of the unperturbed Hamiltonian one may take some simple variational Ansatz. The simplest potential mimicking confinement can be chosen as the harmonic potential, allowing to get the results in the analytic way. This is the way how we will proceed when analyzing the instanton effects on doubly heavy systems. Taking into account all relevant parts from the instanton induced interactions (3.3) and using the trial wave function one may find a following correction to the heavy-meson spectrum (in the large

N_c)

$$< H_{qQ} > \sim -C_F \left(\frac{\Delta M_Q \Delta M_q}{2nN_c} \right) |\psi(\vec{0})|^2 . \tag{3.4}$$

Since we are here interested only in the order of magnitude of the effect, we may simplify further our estimations. Instead of using the explicit form of the wave function depending on our optimal variational parameter, we apply the Van Royen and Weisskopf construction [60] where the meson decay constant is related with the wave function at the origin as

$$|\psi(\vec{0})|^2 \approx \frac{f_H^2 M_H}{12} . \tag{3.5}$$

This implies the value of the mass splitting between the HG-multipelt (e.g. PS-S) to the first order in the instanton effect

$$\Delta = \frac{\Delta M_Q \Delta M_q}{3n_*} |\psi(\vec{0})|^2 \approx \frac{\Sigma}{3} , \tag{3.6}$$

where the quark masses are shifted by $\Delta M_Q = 86$ MeV and $\Delta M_q = 420\text{MeV} \equiv \Sigma$ for the heavy and light quarks, respectively. In (3.6) have been used the upper bound for the decay constant $f_H = 290$ MeV and the mass of heavy mesons $M_H = 2$ GeV. Numerically, the chiral shift reads 140 MeV, a value 3 times smaller comparing to the data for the D mesons. We do not expect an exact agreement with the data, taking into account the nature of the model and simplifying assumptions. The models seems to lead to the right scale of the effect (few hundreds of MeV) and shows in a transparent way the physics of the effect.

We can now study the effect of the instantons on subleading terms in $1/m_Q$ expansion, i.e. the spin corrections to (3.1) using the results presented in [59]. Spin dependent part is given by

$$\mathcal{L}^s_{qQ} = + \left(\frac{\Delta M^s_Q \Delta M_q}{8nN_c} \right) \bar{Q}\lambda^a \sigma^{\mu\nu} Q \bar{q} \lambda^a \sigma_{\mu\nu} q , \tag{3.7}$$

where ΔM^s_Q is the mass shift to the heavy constituent quark mass caused by its spin (for m_c)

$$\begin{aligned} \Delta M^s_Q &= \frac{16\pi n\rho^2}{m_Q} \int dx \left(x \frac{\sin f_x}{(1+x^2)} \right)^2 \\ &\approx 3\text{MeV} \end{aligned} \tag{3.8}$$

with the $f_x = |x|\pi/\sqrt{(1+x^2)}$ followed from the one instanton configuration in the singular gauge. Going further, a rerun of the Fierzing (3.7) procedure yields

$$\begin{aligned}\mathcal{L}^s_{qQ} = & -\left(\frac{2\Delta M^s_Q \Delta M_q}{2nN_c}\right) \\ & \times \left(\bar{Q}q\bar{q}Q - \bar{Q}i\gamma_5 q\bar{q}i\gamma_5 Q - \frac{1}{6}\bar{Q}\sigma^{\mu\nu}q\bar{q}\sigma_{\mu\nu}Q\right) . \end{aligned} \tag{3.9}$$

It turns out that the spin effects are attractive in the 0^- and tensor channels, and repulsive in the 0^+ channel. Additionally, there is no spin-effect in the $1^\pm$ channels. Analogically to (3.6) we may calculate the splitting induced by the spin part. The expression (3.9) implies a reduction in the $0^- - 1^-$ induced U(1) splitting by

$$\Delta_s = \frac{2\Delta M^s_Q \Delta M_q}{n_*} |\psi(0)|^2 \tag{3.10}$$

and an enhancement in the $0^+ - 1^+$ by a similar amount. To compare our induced shifts we look at

$$\frac{\Delta_s}{\Delta} = \frac{6\Delta M^s_Q}{\Delta M_Q} \approx \frac{\Sigma}{16} . \tag{3.11}$$

At this point, we conclude that our spin correction is of the order $\Delta_s \approx 24$ MeV which is small (as expected) and consistent with the constituent quark model estimate of 27 MeV [62]. As before, since we were interested only in the magnitude of the effect, we state that in the case of subleading terms the instanton physics also seems to capture the right scale of the effect. For completeness we mention that the spin induced hyperfine splitting $0^- - 0^+$ is two times bigger then (3.7). In the next section we will present analogical discussion of the case of the heavy-light baryons.

3.3 One-Heavy and Two-Light

Baryons containing heavy (charm or bottom) and light quarks may be analyzed in a similar fashion using the induced interactions derived in [59].The simplest object which is new at the baryonic level is the vertex of the type Qqq where we have now different light flavors. The corresponding instanton induced interaction between one-heavy and two-light quarks is given by [59]

$$\mathcal{L}_{qqQ} = -\left(\frac{\Delta M_Q \Delta M_q^2}{2n^2 N_c^2}\right)$$

$$\times(\bar{Q}Q\,(\det(\bar{q}_L q_R) + \det(\bar{q}_R q_L)) \\ +\frac{1}{4}\bar{Q}\lambda^a\,Q\,(\det(\bar{q}_L\lambda^a\,q_R) + \det(\bar{q}_R\lambda^a\,q_L)))\ . \qquad (3.12)$$

Undoubtedly, the three quarks states have much more complicated structure in comparison to the heavy-light mesons from the previous section. For instance, in above expression we may recognize the contribution following from the 't Hooft vertex for two light flavors ($q = u, d$), i.e.

$$\mathcal{L}_{qq} = \left(\frac{\Delta M_q^2}{nN_c^2}\right)[\det(\mathrm{\bar{q}_L q_R}) + \det(\mathrm{\bar{q}_R q_L})]\ . \qquad (3.13)$$

It is easy to estimate the strength of the vertices appearing (3.12) using mean-field approximation. Then this effective Lagrangian density yields a one-body interaction of the form

$$\mathcal{L}^1_{qQ} = \langle\bar{q}q\rangle\,\frac{\Delta M_Q \Delta M_q^2}{n^2 N_c^2}\,\bar{Q}Q\,\bar{q}\,q\ , \qquad (3.14)$$

because only the first contribution in (3.12) gives non-vanishing vacuum contribution in leading order in $1/N_c$. Using the value of the vacuum condensate $\langle\bar{q}q\rangle = (-240\,\mathrm{MeV})^3$, one may find that the splitting (3.6) is now

$$\frac{\Delta_1}{\Delta} = \frac{\Delta M_q\,\langle\bar{q}q\rangle}{n_*}\ . \qquad (3.15)$$

Collecting all the formulae together we get the approximation to the shift as

$$\Delta_1 = \frac{6}{5}\Sigma\ , \qquad (3.16)$$

i.e. again of the order of few hundreds of MeV.

3.4 Concluding Remarks

We presented in this Chapter a simple toy model for the heavy-light hadrons with the $U_A(1)$ breaking through the instanton effects. In particular, construction of the proper effective Lagrangian densities allowed us to obtain estimations for the mass differences between the opposite parity states induced by the anomaly. We have shown that instantons already via the $U_A(1)$ contribute to this splitting, a rather non-trivial and encouraging result. Resulting mass shifts came $\Delta = \Sigma/3$ and $\Delta_1 = (6/5)\Sigma$ in case of the heavy-light mesons and baryons, respectively. We do not consider exact value of numerical prefactors as significant comparing to experimental data – the scale of the effect turned out to be indeed related to spontaneous breakdown of the chiral symmetry and numerical prefactors are "natural", i.e. came out not magnified or dwarfed by huge numerical values. We may therefore conclude that this simple exercise is qualitatively successful in confrontation with the chiral doublers scenario [33, 34]. What is important, we remind that, if the nature of this mechanism is generic, more chiral doublers for all the heavy-light hadrons are expected to appear. In particular, we will repeat similar reasoning based on instanton induced vertices for the exciting case of the doubly heavy baryons trying to understand experimental signatures observed by SELEX experiment.

Before we will shift to the doubly heavy baryons problem, let us first, encouraged by the conclusions of the toy-model, take the full advantage of the instanton liquid vacuum picture. In the next Chapter we will present the results of realistic numerical simulations of the heavy-light mesons in the instanton vacuum for physical values of the mass parameters.

Chapter 4

Heavy Mesons in the Instanton Liquid Model

In this Chapter we focus on phenomenological studies of instanton effects for heavy masons spectrum in a framework of the Instanton Liquid Model (ILM). Several previous works [70, 72] have elucidated the fundamental role of the instantons in QCD vacuum and their influence on the quark propagators as building blocks of the pertinent correlation functions. We briefly remind the mechanism considering the case of the light quark propagating through the multi-instanton environment and then we describe the behavior of the heavy quark in this medium. Following Shuryak [72] we construct two-point correlation functions for heavy-light mesons and we present how to perform numerical studies at the distances of the order of few femtometers. The main part of this section contains our new results [58] for heavy-light mesons (for both case of strange and non-strange light quarks) evaluated in ILM. We also test the chiral doublers scenario [33] in such environment, in particular we calculate the mass splitting for the opposite parity states.

4.1 The RILM

The evaluation of the single quark propagator in the multi-instanton background field is a very complicated task. The reason for several difficulties is that even for the dilute set of pseudoparticles (instantons and anti-instantons) we have 12 collective variables per one instanton (anti-instanton). These collective variables correspond respectively to: position of the center of instanton z_I (4 variables), size of the instanton ρ (1 variable), orientations U_I (7 variables, since in the case of the $SU(3)$ instanton gauge field commutes with diagonal λ_8, so relevant color space is a coset $SU(3)/U(1)$). According

to assumptions of the model, we simplify the general instanton ensemble by:
- First, freezing the size for all instantons (and anti-instantons) to be the same during the simulation. During our simulation the size is fixed at $\rho_0 = 0.35$ fm. For comparison, typical value used in literature varies from 0.2 to 0.4 fm. Smaller and larger instantons are excluded by comparing the simulation to experimental data.
- Secondly, our instantons have random positions z_I and orientations U_I, hence we use the name Random ILM (RILM). Since we are not studying topological fluctuations, we fix the instantons and anti-instantons densities to be equal to $N/(2V_4)$ so the total density is given by $n_* = N/V_4 \approx (fm)^{-4}$. We remark that V_4 is four dimensional Euclidean volume. Note that since the size of the instanton is three times smaller than the length of the Euclidean hyperbox, the "packing fraction" of the instantons is low, of the order of 10^{-2}. These parameters reproduce the correct value of the quark condensate, as an output of the simulation we get i.e. $|\langle q\bar{q}\rangle_{RILM}| = (271.08 \text{ MeV})^3$.

At this point, we should provide few technical details of our simulation. The numerical analysis uses Stony Brook Monte Carlo code. We have chosen 128 instantons and 128 anti-instantons closed in the box of the size of $3.36^3 \times 6.72\ (fm)^4$. The orientations are sampled from the invariant group measure. The average propagators $\langle S(x+\tau, x)\rangle$ needed to read out the correlators are calculated by averaging over 50 configurations and over 100 randomly chosen initial points for each configuration. This procedure is repeated for each value of the separation distance τ from the initial point. The light current quark masses (m_u and m_d) are taken equal to 10 MeV and the strange mass reads $m_s = 140$ MeV. Below we show how this simulation leads to reasonable description of the heavy-light correlation functions.

4.2 The Mesonic Correlators

Since the correlation functions for light flavors were one of the main tools used in studies of structure of the QCD vacuum it is natural to follow similar path for the case of heavy-light objects. We consider correlators for mesons containing one infinitely heavy $Q = (c, b, t)$ and one light quark $q = (u, d, s)$ [59, 72]. When looking at the dynamics in the heavy quark limit $m_Q \gg \Lambda_{QCD}$, we see that the heavy source behaves as a static center, in some hadronic analogy to the hydrogen atom. Therefore, the corresponding correlation function is almost reduced to the propagator of the light quark in the vicinity of the static color center. This feature will reflect some degeneracy of correlation functions, i.e. the manifestation of the IW symmetry.

We consider the correlation functions of the type

$$K_\Gamma(x) \;=\; \langle 0|(\bar{Q}\Gamma q)_x(\bar{q}\Gamma^\dagger Q)_0|0\rangle\,, \tag{4.1}$$

where Γ is a set of gamma matrices ($\mathbf{1}$,γ_5,γ_μ,$\gamma_\mu\gamma_5$,$\sigma_{\mu\nu}$) and x (or τ) is the (Euclidean) distance between the two points ($\tau = ix_0 - iy_0$), since very heavy quark does not move in space. As was discussed in [72] the heavy -light correlation function at the small distance x is just a product of free propagators (i.e. $S^0(x)$) given by

$$K_0^\Gamma(x) \;=\; Tr\left[S_q^0(x)\Gamma S_Q^0(x)\Gamma\right]\,, \tag{4.2}$$

where the free parts explicitly read

$$S_q^0(x) \;=\; -\frac{\gamma_0}{(2\pi^2x^3)},\;\; S_Q^0(x) = \frac{(1+\gamma_0)}{2}\delta^3(x)\,. \tag{4.3}$$

We start our analysis from the simplest case when the full heavy quark propagator is taken at the order of (m_Q^0), which implies a form $S_Q = S_\infty + O(1/m_Q)$. It also means that in this case we are treating heavy source as a standard Wilson line

$$S_\infty(x) \;=\; \langle x|\frac{1}{i\nabla_{4,I}}|0\rangle = \frac{(1+\gamma_0)}{2}\delta^3(\vec{x})\theta(\tau)P.\exp\left(\imath\int A_{4,I}dx_4\right)\,. \tag{4.4}$$

In [72], authors argue that the correction due to path ordered exponent is small and can be neglected in this kind of investigation. In the next section we will examine this assumption. Meantime, in all our simulations we have chosen to include the effect of the path ordered exponent. Using all previous definitions we may translate our correlation function (4.1) immediately into

$$K^\pm(x) \;=\; Tr\left[\left(\frac{1\pm\gamma_0}{2}\right)_Q S_q(-x)\right]\,, \tag{4.5}$$

where for a moment we have skipped all factors from heavy quark propagators except those with an information about parity of the state considered. The separation x is chosen along the positive time direction. As was mentioned before, in the large m_Q limit we expect the absence of the spin splitting, so the direction of the heavy quark is unimportant. This certainly implies that the pseudoscalars (P) and vector (V) mesons are degenerate alike the axial (A) and the scalar (S) ones, i.e. our correlators respect IW symmetry. These features are reflected by our notation, where $P = -1$ refers to (P,V) and for (A,S) we have $P = +1$. An explicit form of the light quark propagator one can find in [65, 72]. Following Shuryak [71, 72], we consider the correlation function normalized to the free quark correlator $K_0(x)$ and described in the whole region (not only at small distance) by the parametrization originating from the QCD sum rules. The simplest choice (i.e. with the minimal number of parameters) is the sum of the δ like resonance for the bound state and the "continuum" starting at the energy E_0

$$ImK^{\pm} = 6n_{res}\pi\delta(E - E_{res}) + \theta(E - E_0)\frac{3E^2}{2\pi}\,. \quad (4.6)$$

Above expression can be rewritten in the language of the space-time correlation function

$$\frac{K^{\pm}(x)}{K_0(x)} = 2\pi^2 n_{res} x^3 e^{(-E_{res}x)} + \left(1 + E_0 x + \frac{1}{2}E_0^2 x^2\right) e^{(-E_0 x)}\,. \quad (4.7)$$

The procedure now simple, this final equation has to be fitted to our results from the RILM simulation. All information on heavy-light mesons is encoded now in the three parameters : (n_{res}, E_{res}, E_0), in present investigation treated as a free parameters. The 3-dim density of the light quarks at the origin $(n_{res} = f_Q^2 M_q/12)$ reflects the Van Royen and Weisskopf relation, other two correspond to the position of the resonance E_{res} and to the perturbative threshold energy E_0, respectively. We point out that for each parity we may have in principle a separate set of these parameters.

Now, we show our results obtained for the correlation function of the heavy mesons (D and B) simulated by using RILM. All figures with a sample of curves (K^-/K_0,K^+/K_0) have been obtained for initial conditions defined as in section (4.1) with the heavy quark masses chosen to be infinite. The errors on Fig.4.1 are statistical. As we can see on the Figure, the curves behave exactly opposite to each other - one is going up (those for states with $P = -1$) which means that the pseudoscalar mesons are much lighter whereas the second one (for $P = +1$) is going down (because the scalar mesons are heavier). This is the first confirmation, that indeed the instanton vacuum causes right asymmetry between the states of opposite parity. Moreover, the straight and dashed lines correspond to the three parameters fit (4.7) with a following values of the parameters for each parity

$$n_{res}^- = (1.04 \pm 0.03)\ \mathrm{fm}^{-3},$$
$$E_{res}^- = (595.5 \pm 3.3)\ \mathrm{MeV},$$
$$E_0^- = (981.6 \pm 15.8)\ \mathrm{MeV},$$

$$n_{res}^+ = (5.56 \pm 1.19)\mathrm{fm}^{-3},$$

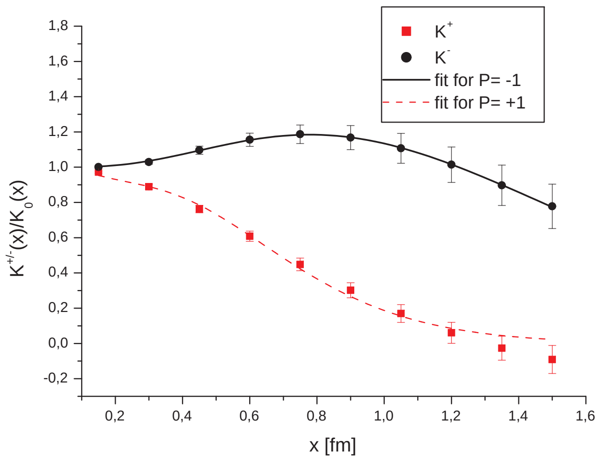

Figure 4.1: Heavy-light correlators normalized to the free quark correlator $K_{\pm}(x)/K_0(x)$ with three parameters fit for mesons (D,B) in heavy quark limit $m_Q \to \infty$. The data points are our results with only statistical errors included, while the dashed curves are fits. Parameters used in fit are listed and commented in text.

$$E^+_{res} = (1244.1 \pm 49.4)\text{MeV},$$
$$E^+_0 = (2156.7 \pm 197.5)\text{MeV}. \quad (4.8)$$

An important comment should be made here, the fitted curves given by eq.(4.7) are quite sensitive respect to the with values of parameters. This complication happens more often in the case of $P = -1$ comparing to $P = +1$. As we have mentioned before, the average correlators were obtained by averaging over 50 different configurations. We have performed the best fits using the data points with the statistical errors only. In the case of D and B mesons, it is visible that the errors of the $K^{\pm}(x)/K_0(x)$ increase with the distance x, which is expected, since we start feeling the finite size effects of the box. Summarizing this part, we may say that in case of nonstrange heavy-light mesons (D, B), the RILM has predicted the scalars much heavier then pseudoscalars and their volume $(1/n_{res})$ is about 7 times larger. One can read out the energies of those states from the fitted values of E_{res}. Moreover, we definitely may say that all values are in reasonable agreement with the previous calculations presented by [72].

As a next step, we have expanded the analysis for the mesons containing the strange quark - i.e. for D_s and B_s. In the same manner, using previous definitions one may construct the figures showing the behavior of the correlation functions for these states as function of the distance (taken from $0 - 1.5$ fm). In addition, using expression (4.7) one may find the best values of parameters as follows

$$\begin{aligned} n^-_{res} &= (0.63 \pm 0.30)\ \text{fm}^{-3}, \\ E^-_{res} &= (533 \pm 49)\ \text{MeV}, \\ E^-_0 &= (756 \pm 221)\ \text{MeV}, \\ \\ n^+_{res} &= (3.39 \pm 0.53)\text{fm}^{-3}, \\ E^+_{res} &= (1096 \pm 33)\text{MeV}, \\ E^-_0 &= (1817 \pm 118)\text{MeV}. \end{aligned} \tag{4.9}$$

Finally, taking into account all of those masses estimated for both $D(B)$ and $D_s(B_s)$ mesons we may try to answer to the question about the existence of the chiral doublers in such an environment. In particular, we need to calculate the splitting between the different parity states and to confront these values with originally predicted and measured in experiments. First of all, let us look at the mass differences: in the non-strange sector we get the value of the split of the order a $M = E^+_{res} - E^-_{res} \approx (649 \pm 53)$ MeV, and the similar shift for strange mesons (D_s, B_s) in the heavy quark limit is about $E^+_{res} - E^-_{res} \approx (563 \pm 82)$ MeV.

In both cases, the mass splittings are overestimated comparing to predictions given by [68, 33] and exceed known experimental values (450 Mev for non-strange and 350 MeV for strange heavy mesons). We may conclude that our splitting between the chiral doublers calculated using RILM is rather close to twice the constituent quark mass. An interesting issue is that the chiral split for the strange hadrons is smaller comparing to non-strange, which at first look is rather counterintuitive, but which reflects qualitatively the experimental situation. Indeed, the difference of the strange and non-strange splits from our simulation is of order of -100 MeV, in agreement with the data.

Let us first try to understand the effect of the mass of the light flavor.

First, we investigate how the estimated parameters (n_{res}, E_{res}, E_0) and our shift are changing when the "light" quark mass m_u is varied from 10 to 200 MeV. First picture showing the behavior of the density of the light quarks tells us that the parameters n_{res} (for $P = \pm 1$) start from the biggest value (about 5) and with increasing mass m_u it tends to be smaller and almost constant (particularly for negative parity states). What is interesting, these values are few times bigger for positive parity states (S,A). For

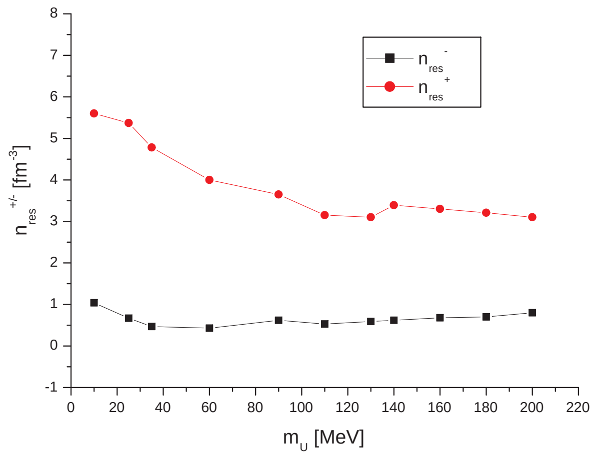

Figure 4.2: Schematic figures of the density of the light quarks at the origin n_{res} calculated for different quark masses m_u and each parity $P = \pm 1$. For pseudoscalar and vector mesons we have few times smaller values then for axial and scalar ones.

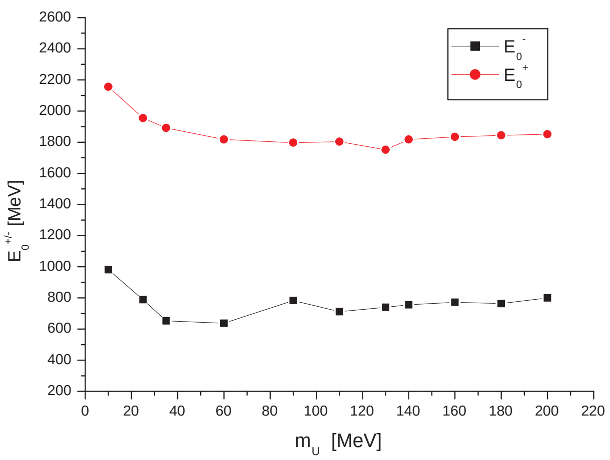

Figure 4.3: The continuum energies starting at the E_0 estimated for each parity as a function of quark mass m_u.

completeness, we present also figures for the threshold energies as a function of quark mass m_u. Another interesting property is visible at the figure showing the dependence of $E_{res}$with respect to the mass of the light quark. In the case of the positive parity states $P = +1$ (E^+_{res}) a curve looks like a straight line, in particular when starting from $m_u \approx 60$ MeV. This new feature implies that the masses for the scalar and axials propagating through the instanton vacuum from this value of m_u are approximately constant. In addition, these tendencies are visible more or less at all figures for the parameters fitted. The last group of Figures show the answer for existence of

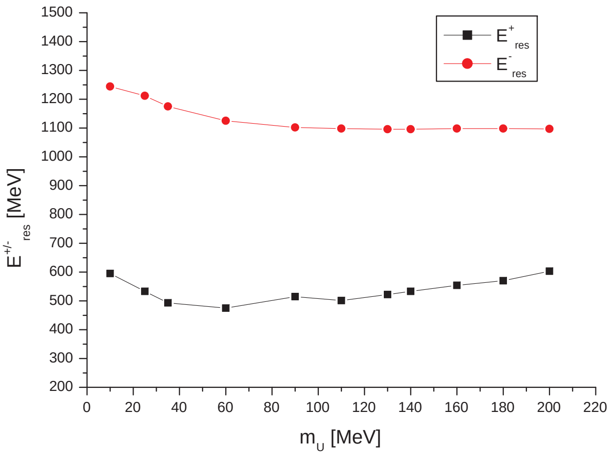

Figure 4.4: A behavior of resonances $E^{\pm}_{res}$ for different values of m_u.

chiral doublers in RILM. We drew a figure with the mass difference (denoted by M) between opposite parity states for light quark mass taken from 10 to 200 MeV. Looking closer into Fig.4.5 we can see that the chiral shift is slowly decreasing with bigger mass m_u. Let us suggest the explanation of this effect. In instanton picture, the constituent mass is the sum of current mass and constituent mass which is a dynamical function of the current one, contrary e.g. to NJL models, when the induced mass is constant. The larger the mass of the quark, the stronger is the suppression of the effects of spontaneous breakdown of the chiral symmetry. It looks that already for the values of the strange mass of order 150 MeV, this suppression is significant and implies that both chiral shifts (i.e. for non-strange doublers versus strange doublers) might be compatible or even inverted, alike in the data and in our simulation.

Despite the results of our simulation are in qualitative agreement with the experimental data, there is an important quantitative difference – the

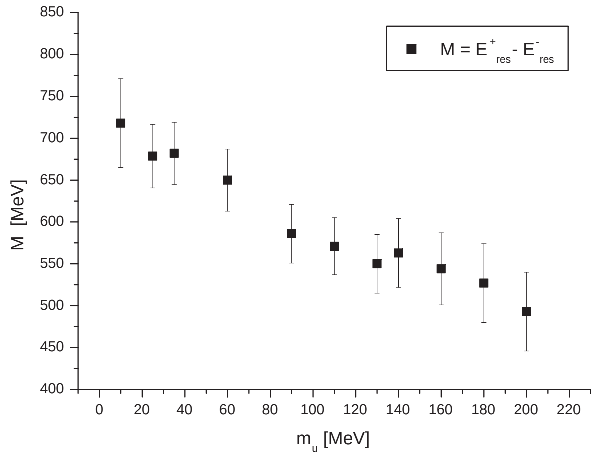

Figure 4.5: Mass difference between $P = \pm 1$ states calculated from $M = E^{+}_{res} - E^{-}_{res}$ where m_u is the light quark mass which was changed from $10 - 200$ MeV.

experimentally measured shifts are 300 MeV smaller comparing to the results of our simulations. At this point we address the crucial question: what is the reason of this difference and how other corrections to our model may improve the outcome of the instanton picture. In the next section we critically examine possible modifications of the RILM and we speculate which other properties of quarks might be responsible for the discrepancy of our prediction comparing to the data.

4.3 Discussion of the Results

In this section we would like to to critically examine our models looking for some possible loopholes.

First of all, to make our predictions more realistic we have to distinguish between the charm and the bottom quarks, i.e. to consider finite mass heavy quarks. It clearly suggests that we have to to include $1/m_Q$ corrections in the heavy propagator. However, in [59] the authors pointed out that those corrections are small in the case of the chiral shift. Before proceeding farther, let us focus for a moment on the definition of the infinitely heavy propagator S_∞. In previous part we have mentioned that in some papers authors claimed that the influence of the path ordered exponent in the definition of the heavy quark field (4.4) can be neglected. To verify this, we present a figure Fig.4.6 with special combinations of the correlation functions (for each parity) which were calculated in the presence or absence of the gauge field. This Figure shows how the path exponent may affect the correlators. The most intriguing conclusion is that the correlation functions for states with $P = -1$ are much more sensitive to the shape of the the instanton field. The curve goes down symmetrically when the path exponent from (4.4) is turned on. On the other hand, the fitted parameters from expression (4.7) are almost equal for both cases. In positive parity states we have found bigger differences for the estimated masses, which means that when the path order exponent is turned off the masses are smaller (about 50 MeV). But this effect has less than 10% effect on the chiral shift (649) MeV. Moreover, in all our simulations we were carefully including the effect of the path exponent, so clearly the details of the path ordering are not responsible for the overshooting of the chiral shift. Now we can come back to the issue of finite mass corrections to

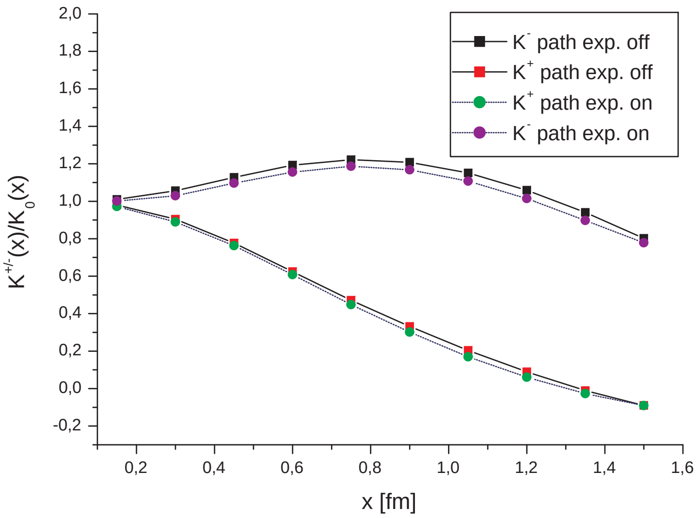

Figure 4.6: A sketch of $K_{\pm}(x)/K_0(x)$ when the path ordered exponent is turn on (off). For clearness we do not show the error bars. The dashed lines with circles correspond to include the exponent. As we see, the dashed curve is a bit lower.

the heavy propagator. Despite we expect in general small influence of the $1/m_Q$ corrections, some of them might be very sensitive to the size of the instanton. We remind that in literature one may find the instanton sizes

which vary from 0.2 to 0.4 fm. Let us briefly analyze finite size and finite mass effects on our simulation.
Following [59], we may add the next part of the order $1/m_Q$ in the definition of the heavy propagator, i.e.

$$S_Q = S_\infty + S_\infty \hat{O} S_\infty + \dots \, . \tag{4.10}$$

Here, S_∞ refers to the free part, the second term is correction of the order $\vartheta(1/m_Q)$ with the $\hat{O}$ operator defined as

$$\hat{O} = -\frac{\vec{\nabla}^2}{2m_Q} - \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q} \, , \tag{4.11}$$

reflecting two obvious lowest corrections: kinematic effect of the recoil and chromomagnetic interaction with the spin of the heavy quark. Obviously, this extended version of the heavy quark propagator may be also analyzed in RILM. To do this, we iterate the inverse of our infinitely heavy propagator S_∞ in powers of a low instanton density $n = n_*/2N_c$ [59]

$$S_\infty^{-1} \;\; = \;\; S_*^{-1} + \imath n \Theta_0 + \frac{\imath n}{\rho m_Q}\Theta_1 + \dots \, , \tag{4.12}$$

where two new contributions are given by

$$\Theta_0 = \int d^4 z_I \mathrm{Tr}\left[S_*^{-1}\left((\imath\gamma^4\nabla_{4,I})^{-1} - S_*\right)\right] \tag{4.13}$$

and

$$\Theta_1 = \int d^4 z_I \mathrm{Tr}\left[S_*^{-1}\left((\imath\gamma^4\nabla_{4,I})^{-1} - S_*\right) \times O_1(\imath\gamma^4\nabla_{4,I} - S_*)^{-1} S_*^{-1}\right] . \tag{4.14}$$

We remind the Tr stands for the trace over color indices and $O_1 = -\vec{\nabla}^2/2$. Therefore, using all above components one can investigate contributions followed from instanton induced shift in the heavy quark mass, i.e. $\Delta M_Q = \Delta M_0 + \Delta M_1$ obtained for a large times. Luckily, in the case of our model, we can make analytic estimations of the effect. The first one is a well known contribution calculated a couple years ago by [70]

$$\begin{aligned} \Delta M_0 \;\; &= \;\; \langle x_{-\infty}|\Theta_0|x_{+\infty}\rangle = \frac{N}{V_4 N_c} 16\pi\rho^3 \int_0^\infty dx \; (x\cos(f_x/2))^2 \\ &\simeq \;\; 16\pi n \rho^3, \end{aligned} \tag{4.15}$$

where the one instanton configuration appeared in $f_x = \pi|x|/\sqrt{1+x^2}$. Now we definitely may say that, the mass shift at this order strongly depends on the average size of instantons, i.e. $\rho = \rho_0$. As was mentioned at the beginning of this Chapter, the instanton size ρ_0 is usually varied from $(0.2 - 0.4)$ fm. According to these constraints, in our calculations we were using the value about 0.35 fm. Hence, one can easily obtain that the first shift in the heavy quark mass ΔM_0 is about 70 MeV and seems to be a significant value. The second one ΔM_1 [59] is of the order $(1/m_Q)$ and takes the following form

$$\begin{aligned}\Delta M_1 &= \langle x_{-\infty}|\Theta_1|x_{+\infty}\rangle = \qquad (4.16)\\ &= \frac{8\pi n\rho^2}{m_Q}\int_0^\infty dx\ x^2\left((\partial_\mu \cos f_x)^2 - (\partial_\mu \sin f_x)^2 + (\rho A_\mu)^2(\cos^2 f_x - \sin^2 f_x)\right)\end{aligned}$$

From above expression, using the mass of charmed quark we get the effect of the order $\Delta M_1 \simeq 28$ MeV. For completeness, we present also the values of the heavy mass shift ΔM_Q estimated for three different size of instantons ρ_0,i.e. 0.2 fm, 0.35 fm and 0.4 fm, respectively

$$\begin{aligned}\Delta M_Q &\simeq 25\ \text{MeV}\,,\\ \Delta M_Q &\simeq 100\ \text{MeV}\,,\\ \Delta M_Q &\simeq 155\ \text{MeV}\,. \qquad (4.17)\end{aligned}$$

Undoubtedly, the choice of the instanton size has a significant effect on these values.

Nevertheless, by taking into account two calculated contributions from the finite mass effects and modulo sensitivity of the results on the instanton size, we conclude that the $1/m_Q$ corrections (which are of order of 100 MeV) are too small to explain the overshooting of the shift in the instanton liquid picture. Moreover, our analysis leads us to the conclusion that even if we include the next step in m_Q, this will not change the estimated chiral shift as much as we need. Obviously, there were other assumptions in our model, like for instance the shape of the spectral function. However, we were expecting to measure the fundamental effect of the QCD, and such effect should not depend on the particular Ansatz and choice of the parameters.

If we look at our result from the perspective of the above discussion, we conclude that the obtained value of the chiral shift of order 650 MeV for non-strange doublers and 560 MeV for strange doublers might not be at all so puzzling. Obviously, comparing these results to the current experiments, we see that the results are overestimated. However, we have to remember about major shortcoming of the instanton liquid picture - the model was invented to include chiral dynamics, and the model does not take into account the effect of the confinement. From this perspective, the major effect for heavy-light hadrons in this model comes from anticommutation relation for the

Dirac-contracted four-velocity of the heavy quark and γ_5. Since this effect is opposite for the scalar and pseudoscalar sources (as well for axial and vector sources in the IW limit), chiral shifts are opposite, each of order of constituent mass. Similar argument [34] based on analogue of the Goldberger-Treiman (GT) relation for heavy-light systems shows that the leading order in $1/m_Q$ mass splitting (ΔM) between the non-strange parity partners is given by expression

$$\Delta M \quad = \quad g_\pi f_\pi \simeq 2m_{c'} \simeq 600\text{MeV}\,, \tag{4.18}$$

where g_π is the coupling constant of $G \rightarrow H\pi$ transition and is analogue of the $g_{NN\pi}$ in the nucleon system. Moreover, the light quark constituent mass $m_{c'}$ is given by $g_\pi f_\pi/2$ where f_π refers to the the pion decay constant. Again, this chiral Goldberger-Treiman analysis gives the result which is quite close to our approximation.

We are now at the position to conjecture, that the main quantitative effect explaining the discrepancy of our prediction to the existing data is related to the confining forces, not included in our model. Indeed, already looking at $1/m_Q$ corrections, we have already seen terms directly relating chromomagnetic field with the spin of the quark. Since confining and chiral phenomena are interrelated, as visible at finite temperature lattice studies, possibly, color effects may also screen considerably the chiral gap. It will be very interesting to analyze this system from the point of view of lattice simulations with genuine chiral fermions. We also hope, that the values of chiral shift for B-heavy-light chiral doublers will be available some day, shedding even more light of this conjectured subtle interplay between confining and chiral effects in case of chiral doublers.

Chapter 5

Doubly Heavy Baryons and QCD Instantons

Double heavy baryons are perhaps the most fascinating hadrons, due to the fact that they experience non-trivial color tube configuration, they probe spontaneous breakdown of the chiral symmetry and simultanously are subjected to restrictions from approximate Isgur-Wise and Savage-Wise symmetries. Unfortunately, these "dream" hadrons for several years have escaped the experiments. One notable exception is the mentioned already SELEX experiment at Fermilab, but five signals observed were never confirmed by other collaborations, and the nature of the data excludes the precise quantum number assessment. First major problem one immediately encounters when looking at the data is similar split between the lowest observed two pairs of states of order 78 MeV. Since this value is only 30% lower then the value of the spin split based on SW symmetry between corresponding total spin states $S = 3/2$ and $S = 1/2$ related to the D^* and D mass difference, one is tempted to conjecture that already at the value of charm quark mass SW symmetry is working. This implies a strong assumption, that color source of the diquark is very small, almost pointlike. We will carefully verify this point. We begin this chapter demonstrating, how one can deliver relevant vertices for heavy-heavy-light systems in the presence of the instantons. The presented calculation can be viewed as a analytic alternative to the numerical calculations of the type presented in the previous chapter. Since we start this part from presenting the general framework for calculations the effective vertices, hopefully this part also sheds some light on the details of the results used in our toy-model approximation. After obtaining the effective vertices, we simplify our results in order to get an estimate for doubly-heavy system in the guise of the toy-model presented before. As a next step, we will challenge the major assumption on the point-like nature of the diquark and the separation of the scales. We will make connection with popular potential models of confinement in order to estimate the scale of possible diquark excitations. We will also check what is the effect of non-perturbative interactions of the instanton-type on the potential models spectra. Finally, we will address the issue of the chiral doubling in the doubly heavy systems hadrons.

5.1 Quark Propagator and the Doubly Heavy Baryons

As was discussed in [59] the heavy-light hadrons in a dilute gas of instantons can be analyzed with the help of a correlation function based on systematic expansion in the inverse powers of heavy quark mass $1/m_Q$ using the Foldy-Wouthuysen construction [67]. In agreement with those arguments we will expand this picture to include the baryons with two heavy quarks. Our construction relies of the inverse correlation function C, expressed in terms of the propagators of the light quark field S_1 and propagators for the heavy quark field S_Q. Similar arguments like in the case of heavy-light mesons [59, 65] lead to the correlation function of the heavy-light system in the planar approximation ($N_c \to \infty$)

$$\begin{aligned} C^{-1}_{qQQ} &= S_q^{-1} \otimes S_Q^{-1,T} \otimes S_Q^{-1,T} \\ &\quad - \sum_{I\bar{I}} \left\langle [S_q - \not{A}_I^{-1}]^{-1} \otimes [S_Q - \not{A}_{I,4}^{-1}]^{-1} \otimes [S_Q - \not{A}_{I,4}^{-1}]^{-1} \right\rangle \end{aligned} \quad (5.1)$$

with A_I related to the gauge potentials for individual instantons. As we see, the inverse of the correlator contains the light quark propagator which can be written using expression

$$S_q^{-1} = S_0^{-1} - \sum_{I,\bar{I}} \left\langle \left[\not{A}_I^{-1} - S_q \right]^{-1} \right\rangle , \quad (5.2)$$

where we have denoted the propagator of the massless quark as $S_0^{-1} = \imath \gamma_\mu \partial_\mu$. In the same way, we are constructing the propagator of the heavy quark. Thus, the infinitely heavy quarks satisfy a following integral equation

$$S_Q^{-1} = S_*^{-1} - \sum_{I,\bar{I}} \left\langle \left[\not{A}_{I,4}^{-1} - S_Q \right]^{-1} \right\rangle . \quad (5.3)$$

Here, a few comments are in order. First visible difference between above equations is coded in label 4, which means that in the case of the heavy quarks we are looking only along time direction. The propagator contains only $\not{A}_4 = \gamma_4 A_4$ and the free part is denoted by $S_* = \imath\gamma_4\partial_4$. In both cases the sum is running over all considered pseudoparticles (i.e. instantons and anti-instantons) and $< ... >$ refers to the averaging over their positions z_I ($z_{\bar{I}}$) and the $SU(N_c)$ color orientation (denoted by U_I) hiding in the instanton field

$$A_I(x) \quad = \quad U_I A(x - z_I, \rho_I) U_I^\dagger \, . \tag{5.4}$$

Note that this procedure restores the Lorentz and gauge symmetries of the QCD vacuum. More details about group averaging U_I will be presented in the next section and outlined in Appendix B, section 8.4. Other interesting examples are presented in [59, 69]. Proceeding further, due to the small instanton density $n_* = N/V_4 \sim (\text{fm})^{-4}$ we can iterate the inverse of correlation function (5.1) in powers of density. The result of this rather complicated procedure reads in the leading order of n

$$\begin{aligned} C^{-1}_{qQQ} \quad &= \quad S_q^{-1} \otimes S_Q^{-1,T} \otimes S_Q^{-1,T} \\ &\quad -\imath n N_c \int d^4 z_I \text{Tr}\left([L]_I \otimes [H]_I \otimes [H]_I\right) \end{aligned} \tag{5.5}$$

with the light and heavy quark parts kernels are

$$\begin{aligned} [L]_I \quad &= \quad S_0^{-1} \left(\frac{|\Phi_0\rangle\langle\ \Phi_0|}{\imath\sqrt{n}\Sigma_0} - S_0 \right) S_0^{-1} \, , \\ [H]_I \quad &= \quad S_*^{-1} \left(\frac{1}{\imath\gamma^4 \nabla_{4,I}} - S_* \right) S_*^{-1} + \vartheta \left(\frac{1}{m_Q} \right) \, . \end{aligned} \tag{5.6}$$

Result for antiinstantons come out by replacing $I \to \bar{I}$. In the expression (5.6), the fermionic zero mode state Φ_0 has appeared. The explicit form in the so-called singular gauge was found by 't Hooft [54]

$$\Phi_0 \quad = \quad \frac{1}{\pi} \frac{\rho}{(x^2 + \rho^2)^{3/2}} \frac{\not{x}}{|x|} \gamma_\pm \varphi \, . \tag{5.7}$$

Here, φ refers to the color and the Dirac spinor, $\gamma_\pm = (1 \pm \gamma_5)/2$ are matrices for instantons/anti-instantons. This is the main building block of the spontaneous breakdown of the chiral symmetry, Planar resummation generates a new mass from the overlap between the zero modes, responsible for the appearance of the constituent of order $\sim (420\text{MeV})^{-1}$. For more detailed discussion we refer to the one of the thorough reviews [68].

5.2 Effective Action

After this general introducion of the formalism we concentrate on the calculation of the effective cubic vertex for HHL quarks. Other types of vertices, as HLL, HL, $H\bar{H}$, $L...L$ have been already constructed in the literarute, the last case being the celebrated 't Hooft determinant. In the spirit of the 't Hooft interaction between the light effective fields $\mathbf{q}$ we try to construct effective vertex for our doubly heavy baryons. To do that, let us first specify our coordinates: we chose (x, y) and $(x', y', x", y")$ to be the coordinates related with the one light and two heavy quarks, respectively. Thus, looking at the inverse of the correlator (5.5) we get the effective vertex

$$\Gamma^{ace}_{bdf}(x,y,x',y',x",y") = -\imath N_c \int d^4z \int dU_I \times (U^a_i \langle x| \, [L_I]^i_j \, |y\rangle U^{\dagger j}_{\ b} \otimes U^c_k \langle x'| \, [H_I]^k_l \, |y'\rangle U^{\dagger l}_{\ d} \otimes U^e_m \langle x"| \, [H_I]^m_n \, |y"\rangle U^{\dagger n}_{\ f}), \tag{5.8}$$

where U_I's refer to the $SU(3)$ color matrices.
This effective vertex can be viewed as originating from the effective action for the doubly heavy baryons $\mathcal{L}_{qQQ}$

$$\Gamma^{ace}_{bdf}(x,y,x',y',x",y") = \frac{\delta^6 \mathcal{L}_{qQQ}}{\delta q(x)^a \delta q^\dagger_b(y) \delta Q^c(x') \delta Q^\dagger_d(y') \delta Q^e(x") \delta Q^\dagger_f(y")}. \tag{5.9}$$

Our goal is to read out the structure of this Lagrangian. Note that the crucial combination comes out from the instanton, which is simultaneously "seen" by both heavy quarks and the light quark. It turns out, that it is convenient to express light and heavy propagators (5.6) in the instanton background in momentum space. To avoid repetitions from previous works, we introduce the light quark contribution to the action in a form

$$\mathcal{L}_q \simeq \int d^4z \int dU_I \; \mathbf{q}^\dagger_\alpha(k_1) \, \frac{1}{8} \, k_1\phi'(k_1) \; k_2\phi'(k_2) \left(\gamma_\mu \gamma_\nu \frac{1+\gamma_5}{2}\right) \times (U_I[\tau^-_\mu \tau^+_\nu]U_{I'})_{\alpha\beta} \; \mathbf{q}_\beta(k_2) e^{i(k_2-k_1)z} \int d\not{k}_1 \int d\not{k}_2 \,, \tag{5.10}$$

where $\int d\not{k}_i = d^4k_i/(2\pi)^4$ and the 't Hooft symbols are defined by $\tau^-_\mu \tau^+_\nu = \delta_{\mu\nu} + \imath \bar{\eta}^a_{\mu\nu} \tau^a$. For completeness, we remind also the construction of the heavy part. In the previous sections, we have described the heavy quark propagator using the Wilson loop [59, 65]

$$\langle x|\frac{1}{\imath\nabla_{I,4}}|x'\rangle = \delta^3(\vec{x}-\vec{x}')\theta(t_{x'}-t_x)\frac{1+\gamma_4}{2}P.\exp\left(\imath\int_{t_x}^{t'_x} dx_4 A_{I,4}\right). \quad (5.11)$$

It is obvious that in above expression we need to evaluate the path exponent in the singular gauge (see Appendix A) defined by

$$A^a_{I\mu} = U_I\bar{\eta}^a_{\mu\nu}\tau^a U_I^\dagger \frac{x_\nu\rho^2}{x^2(x^2+\rho^2)} \quad (5.12)$$

with a parameter ρ stands for the size of an instanton and the initial position chosen at $z_{I4} = 0$. This issue was also discussed in [65]. After substituting (5.12) into (5.11), the heavy part reads

$$\begin{aligned}\mathcal{L}_{QQ} \simeq & \int d^4z_I \int dU_I \mathbf{Q}^\dagger_{\alpha'} \frac{1+\gamma_4}{2}\,(l\omega)\,\left[F_1 + U_I(\vec{\tau}\vec{\omega})U_I^\dagger F_2\right]_{\alpha'\beta'} \mathbf{Q}_{\beta'} \quad (5.13)\\ & \int \frac{d^3\omega}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\int\frac{dl}{(2\pi)}\frac{d\omega}{(2\pi)} e^{-\imath\vec{z}\vec{\omega}} e^{-\imath z_4(l-\omega)} \times (\text{heavy quark } \mathbf{Q'})\,.\end{aligned}$$

At this point, let us explain in more details the above formula. We have defined a special set of coordinates: time dependent (l,ω) which refers to $(x_4 - z_4, |x_4 - z_4|)$ and the space dependent like $(\vec{k},\vec{\omega},\vec{z})$. As usual, $\vec{\tau}$ are a standard set of Pauli matrices. Moreover, a new F is just a result followed from integration of the path exponent, connected with $F_1 = \cos(F/|\vec{x}-\vec{z}|^{-1}) = \cos(\widetilde{F})$ and $F_2 = \sin(F/|\vec{x}-\vec{z}|^{-1}) = \sin(\widetilde{F})$. Finally, we will combine together the light and heavy sectors, i.e. (5.11) with (5.14) by integration over instanton position (z_I, z_{I4}) and color orientations. Integrations over position give us only the Dirac delta functions which is clearly a manifestation of conservations of energy and momentum by our vertices. The last but non-trivial step comes from the color structure. As was mentioned before, we need to integrate the final Lagrangian density $\mathcal{L}_{qQQ}$ over the $SU(3)$ color coordinates (U_I-integration). This integration is equivalent to finding all projections onto the singlets of the group

$$\int dU[U^a_i U^{\dagger j}_b]^m \;=\; P(\mathbf{3}\otimes\bar{\mathbf{3}})^m \to \mathbf{1}\,. \quad (5.14)$$

In practice this problem reduces to finding all projections of the product of m octets onto the singlet $P(\otimes\mathbf{8}) \to \mathbf{1}$. Looking into literature we may find many useful methods for small values of $n = 1, 2, 3$ [69]. For our purposes, the effective action requires all these three solutions because we have a following color dependent structure

$$\left[U_I(\tau_\mu^- \tau_\nu^+)U_I^\dagger\right]_{\alpha\beta} \times \left[\cos(\widetilde{F}) + U_I(\tau^a\omega^a)U_I^\dagger \sin(\widetilde{F})\right]_{\alpha'\beta'} \times \left[\cos(\widetilde{F}') + U_I(\tau^b\omega^{b'})U_I^\dagger \sin(\widetilde{F}')\right]_{\alpha"\beta"}, \quad (5.15)$$

where the first bracket comes from the light part and the last parts involve an information about the heavies. The trivial term comes from $m = 1$ case, providing some sort of the renormalization of the disconnected part of the Green's function, i.e. $\sim \frac{2}{Nc}\delta^\alpha_\beta \delta_{\mu\nu} \cos(\widetilde{F})\cos(\widetilde{F}')$. It is simply a product of Kronecker deltas with the color $(\alpha\beta)$ and the Dirac $(\mu\nu)$ indexes, modulo a coefficient. Note that a contribution from one octet leads to zero. An interesting interaction between two different sectors will appear for $m = 2$ and $m = 3$. The most complicated term binds together the three flavors and the projection of the three octets onto the singlets is given by

$$\frac{N_c}{8(N_c^2-4)(N_c^2-1)} d_{ijk} d_{abc} [\lambda^i]^\alpha_\beta [\lambda^j]^{\alpha'}_{\beta'} [\lambda^k]^{\alpha"}_{\beta"} \times [\lambda^a]^{h_1}_{h_2} [\lambda^b]^{h_3}_{h_4} [\lambda^c]^{h_4}_{h_5} + \frac{1}{8(N_c^2-1)N_c} f_{ijk} f_{abc} [\lambda^i]^\alpha_\beta [\lambda^j]^{\alpha'}_{\beta'} [\lambda^k]^{\alpha"}_{\beta"} \times [\lambda^a]^{h_1}_{h_2} [\lambda^b]^{h_3}_{h_4} [\lambda^c]^{h_4}_{h_5}, \quad (5.16)$$

where we have a combination of the color Gell-Mann matrices λ^a ($a = 1, ..., N^2-1$) with normalization $\mathrm{Tr}\lambda^a\lambda^b = 2\delta^{ab}$. All these projections may be constructed from a proper combinations of the standard structure constant d^{abc} (symmetric) and f^{abc} (antisymmetric). We remind, they are defined by a anticommutator $\{\lambda^a\lambda^b\} = 2d^{abc}\lambda^c + \frac{4}{3}\delta^{ab}$ and commutator $[\lambda^a\lambda^b] = 2\imath f^{abc}\lambda^c$, respectively. To deal with $SU(2)$ color one should set $f^{ijk} = \epsilon^{ijk}$ and just omit terms with symmetric d^{abc}. To summarize, we list terms appearing in the effective Lagrangian:

$$\int dU (U_I \tau_\mu^- \tau_\nu^+ U_I^\dagger) \cos(\widetilde{F})\cos(\widetilde{F}') = \frac{2}{Nc}\delta^\alpha_\beta \, \delta_{\mu\nu} \cos(\widetilde{F})\cos(\widetilde{F}') \quad (5.17)$$

for $m = 1$, then first relevant term with an interaction between light and heavy degrees (with $m = 2$)

$$\int dU \left(U_I[\tau_\mu^- \tau_\nu^+]U_I^\dagger\right)_{\alpha\beta} \left(U_I[\tau^a\omega^a]U_I^\dagger\right)_{\alpha"\beta"} \sin(\widetilde{F}')\cos(\widetilde{F}) \quad (5.18)$$

$$= \frac{2}{4(N_c^2-1)} [\lambda^i]^\alpha_\beta [\lambda^i]^{\alpha"}_{\beta"} \times \left[\frac{2}{\sqrt{3}}\delta_{\mu\nu}\delta_{b8} + 2\imath\bar{\eta}^b_{\mu\nu}\right] \omega'^b \sin(\widetilde{F}')\cos(\widetilde{F})$$

and finally from the interaction of three flavors, after lengthy calculation we find out that only two terms survive, i.e.

$$\frac{1}{(N_c+2)(N_c^2-1)}\ \delta_{\mu\nu}\omega^b\omega'^b\ d_{ijk}[\lambda^i][\lambda^j][\lambda^k]\sin(\widetilde{F})\sin(\widetilde{F}') + \frac{1}{N_c(N_c^2-1)}\ \imath\epsilon_{abc}\bar{\eta}^a_{\mu\nu}\omega^b\omega'^c\ f_{ijk}[\lambda^i][\lambda^j][\lambda^k]\sin(\widetilde{F})\sin(\widetilde{F}')\,. \quad (5.19)$$

For simplicity, in above expression we have skipped an indexes $(\alpha,\beta...)$ and reminded that for indexes $\{a,b,c\}=\{1,2,3\}$ the antisymmetric structure constant can be replaced by $f_{abc}=\epsilon_{abc}$.

In this way we have collected all essential elements needed for constructing the effective actions for the doubly heavy baryons

$$\begin{aligned}\mathcal{L}_{qQQ} \simeq\ & \frac{\Delta M_q \Delta M_Q^2}{2n^2N_c^2}\int\left(\mathbf{q}^\dagger_\alpha(k_1)\ \frac{1}{8}k_1\phi'(k_1)\ k_2\phi'(k_2)(\gamma_\mu\gamma_\nu\frac{1+\gamma_5}{2})\mathbf{q}_\beta(k_2)\right)\\ & \times[...]\times\left(\mathbf{Q}^\dagger_{\alpha'}(\vec{k},l)\frac{1+\gamma_4}{2}\ (l\omega)\ \mathbf{Q}_{\beta'}(\vec{k}-\vec{\omega},\omega)\right) \qquad (5.20)\\ & \left(\mathbf{Q}^\dagger_{\alpha"}(\vec{k'},l')\frac{1+\gamma_4}{2}\ (l'\omega')\ \mathbf{Q}_{\beta"}(\vec{k'}-\vec{\omega'},\omega')\right)\\ & (2\pi)^4\delta^3(\vec{k_2}-\vec{k_1}-\vec{\omega}-\vec{\omega'})\ \delta(k_2-k_1+\omega-l+\omega'-l')\\ & \int\frac{d^3\omega}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\int\frac{dl}{(2\pi)}\frac{d\omega}{(2\pi)}\int\frac{d^3\omega'}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\int\frac{dl'}{(2\pi)}\frac{d\omega'}{(2\pi)}\int d\not{k}_1\int d\not{k}_2\end{aligned}$$

with the bracjet [...] contains all nontrivial information on the color structure (see Appendix 8.4)

$$\begin{aligned}[...] =\ & \frac{2}{Nc}\delta^\alpha_\beta\ \delta_{\mu\nu}\cos(\widetilde{F})\cos(\widetilde{F}')\\ & +\frac{1}{(N_c^2-1)}[\lambda^i]^\alpha_\beta[\lambda^i]^{\alpha"}_{\beta"}(\imath\bar{\eta}^b_{\mu\nu})\omega'^b\sin(\widetilde{F}')\cos(\widetilde{F})\\ & +\frac{1}{(N_c^2-1)}[\lambda^i]^\alpha_\beta[\lambda^i]^{\alpha'}_{\beta'}(\imath\bar{\eta}^b_{\mu\nu})\omega^b\sin(\widetilde{F})\cos(\widetilde{F}')\\ & +\frac{2}{N_c(N_c^2-1)}[\lambda^i]^{\alpha"}_{\beta"}[\lambda^i]^{\alpha'}_{\beta'}\delta^\alpha_\beta\delta_{\mu\nu}\omega^j\omega'^j\sin(\widetilde{F})\sin(\widetilde{F}') \qquad (5.21)\\ & +\frac{1}{(N_c+2)(N_c^2-1)}\ \delta_{\mu\nu}\omega^b\omega'^b\ d_{ijk}[\lambda^i][\lambda^j][\lambda^k]\sin(\widetilde{F})\sin(\widetilde{F}')\\ & +\frac{1}{N_c(N_c^2-1)}\ \imath\epsilon_{abc}\bar{\eta}^a_{\mu\nu}\omega^b\omega'^c\ f_{ijk}[\lambda^i][\lambda^j][\lambda^k]\sin(\widetilde{F})\sin(\widetilde{F}')\,.\end{aligned}$$

The above expression (5.21) derived by us has several intersting features. It can be seen that contracting a pair of the light flavors $(q^{\dagger}_{\alpha}q_{\beta})$ in the effective Lagrangian for the doubly heavy baryons gives us additional factor $V' = (\frac{1}{2}V_0 < \bar{q}q >)$. Moreover, in our new two-body interaction, one may immediately recover a significant instanton-induced interaction for heavy mesons $\bar{Q}Q$

$$\mathcal{L}_{QQ} \sim \frac{2V'}{N_c}\bar{\mathbf{Q}}\bar{\mathbf{Q}}'\left(\cos(\widetilde{F})\cos(\widetilde{F}') + \frac{\omega^j\omega'^j}{(N_c^2-1)}\lambda_1^i\lambda_2^i\sin(\widetilde{F})\sin(\widetilde{F}')\right)\mathbf{Q}\mathbf{Q}'\,. \tag{5.22}$$

In the next section we will focus our attention on the possible influence of the derived action on the spectroscopy of the doubly heavy baryons.

5.3 Spectrum of the Doubly Heavy Baryons

We would like now to analyze the features of the spectrum of the doubly heavy baryons reported by SELEX Collaboration [7] using our instanton model. Taking into account the complicated structure of the interaction, we choose a simple scenario. Since we are aware that instanton model does not provide sufficiently strong confining forces, we mimic confinement by a harmonic potential. Then variational calculation allows us to simply parameterize the binding. The Hamiltonian involving instanton vertices calculated before will be used as a perturbation, and by calculating expectation values in the lowest instanton density approximation we will get the relevant splits. Surprisingly, above calculations will be of the "back of the envelope" kind, since we will employ couple of tricks. Nevertheless, we will still be able to identify major advantages and the weak-points of the presented approach. In order to show the versatile nature of the instanton calculus, in the subsequent chapters we will confront our estimation with the results of potential models.

Before we will turn into the calculation, let us make an important remark, which will also allow us to fix the notation. Till this moment, in our considerations we did not specify the detailed flavor setting of the doubly heavy baryons. If the heavy quarks composing the diquark subsystem (with $\bar{3}$ color) have identical flavors ($Q = c, b$), then it is necessary to take into account the Pauli principle. Due to this constraint, the sum of heavy quark spins $S_d = 0$ is forbidden for symmetric wave functions of the diquark ψ_d with the orbital angular momentum $L_d = 0, 2, ...$ (as usual labeled by S, D,..). Therefore, the ground state $1S1s$ $(n_dL_dn_ql)$ of the doubly heavy baryons Ξ_{QQ} has the total angular momentum given by $J^P = (\frac{1}{2}^+, \frac{3}{2}^+)$. Similarly, for the antisymmetric, odd functions Ψ_d (i.e., with the orbital angular momentum $L_d = 2n + 1 = 1, 3, ...$) the total spin of diquarks $S_d = 1$ is forbidden. All these problems disappear when we consider the different flavors inside the heavy diquark $(Q'Q)$. For example the ground state of $\Xi_{Q'Q}$ will have $J^P = (\frac{1}{2}^+; \frac{1}{2}^+, \frac{3}{2}^+)$. Also this case the SW "supersymmetry" is broader comparing to the case of identical heavy flavors. Since there is only one signal and not confirmed signal for the "triple scoop" baryon *scb*, and it will turn out that charm is not heavy enough to believe into SW symmetry, we will not study this case in this work.

5.3.1 Variational estimate

We use variational model developed in [59]. We explain the notation and the main idea considering much simpler case of the mesons, then we move towards double heavy baryons. In case of heavy-light charmed mesons, the Hamiltonian reads

$$H = \frac{\vec{p}_q^2}{2m_q} + \frac{\vec{p}_Q^2}{2m_Q} + \frac{1}{2}M\omega^2|\vec{r}_q - \vec{r}_Q|^2 + H^{(2)}\,. \tag{5.23}$$

Here M is the reduced mass of the heavy-light system, $m_q = \Delta M_q \sim 420$ MeV and $m_Q = m_c + \Delta M_Q \sim (1350+86)$ MeV. The harmonic potential is responsible for the confinement. The instanton-induced interaction $H^{(2)}$ plays the role of the perturbation. The simplest choice for the trial wavefunction is a Gaussian

$$\psi(\chi) = \left(\frac{2\alpha}{\pi}\right)^{3/4} e^{-\alpha\chi^2}\,, \tag{5.24}$$

where $\vec{\chi} = \frac{1}{\sqrt{2}}(\vec{r}_q - \vec{r}_Q)$. Minimization of the expectation value of (5.23) with respect to variational parameter α yields $\alpha = \frac{1}{2}M\omega$ leads to the confining energy reads $\mathcal{E}_\alpha = \frac{3}{2}\omega$, as expected from elementary quantum mechanics. The size $r = \sqrt{\frac{1}{2\alpha}}$ of the ground state is a function of the reduced mass M, which allows to fix the parameters by the size of the heavy-light system. For example [59], if $r_{qQ} = 0.6\,\text{fm}$, then the size of the the heavy-heavy system is $r_{Q\bar{Q}} \simeq 0.4\,\text{fm}$ for charmed state and $r_{Q\bar{Q}} \simeq 0.35$ fm for the bottomium. The correction comes from two-body instanton-induced interactions, which for system $Q\bar{q}$ reads [59]

$$H^{(2)}_{qQ} = \Big(\frac{\Delta M_Q \Delta M_q}{2nN_c}\Big)\left(1 + \frac{1}{4}\lambda^a_q\lambda^a_Q\right)\delta^3(\vec{r}_q - \vec{r}_Q)\,. \tag{5.25}$$

The spectrum of the heavy-light system comes from formula

$$M_{qQ} = \langle H^{(0)} + H^{(1)} + H^{(2)} \rangle\,, \tag{5.26}$$

where first term $H^{(0)}$ represents the binding energy $\mathcal{E}_\alpha$ and the current masses of the quarks, the second is an instanton-induced dressing ($H^{(1)}$) stands for the induced instanton mass for the light and heavy quarks and $H^{(2)}$ provides the mechanism for splitting by instanton-induced heavy-light vertices. It is rather interesting that such a simple model leads to the results which are in satisfactory agreement both with experiment and with several potential models.

The extension to double heavy baryons goes as follows. The first, kinematic part is easy. One introduces the Jacobi coordinates, i.e. in addition to $\vec{\xi}$ corresponding to the difference of heavy quarks coordinates we also have $\vec{\eta} = \sqrt{\frac{1}{6}}(\vec{r}_Q + \vec{r}_Q - 2\vec{r}_q)$. The trial function is the product of two Gaussians for these coordinates, so the binding harmonic Hamiltonian separates into the sum of two harmonic oscillators, depending on variational parameter reflecting the total size of the system (dispersion of the Gaussian). The dynamical part is less trivial. On top of Hamiltonians $H^{(1)}$ and $H^{(2)}$ we have now to take into account Hamiltonian $H^{(3)}$, calculated in the previous chapter. At first, this looks weird, since the averaging over color matrices scales like $1/N_c^m$, where m is number of quarks entering the relevant vertex, so $H^{(3)}$ is $1/N_c$ suppressed comparing to $H^{(2)}$. This is certainly true for the infinite number of colors, but for $N_c = 3$ these "sub-leading" term is crucial for understanding the binding. Let us explain it in more detail. In case of our simple Ansatz, the addition of one more quark into the vertex (i.e. $m \longrightarrow (m+1)$ for color averaging), brings about in the $(m+1)$-body contribution to the energy an additional overall factor of $\mathcal{R}_q$ for a light quark, and $\mathcal{R}_Q$ for a heavy quark, in comparison the m-body interaction. These R factors [59] read explicitly

$$\mathcal{R}_{q,Q} = -2\Big(\frac{\Delta M_{q,Q}}{2nN_c}\Big)\left(\frac{1}{\sqrt{\pi}r}\right)^3. \tag{5.27}$$

To see the role of the $\mathcal{R}_q$ factor, let us recall the double-heavy vertex generated by the instantons

$$H^{(2)}_{QQ} = \Big(\frac{\Delta M_Q \Delta M_Q}{nN_c}\Big)\left(1 + \frac{3}{32}\lambda_1^a\cdot\lambda_2^a - \frac{9}{32}\sigma_1\cdot\sigma_2\,\lambda_1^a\cdot\lambda_2^a\right)\delta^3(\vec{r}_1 - \vec{r}_2) \tag{5.28}$$

This interaction is expected to be overall attractive and binding. The expectation value of the $H^{(3)}$ term compares to $< H^{(2)}_{QQ} >$ multiplied by $\mathcal{R}_q$, which is negative. The binding in the instanton vacuum picture is therefore the result of subtle interplay between between two-body and three-body forces, since e.g. for sizes of order .9 fm both terms are of similar magnitude. Since the $\mathcal{R}$-factor has an inverse cubic power of the typical size of the baryon, the resulting spectra are extremely sensitive to the size of such object, on top of very strong dependence of the heavy quark mass instanton-induced shift on the average size of the instanton. For typical sizes of the instanton $\rho = 0.3 \pm 0.1$fm, it is impossible to fit the spin-shift of the SELEX data for Ξ_{cc} (equal to 78 MeV) for average size of the system considerably smaller than 0.8 fm [1]. Huge variation of the predicted spectra as a function of the size of the instanton tells us also that the unknown role of color screening and confining forces, missing in the instanton picture, is very important for understanding the spectra of the doubly heavy baryons.

Finally, we would like to come to the question, how, if at all, SW symmetry is fulfilled at the level of the charm quark. We would like to point out, that approaching the infinite heavy quark limit, we have a natural separation of the several scales. Since infinitely heavy diquark is point-like, excitations in the finite mass heavy quark-quark systems (orbital excitations of the diquark) have to scale like m_Q. Excitations related to light degrees of freedom are independent of the mass of the heavy quark, so they scale like m_Q^0. Finally, spin effects scale like m_Q^{-1}, so for sufficiently heavy mass these scales are well separated. Similar analysis including additionally the effect of the finite velocity of the heavy quark was presented recently in [52]. We would like here to perform similar analysis, taking into account effects from instantons and complementing the speculations [52] by dynamical models of the potential type.

5.3.2 Diquarks, charmonia and double-heavy baryons

In this section we would like to use the existing data for charmonia system to extract certain information on doubly heavy systems. Our motivation is as follows:
First, we choose a standard potential model describing well charmnia states (at least till the recent BaBar/Belle revolutions) as a gauging device for our speculations. We also study the effect of instanton on these potentials. Then, using the experimental data and popular argument, based on Casimir scaling, that quark -quark interactions are twice weaker comparing to quark-antiquark interaction, we calculate the scale of the typical excitations of the diquark, ignoring the issue that such system is not at color neutral. Finally, we build a color neutral, double heavy baryon, by solving the pertinent

[1]We would like to thank Bartosz Fornal for independent verification of this fact.

bound-state equation for a doubly heavy diquark and light quark. Since we perform the above calculations for the realistic masses of charm and bottom quark, we hope that the numerical results for the bound states will teach us something on the interplay of the three scales discussed above.

We start from considering the doubly heavy diquark and from computation the energy levels. Our considerations are very similar to those well known in literature, where the charmonium or bottomium states have been examined [73]. In the similar manner, to obtain the spectrum of the heavy diquark (QQ) we use the nonrelativistic Schrödinger equation with a proper selection of the potential. For simplicity, in the first part of present calculations we decided to take a combination of the scalar and vector potentials given by

$$V(r) = V_s(r) + \frac{1}{2}V_v(r) = (br + c) + \frac{1}{2}\left(-\frac{4}{3}\frac{\alpha}{r}\right), \qquad (5.29)$$

where for the anti-triplet quark state, we have introduced the color factor 1/2 in front of usual Coulombic potential $V_v(r)$ corresponding to the quark-antiquark pair. It is a consequence of a following expression $V_{QQ} = \frac{1}{2}V_{Q\bar{Q}}$. When applying potential models, several possible choices of input parameters can be made. Typical range of the vector potential parameter α is about $(0.28 - 0.44)$, and for the value of the mass we use $m_c = (1.3 - 1.7)$ GeV. Unfortunately, only one of these parameters seems to be well defined in literature, i.e. the so-called string tension, b, due to model independent information from the lattice. In our numerical calculations we combined the values used in papers [73, 74]. For completeness, all parameters with references are summarized in Table (5.1).

Table 5.1: Parameters of the potential model.

Parameter	Value	Ref.
α	0.44	[74]
m_b	4700 MeV	
m_c	1350 MeV	
m_q	385 MeV	[74]
b	0.18 GeV^2	[73]
c	0.02 GeV	[73, 76]

Therefore, following [74], our first task is to solve numerically the Schrödinger equation

$$\left(2m_Q - \frac{\nabla_d^2}{m_Q} + V(r)\right)\psi_d \quad = \quad M_d\psi_d \tag{5.30}$$

and to look at the excitation spectra. Here, the masses of the doubly heavy diquark at zeroth order is determined by M_d and the nabla operator for a spherically symmetric system has a standard form, i.e. $\nabla_d^2 = \frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} - \frac{\vec{L}_d^2}{r^2}$. Let us only remark that this part is evaluated without the spin-dependent corrections. Our aim is to obtain a reasonable spectrum for a few significant levels with the potential $V(r)$ chosen in two different ways. On top of the standard linear potential we modify the shape of $V(r)$ by taking into account the instanton contribution due to the Wilson line. It can be done by replacing of the scalar part of our potential $V_s(r)$ by the potential between the heavy quarks, i.e

$$\begin{aligned} V_s(r) \quad \to \quad & V_{inst}(r) \\ & = 2\frac{N}{VN_c}\int d^3z[1 - \cos\left(\frac{\pi|\vec{r}-\vec{z}|}{\sqrt{(\vec{r}-\vec{z})^2+\rho^2}}\right)\cos\left(\frac{\pi|\vec{z}|}{\sqrt{|\vec{z}|^2+\rho^2}}\right) \\ & + \frac{(\vec{r}-\vec{z})}{|\vec{r}-\vec{z}|}\frac{(\vec{z})}{|\vec{z}|}\sin\left(\frac{\pi|\vec{r}-\vec{z}|}{\sqrt{(\vec{r}-\vec{z})^2+\rho^2}}\right)\sin\left(\frac{\pi|\vec{z}|}{\sqrt{|\vec{z}|^2+\rho^2}}\right)]. \end{aligned} \tag{5.31}$$

Certainly, this modification can be viewed only as a approximation, as we all know [56], that this kind of potential is rather quadratic for small distances ($r \to 0$) then rises linearly to approach soon the constant values, there is rather binding than confining. Moreover , it can be easily checked that the shape of an instanton-induced potential strongly depends on the inverse of high power of the instanton size ρ. More details about this feature, in particular the example of curve obtained for size of instanton about 0.35 fm was studied in [70]. What is interesting, the confrontation of these two scenarios (i.e. energies M_d and $M_{d,I}$) gives us an information about the role of instantons in the diquark spectrum. The estimated energies of the doubly heavy diquark (QQ) for the various of levels are listed in Table 5.2.

Before proceeding further, let us analyze some features of the obtained spectra. First, we notice that the spectra for instanton potential are considerably below the linear one. The reason is that instanton potential is weaker comparing to the linear one with proper string tension, a fact reflecting lack of confining forces in the instanton vacuum picture. Comparing the results for charm and bottom quarks, we see that the effects are less dramatic for bottom quarks. Again, this is not surprising, since their wave functions are more localized around the origin and less sensitive to larger distance behavior of the potential. The effect weakens for highly excited states, when extended wave functions probe considerably larger distances. An important message comes from the size of the gap for the first diquark excitation: in case of the charm quark, the gap is of order of 450 MeV, therefore comparable to the chiral shift observed from BaBar and Belle data for chiral doublers. Clearly there is no separation of scales at the level of charm quark and the application of constraints from SW symmetry is doubtful. For the case of the bottom quark, the result is exciting - the corresponding gap is of order of 140 MeV, and since similar chiral shift of order of 400 MeV is expected for b as for c quark, there is some hope, that perhaps at the level of the mass of the b quark Savage-Wise symmetry may we approximately working. If indeed, this may have dramatic consequences for e.g. appearance of exotic states including bottom and light quarks.

Before proceeding towards the analysis of the spin effects, let us complete the potential model for heavy-heavy-light baryons. As a next step we have to introduce second Schroedinger equation, this time binding a heavy diquark to a light quark. Such equation can be solved only numerically. We have performed this task in analogy to the discussion presented in [74]. Relevant equation takes the following form

$$\left[M_{d,I} + m_q + \frac{p^2}{2m_q} + \frac{p_d^2}{2M_{d,I}} + V_1(r)\right]\psi_q(r) \quad = \quad M_B^{(0)}\psi_q(r) \qquad (5.32)$$

with the potential given by

$$V_1(r) \quad = \quad (br+c) + \left(-\frac{4}{3}\frac{\alpha}{r}\right) . \qquad (5.33)$$

In light of the previous discussion, we have ignored instanton effects in the scalar potential as to weak to provide proper binding. Above equation connects the light quark with the doubly heavy diquark and leads us to the spectrum of the doubly heavy baryons like Ξ_{cc} (energies are denoted by M_B). Thus, we have completed the calculation of the binding force. In the next section we will describe how the spin splitting should be inserted and worked out for the quark-diquark systems.

5.3.3 Spin-dependent corrections

The diquark levels calculated in the previous chapter did not take into account the spin effects. Now, we would like to take into account the spin-spin

Table 5.2: The spectrum of the doubly charmed diquarks (cc) without the spin-dependent splitting.

Diquark level ($n_d L_d$)	Mass M_d[GeV]	Mass $M_{d,I}$ [GeV]
$1S$	3.23	2.79
$2S$	3.79	3.08
$3S$	4.22	3.36
$1P$	3.59	2.97
$1D$	3.87	3.11

Table 5.3: A few relevant states of the doubly heavy diquarks (bb) without the spin-dependent splitting.

Diquark level ($n_d L_d$)	Mass $M_{d,I}$ [GeV]
$1S$	9.43
$2S$	9.59
$3S$	9.74
$1P$	9.53
$1D$	9.61

and spin-orbit interactions, causing the splitting between the levels of the quark-diquark system. As far as we know, this issue was discussed in few papers [74]. In accordance with those results, we specify the additional potential as

$$\begin{aligned} V_{SD}(r) &= \frac{1}{4}\left(\frac{\vec{l}\cdot\vec{S}_d}{2m_Q^2}+\frac{2\vec{l}\cdot\vec{S}_q}{2m_q^2}\right)\left(-\frac{dV_1(r)}{rdr}+\frac{8}{3}\frac{\alpha}{r^3}\right) \\ &+\frac{1}{3}\frac{\alpha}{m_Q m_q}\frac{(\vec{l}\cdot\vec{S}_d+2\vec{l}\cdot\vec{S}_q)}{r^3}+\frac{4}{3}\frac{\alpha}{3m_Q m_q}(\vec{S}_d+\vec{L}_d)\cdot\vec{S}_q[4\pi\delta(r)] \\ &-\frac{1}{3}\frac{\alpha}{m_Q m_q}\frac{1}{r^3}\frac{1}{(4\vec{l}^2-3)}\times[6(\vec{l}\cdot\vec{S})^2+3(\vec{l}\cdot\vec{S})-2\vec{l}^2\vec{S}^2 \\ &-6(\vec{l}\cdot\vec{S}_d)^2-3(\vec{l}\cdot\vec{S}_d)+2\vec{l}^2\vec{S}_d^2], \end{aligned} \qquad (5.34)$$

where $\vec{L}_d=\vec{L}_1+\vec{L}_2$, $\vec{S}_d=\vec{S}_1+\vec{S}_2$ are the orbital momentum of the diquark and the summed spin of the heavy quarks inside the diquark, respectively. The interaction between the light constituent quark and the doubly heavy diquark appeared in the total spin, i.e. $\vec{S}=\vec{S}_q+\vec{S}_d$. Let us remind that the first term in (5.34) refers to the relativistic corrections to the potential $V_1(r)$, three others come from the one gluon exchange between the quarks.

From above spin-dependent potential one may recover the first perturbative correction to the spectrum of the doubly heavy baryons

$$M_B^{(1)} = \int d^3r \psi^*(\vec{r}) V_{SD}(r) \psi(\vec{r}) , \tag{5.35}$$

so the bound state mass to the first order can be written as

$$M_B = M_B^{(0)} + M_B^{(1)} . \tag{5.36}$$

Table 5.4: The mass spectrum of the doubly charmed baryons Ξ_{cc}.

$(n_d L_d n_q l) J^P$	Mass [GeV]	Ref.[74]	Ref.[75]
(1S 1s) $1/2^+$	3.667	3.478	3.620
(1S 1s) $3/2^+$	3.832	3.610	3.727
(1S 1p) $1/2^-$	4.120	3.927	4.053
(1S 1p) $3/2^-$	4.229	4.039	4.101
(1S 1p) $5/2^-$	4.233	4.047	4.155
(1S 1p) $1/2'^-$	4.241	4.052	4.136
(1S 1p) $3/2'^-$	4.205	4.034	4.196

As usual, a general formulation for this kind of interaction, where diquark and light quark have different masses is given by the scheme of jj coupling. It is convenient to use following relations for the given values of $\vec{J}$ and $\vec{S}$

$$|J; J_q\rangle = \sum_S (-1)^{(J+l+S_d+S_q)} \sqrt{(2S+1)(2J_q+1)} \left\{ \begin{array}{ccc} S_d & S_q & S \\ l & J & J_q \end{array} \right\} |J; S\rangle \tag{5.37}$$

or

$$|J; J_q\rangle = \sum_{J_d} (-1)^{(J+l+S_d+S_q)} \sqrt{(2J_d+1)(2J_q+1)} \left\{ \begin{array}{ccc} S_d & l & J_d \\ S_q & J & J_q \end{array} \right\} |J; J_d\rangle , \tag{5.38}$$

where the braces stand for the Wigner 6j-symbols, commonly used in the coupling of three angular momenta. Apart from the phase, 6j-symbols are equal to the Racah's W-coefficients. In the current problem, the total angular momentum of the doubly heavy baryons involves $\vec{J} = \vec{J_q} + \vec{S_d}$, where

$\vec{J}_q = \vec{l} + \vec{S}_q$ is the light quark total angular momentum and $\vec{J}_d = \vec{l} + \vec{S}_d$. As explained previously, for the case of identical heavy flavors, we identify the heavy quarks by the values od $\vec{S}_d$, i.e. the sum of the heavy quarks spin $S_d = 1$ for even L_d, as well as for odd $S_d = 0$. In order to calculate the level shifts between baryon states, appearing due to the spin-dependent terms, one has to average expression (5.34) over the corresponding wave functions. Moreover, the potential terms of the order $1/m_Q^2$ and $1/m_Q m_q$ lead to the mixing of baryon states with the same values of the total angular momentum J but different light quark total momentum J_q. For instance, it has to be taken into account in case of state with the diquark in a ground state (S-wave) and the light quark in p-wave state, i.e. denoted by $(1S1p)$. This is why it is convenient to use two separate bases for the $|J; J_q >$ states, depending on which type of spin operator we would like to take the expectation value. Despite this procedure is similar to standard procedures in atomic or molecular spectroscopy, technically it is quite involved. Luckily, the hyperfine splitting for the system of the light quark-diquark is easier to compute, using a following formula

$$\Delta_{HF} = \frac{2}{9 m_Q m_q} \alpha \left[J(J+1) - J_d(J_d+1) - \frac{3}{4} \right] |\psi_q(0)|^2 , \quad (5.39)$$

where $\psi_q(0)$ identifies the radial wave function at the origin. According to this expression one may immediately calculate the masses of the ground states $(1S1s)$ for the doubly charmed baryons. All relevant states obtained using our simple potential model are presented and compared to other models predictions in Tables 5.4 and 5.5.

Table 5.5: The mass spectrum of Ξ_{bb} baryons.

$(n_d L_d n_q l) J^P$	Mass [GeV]	Ref.[74]	Ref.[75]
(1S 1s) $1/2^+$	10.346	10.093	10.202
(1S 1s) $3/2^+$	10.401	10.113	10.237
(1S 1p) $1/2^-$	10.788	10.541	10.632
(1S 1p) $3/2^-$	10.812	10.567	10.647
(1S 1p) $5/2^-$	10.821	10.580	10.661
(1S 1p) $1/2'^-$	10.842	10.578	10.675
(1S 1p) $3/2'^-$	10.872	10.581	10.693

Our results are closer to calculations done in [75], but there are also consistent with the others [74], including the (quenched) lattice prediction for ground state of Ξ_{cc} to be of order of 3600 MeV. The split between the 1/2 and 3/2 state comes out to be of order of 160 MeV, i.e. 30% more comparing to other quoted studies, and twice bigger than in experiment if the observed by SELEX lowest excitation is indeed a 3/2 state. Finally, the conjectured chiral shift of order 400 MeV matches the results of our calculations, i.e. 450 MeV and is in reasonable agreement with the 340 MeV guess based on the SELEX data. In the case of the predictions for double heavy Ξ_{bb}, the split between 3/2 and 1/2 states comes out to be of order of 60 MeV, as expected comparing to similar split of 160 MeV for charmed state, since this split is a $1/m_Q$ effect and $m_b \sim 3m_c$. The opposite parity split for bottom quarks comes again about 450 MeV. If confirmed by future experiment, it will certainly pose the theoretical challenge to decide if the similar splits are accidental or they reflect the chiral doubling nature of the heavy-light systems.

Chapter 6

Summary

This work was triggered by several new experimental results in heavy-light hadrons spectroscopy. Guided by the underlying idea of chiral doubling, we tried to present a unified picture for all hadrons exhibiting this phenomenon, using as a tool instanton vacuum picture. In the process of this work, we have obtained several new results, like stronger arguments for chiral doubling based on anomaly (Chapter 3), numerical studies of the heavy-light mesons in the framework of Stony Brook instanton model for the light hadrons (Chapter 4) and the structure of effective Lagrangian describing heavy-heavy-light baryons in the instanton picture (Chapter 5). We have also verified and checked several other results, like the calculation of doubly heavy baryons in potential models and we have examined the criticism of Savage-Wise symmetry at the presently accessible experimental scales. Here we would like to present our conclusions and prospects of the heavy-light hadrons according to several ideas of interest.

- Fundamental understanding of the heavy-light hadrons

  The fundamental understanding of heavy-light hadrons from first principles remains a challenge, since the nature of the QCD vacuum providing both confinement and spontaneous breakdown of the chiral symmetry is still mysterious. It looks that the only hadronic system for which confinement forces are not crucial is the pion, because its Nambu-Goldstone nature seems to dominate the dynamics. But already its strange cousin, kaon, is more complicated, since the mass of the s quark is sufficiently heavy to be comparable with the fundamental constant of the QCD, Λ_{QCD}, obscuring natural m_s/Λ expansion. For other light hadrons, since the spontaneous breakdown of the chiral symmetry generates the dynamical constituent mass of order 350 MeV, one can easily approach the correct values of masses of vector mesons or Δ on the basis of constituent models, but at the price of loosing the confinement, like in NJL models or instanton vacuum picture. The other extreme are purely heavy systems, when spontaneous breakdown of the chiral symmetry does not seem to play the crucial role. Since the spectroscopy of HHH-system, the Holy Grail of experimentalists, is still the dream, one has to study $H\bar{H}$ systems. Recent experimental results have clearly demonstrated, that the announcement of the success of the potential models for charmonia was premature. This leaves us with challenging possibility, that the effects of light degrees of freedom might still be crucial for understanding states considered till today as nonrelativistic. Perhaps the overlap of confining and chiral domains is most visible for heavy-light systems, as recent data have been shown. Our results seem to corroborate this picture, since the obtained results show that for the charm quark we are still far from the factorized physics for heavy and light degrees of freedom. It is visible from our instanton simulations for heavy-light mesons, when we have obtained the value of the chiral shift of order of 600 MeV, therefore close to twice the constituent mass of the light quark. Indeed, each light quark dresses via propagation of the vacuum, but since the relative sign is opposite due to the chiral properties of the chiral doublers, naive split is twice the value of the constituent mass. The fact that experiments show the value which is considerably smaller confirms the fundamental role of color screening interactions for heavy-light systems. It will be therefore exciting to see what are the experimental results for the chiral split in the case of B mesons. Above arguments prevented us from repeating the full scale simulations for the baryons, including especially doubly heavy baryons, when the flow of color forces joining the diquark with the light quark is certainly very complicated. It remains a challenge for lattice calculations to understand better the configuration of the flux tube in such cases. On the other side, we have shown that major spin effects can be described quite successfully at the phenomenological level using the concept of effective Lagrangians originating from instanton picture. This is in agreement with the old ideas of Shuryak and Rosner for light degrees of freedom and confirms and extends the similar analysis for heavy-light degrees of freedom started in [59]. From computational point of view, this approach is an interesting alternative to several potential models dominated by one-gluon exchange potentials, as we were advertising in this work.

- Experimental results

The success of our approach relies heavily on experimental data, and it is disappointing that there are several experimental obstacles to gain minimal set of data to verify such a unified picture of heavy-light baryons as we have attempted. The first announcements of chiral doublers for $c\bar{s}$ were spectacular and they triggered worldwide a renowned interested in heavy-quark spectroscopy. Till today there are well established and confirmed by few experiments. Unfortunately, this is not the case of the corresponding B_s mesons, which till today have not yet been seen. This experimental signal is crucial for discriminating between potential models and chiral doublers scenario. It will be also great if high quality data for nonstrange opposite parity mesons will come, including the exciting states.

Even more dramatic situation is in the case of the double heavy baryons. Only 5 types of ccL configurations were observed, all by the same experiment (SELEX) [7], and the quantum numbers of the observed states were not established. Till today the SELEX data were not confirmed by other experiments, including negative results from e^+e^- experiments. In this work we have assumed that the SELEX data and their conjecture on the nature of spin are the correct ones, but it will be certainly very important to get an independent experimental confirmation. At this point, let us mention an intriguing possibility, that the mechanism of formation of the doubly heavy states may depend on the type of the initial reactions, i.e. may require some particular and unknown mechanism of coalescence. This may partially explain why the states visible at hadronic collisions at Fermilab were not observed in electron-positron colliders.

Last but not least, we would like to comment on the Z^+ state observed by Belle [16]. Since this object was observed in the decay onto excited charmonium and the charged pion, this signal, if confirmed, is the clear evidence of the structure beyond the constituent quark model, i.e. the charmed tetraquark of the type $c\bar{c}u\bar{d}$. If confirmed, result is sensational, pointing at possible non-trivial involvement of the color and chiral forces, as speculated in this work. However, taking into account a long-standing confusion concerning the light pentaquark and the fact that Belle result is in discrepancy with the BaBar analysis, one has to wait until the experimentalists will reach the ultimate conclusion.

- Savage-Wise symmetry

  Savage-Wise symmetry [51] relating such ingredients of heavy light hadrons as $\bar{H}$ to HH, i.e relating HHL to $\bar{H}L$ and HLL to $\bar{H}\bar{H}LL$ is an intriguing theoretical idea. One of the results of this work is the expectation, that the mass of the charm quark is too light to make this symmetry working. Some of our arguments based on analysis of the bound states have shown, that certainly the scale of the excitations of light degrees of freedom overlaps strongly with the scale of the internal excitations of the diquark, therefore falsifying the point-like nature of the diquark assumed by Savage and Wise. Our results were then confronted with the SELEX data, modulo the above mentioned reservations. Similar analysis of SELEX data was performed in [52]. Our conclusion is stronger, we expect that 30% agreement with the predicted split based on SW symmetry might be accidental. We also do not expect that SW symmetry will manifest itself at the level of doubly heavy "three scoop baryons" of the type bcL, since the mass of the charm quark is not heavy enough. On the positive side, we speculate that bbL baryons (not observed yet) may show remnants of the SW symmetry, since the bb system is considerably smaller. This speculation opens the possibility for exotic states like tetraquark related to "ordinary" bLL baryons.

- Exotics

  We have already mentioned the significance of the Z^+ signal (if confirmed) and the exotic consequences of the SW symmetry at the b quark scale. Here we would like to provide an additional argument for the possibility of heavy exotic states based on our analysis of multiquark vertices induced by the instanton vacuum. Such vertex is fundamental at the level of light quarks ('t Hooft determinant), therefore we may expect that its heavy-light counterparts might be equally important for hadronic physics. Two body interaction differs by sign comparing to three body interactions, and for the typical sizes of heavy hadron below 1 fm the magnitude of two terms is comparable. Similarly, four body interaction induced by instantons has opposite sign comparing to three-body force. This implies that for exotic configuration there is a subtle interplay that may lead to weak binding. Let us assume that such configuration is almost bound at the charm level. If we will replace now the charm by bottom, potential part will stay the same, but the positive kinetic part will be three times smaller, so overall the binding will be considerably stronger. Therefore it is plausible that the exotic heavy states will manifest themselves only at the level of the bottom quark.

Summarizing, we are convinced that heavy-light systems open us a unique insight onto the hadronic world. We also believe that better experimental data will strengthen the theoretical rigor of similar analysis, as it happened historically in the case of chiral perturbation theory and usual parity Heavy Quark Effective Theory. There is a considerable hope that experimental breakthrough may soon happen. One intriguing possibility is the further upgrade of the KEK machine, so Belle will start probing higher and higher b-quark spectroscopy, hopefully so successfully as in the case of charm quark. New signals are also expected from LHC at CERN, where the physics of charm and bottom hadrons plays the major role and when new mechanisms of production of heavy-light states may take place in extreme conditions. Finally, the statistics and branching ratios for existing heavy-light particles will be upgraded by experiments at Fermilab, Stanford (BaBar), BES (Beijing) or PANDA (GSI Darmstadt). We are impatiently waiting for these events.

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Chapter 7

Appendix A

7.1 Fierz Transformation

In this Appendix we present the details of Fierz transformation used in Chapter 3 for rearranging our Lagrangian densities. Since we want to work with color-neutral objects ("white"), we start from basic relations allowing us manipulate the flow of colors Fig.7.1. The figure depicts the relation for color

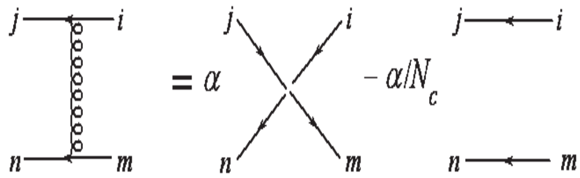

Figure 7.1: Basic relations for color indexes. The value of α follows from used normalization of color matrices, i.e. $Tr\lambda^a\lambda^b = \alpha\delta^{ab}$.

matrices which can be explicitly decoded as

$$\sum_{a=1}^{8}(\lambda^a)_{ij}\,(\lambda^a)_{mn} = \alpha(\delta_{in}\delta_{mj} - \frac{1}{N_c}\delta_{ij}\delta_{mn}) \equiv \alpha(f - \frac{1}{N_c}d)\,. \quad (7.1)$$

We rewrite this relation using the basis of symmetric and antisymmetric tensors, i.e.

$$S \;=\; \frac{1}{2}(d+f)\,,$$

$$A = \frac{1}{2}(d - f)\,, \tag{7.2}$$

where we have suppressed the indices since their flow is obvious. In this basis we have $d = S + A$ and $f = S - A$. Putting above formulae all together we get

$$\begin{aligned}(\delta_{in}\delta_{jm} - \frac{1}{N_c}\delta_{ij}\delta_{mn}) &= \frac{1}{\alpha}\lambda_{ij}\lambda_{mn}\,,\\ \frac{1}{N_c}(\delta_{ij}\delta_{mn} - \delta_{in}\delta_{jm}) &= \frac{1}{N_c}\sum_k \epsilon_{kim}\epsilon_{knj}\,.\end{aligned} \tag{7.3}$$

This immediately implies one of the Fierz color identities used in our calculations

$$\sum_{a=1}^{8}(\lambda^a)_{ij}\,(\lambda^a)_{mn} = \frac{4}{3}\,\mathbf{1}_{in}\mathbf{1}_{mj} + \frac{2}{3}\sum_{k=1}^{3}\epsilon_{kim}\epsilon_{knj} \tag{7.4}$$

with normalization $\alpha = 2$. A second identity may be found by the simple exchange the indexes $j \leftrightarrow n$ in the first part of eq.(7.1)

$$\frac{1}{2}\sum_{a=1}^{8}(\lambda^a)_{in}(\lambda^a)_{mj} = (\mathbf{1})_{ij}\,(\mathbf{1})_{mn} + \frac{1}{3}\,\mathbf{1}_{in}\mathbf{1}_{mj}\,. \tag{7.5}$$

All these relations were used to relocate our fields in color space. To endow the states with correct quantum numbers we also have to apply a Fierz transformations for the spin. Before doing this let us define first our convention. Following [61]

$$\begin{aligned}S &= \mathbf{1}\otimes\mathbf{1} = \Gamma_s\otimes\Gamma_s\,,\\ V &= \gamma^\mu\otimes\gamma_\mu = \Gamma_V^\mu\otimes\Gamma_{\mu,V}\,,\\ T &= \frac{1}{2}\sigma^{\mu\nu}\otimes\sigma_{\mu\nu} = \frac{1}{2}\Gamma_T^{\mu\nu}\otimes\Gamma_{\mu\nu,T}\,,\\ A &= \gamma_5\gamma^\mu\otimes\gamma_\mu\gamma_5 = \Gamma_A^\mu\otimes\Gamma_{\mu,A}\,,\\ P &= \gamma_5\otimes\gamma_5 = i\Gamma_P\otimes i\Gamma_P\,,\end{aligned} \tag{7.6}$$

where the set of all spin operators is expressed in term of the 16-dimensional basis for Dirac matrices, denoted by Γ_β. In the definition (7.6) we omitted for simplicity the spinors ψ. Now, we can write in full glory the Fierz transformation

$$(\bar\psi(4)\Gamma_I^\beta\psi(2))\;(\bar\psi(3)\Gamma_{\beta,I}\psi(1)) \;=\; F_{IK}\;(\bar\psi(4)\Gamma_K^\beta\psi(1))(\bar\psi(3)\Gamma_K^\beta\psi(2)) \tag{7.7}$$

with a numerical matrix F relating the two sets of quantities with $F^2 = 1$. The matrix has the following form

$$F_{IK} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 0 & 2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{pmatrix}. \tag{7.8}$$

Notice that in our case the relevant matrix is actually $-F$ due to the fact that Fermi fields anticommutate. To demonstrate how the Fierz transformation works in practice, According to all our definitions of Fierz coefficients, we apply it to one gluon exchange Lagrangian (3.2), with the following result

$$\mathcal{L}_{qQ} = \frac{g^2}{\Lambda^2}\left(\bar{Q}q\bar{q}Q - \frac{1}{2}\bar{Q}\gamma_\mu q\bar{q}\gamma^\mu Q + \frac{1}{2}\bar{Q}\gamma_\mu\gamma_5 q\bar{q}\gamma^\mu\gamma_5 Q - \bar{Q}\gamma_5 q\bar{q}\gamma_5 Q\right). \tag{7.9}$$

Chapter 8

Appendix B

In this Appendix we provide a few details of the calculations done in Chapter 5. We start from the "dictionary" relating several quantities in Euclidean and Minkowski space-time. Next, we collect several useful relations for the 't Hooft symbol. Then, we outline how the specific "hedgehog" (in spin and color) feature of the instanton gauge field allows to make explicit calculation of the path-ordered exponent in the Wilson loop, leading to the derivation of the instanton-driven potential between two infinitely heavy quarks. Finally, we review one of tricks allowing very effective way of integration over the string of color matrices (U integration) with respect to the Haar measure.

8.1 Conventions

The formulae for transcription from Euclidean to Minkowski space-time are as follows:

- Gamma matrices
  $\gamma_4^E = \gamma_0$, $\gamma_i^E = -i\gamma_i$ where $i = 1, 2, 3$
  and anticommutation relation $\{\gamma_m u^E, \gamma_n u^E\} = 2\delta_\mu \nu$.

- Covariant differentiation:
  $D_4^E = -iD_0$, $D_i^E = -D_i$ $(i = 1, 2, 3)$ hence $D_\mu^E = \partial_\mu - \imath g A_\mu^E$.

- Fermi fields:
  $\psi^E = \psi$, $\psi^{\dagger E} = i\bar{\psi}$.

- Vector potentials:
  $A_\mu^E = -\imath A_0$ and $A_i^E = -A_i$ with $i = 1, 2, 3$.

- Space-time coordinates:
  $x_4^E = \imath x_0$ and spatial $x_i^E = x_i$.

- Action:
  $S^E = -\imath S$

8.2 't Hooft Symbols

So-called 't Hooft symbols η are defined with the help of the $4-vector$ matrices

$$\tau^{\pm}_{\mu} = (\vec{\tau}, \mp\imath) \tag{8.1}$$

and follow from relations

$$\begin{aligned} \tau^{-}_{\mu}\tau^{+}_{\nu} &= \delta_{\mu\nu} + \imath\bar{\eta}_{a\mu\nu}\tau^{a}, \\ \tau^{+}_{\mu}\tau^{-}_{\nu} &= \delta_{\mu\nu} + \imath\eta_{a\mu\nu}\tau^{a}\,. \end{aligned} \tag{8.2}$$

Numerically they read $\eta_{a\mu\nu} = \begin{cases} \varepsilon_{a\mu\nu}, & \text{for } \mu,\nu = 1,2,3, \\ -\delta_{a\nu}, & \text{for } \mu = 4 \\ \delta_{a\nu}, & \text{for } \nu = 4 \\ 0, & \text{for } \mu = \nu = 4 \end{cases}$

Definition for $\bar{\eta}_{a\mu\nu}$ differs from $\eta_{a\mu\nu}$ by a change of the sign of δ. We list several useful relations for these symbols ($a, b = 1,2,3$ and $\mu,\nu = 1,...,4$):

$$\begin{aligned} \eta_{a\mu\nu} &= \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\eta_{a\alpha\beta}\,, \\ \eta_{a\mu\nu} &= -\eta_{a\nu\mu}\,, \\ \eta_{a\mu\nu}\eta_{a\gamma\lambda} &= \delta_{\mu\gamma}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\gamma} + \varepsilon_{\mu\nu\gamma\lambda}\,, \\ \eta_{a\mu\nu}\eta_{a\mu\nu} &= 12\,, \\ \eta_{a\mu\nu}\eta_{b\mu\nu} &= 4\delta_{ab}\,, \\ \eta_{a\mu\nu}\eta_{a\mu\rho} &= 3\delta_{\nu\rho}\,, \\ \eta_{a\mu\nu}\bar{\eta}_{b\mu\nu} &= 0\,, \\ \eta_{a\mu\nu}\bar{\eta}_{b\mu\gamma} &= \eta_{a\mu\gamma}\bar{\eta}_{b\mu\nu}\,. \end{aligned} \tag{8.3}$$

To pass from the above relations to those for $\bar{\eta}_{a\mu\nu}$ it is necessary to make a following substitution $\eta_{a\mu\nu} \to \bar{\eta}_{a\mu\nu}$ and $\varepsilon_{\mu\nu\alpha\beta} \to -\varepsilon_{\mu\nu\alpha\beta}$. The power of the 't Hooft symbols calculus stems from the fact that by construction they are either self-dual (η's) or anti-self-dual ($\bar{\eta}$'s)

8.3 Path Ordered Exponent and Heavy Quarks

In this part we compute the Wilson line in the instanton background and we demonstrate a behavior of the heavy quark potential induced by the instanton as a function of the distance, following Callan et al., Diakonov [70] and others. The propagator for a heavy quark is given by a following expression

$$S_Q(x) = \frac{1+\gamma_4}{2}\delta^3(\vec{r})\theta(\tau)P.\exp\left(\imath\int A_\mu dx_\mu\right), \tag{8.4}$$

where P. denotes a path-ordering along the contour. We choose an infinite straight line along time direction (4-direction), i.e.

$$P.\exp\left(\imath\int A_\mu dx_\mu\right) = P.\exp\left(\imath\int_{-\infty}^{+\infty} A_4^a dx_4\right) \tag{8.5}$$

and we calculate the phase in the presence of one instanton (centered at z_I and of the size ρ_I) in singular gauge

$$A_\mu^a = \bar{\eta}_{\mu\nu}^a \tau^a \frac{(x-z)_\nu \rho^2}{(x-z)^2((x-z)^2+\rho^2)}\,. \tag{8.6}$$

Using formulae for 't Hooft symbols from the previous section with $\bar{\eta}_{4\nu}^a = \delta_{a\nu}$ we find a path exponent in an explicit form

$$P.\exp\left(\imath\int_{-\infty}^{+\infty} A_4^a dx_4\right)|_{z_{I4}=0} = \cos\left(\pi - \frac{\pi|\vec{z}|}{\sqrt{|\vec{z}|^2+\rho^2}}\right) + \imath\frac{\vec{z}\vec{\tau}}{|\vec{z}|}\sin\left(\pi - \frac{\pi|\vec{z}|}{\sqrt{|\vec{z}|^2+\rho^2}}\right). \tag{8.7}$$

Let us remark that anti-instantons will double this contribution, i.e. $A_4^a = A_{\pm,4}^a$. To get the explicit color dependence we need to sandwich the second term between U and $U^\dagger$ color matrices. Now we are ready to extract the potential from the Wilson loop. Following [65] we define the potential as

$$V(x)_{inst} = \frac{N}{(2N_cV)}\int d^3z\ \mathrm{Tr_c}[1 - L(\vec{x}-\vec{z})L^\dagger(-\vec{z})]\,, \tag{8.8}$$

where L are corresponding path-exponents for infinitely quark and antiquark separated by $\vec{x}$. Short calculation gives the following form

$$V(x)_{inst} = 2\frac{N}{(N_cV)}\int d^3z[1 - \cos\left(\frac{\pi|\vec{x}-\vec{z}|}{\sqrt{(\vec{x}-\vec{z})^2+\rho^2}}\right)\cos\left(\frac{\pi|\vec{z}|}{\sqrt{z^2+\rho^2}}\right) + \frac{(\vec{x}-\vec{z})}{|\vec{x}-\vec{z}|}\frac{(\vec{z})}{|\vec{z}|}\sin\left(\frac{\pi|\vec{x}-\vec{z}|}{\sqrt{(\vec{x}-\vec{z})^2+\rho^2}}\right)\sin\left(\frac{\pi|\vec{z}|}{\sqrt{z^2+\rho^2}}\right)]\,. \tag{8.9}$$

8.4 U-integration

This part of the Appendix contains a complete set of relations used in the last part of our calculation for $\mathcal{L}_{qQQ}$. To obtain the effective vertex for doubly heavy baryons we need in particular to average over color matrices U_I of the instanton "seen" by all three quarks. Instead of using popular textbook techniques for and integration over the color space $SU(N_c)$ we rely on the graphical techniques [59, 69] based on constructing proper projection operators onto singlets. Averaging over the color-orientation matrices requires the knowledge about integrals of the type

$$\int dU[U_i^a U_b^{\dagger j}]^m \tag{8.10}$$

with invariant measure $\int dU = 1$ for several natural numbers m. The trick is based on the fact, that integrating over colored orientations chooses the color singlets, since any non-trivial direction in color is averaged to zero when the sum over orientations is performed. We may therefore use the following correspondence

$$U_i^a U_b^{\dagger j} \leftrightarrow \frac{1}{N_c}\delta_b^a \delta_i^j + [\lambda^i]_b^a [\lambda^i]_i^j \,, \tag{8.11}$$

where λ^a are eight color matrices ($a = 1, ..., 8$) with a standard normalization $\mathrm{Tr}\lambda^a\lambda^b = 2\delta^{ab}$. The problem reduces therefore to the classification of all projections of the product of m octets onto singlets. For $m = 1$, since $3 \otimes \bar{3} = 1 \oplus 8$, only singlet survives, since averaging over octet is zero. For $m = 2$, additionally we need the projection of the product of two octets onto singlet, i.e. $8 \otimes 8 \to 1$. Relevant projectors are

$$P_1 = \frac{1}{N_c}, \quad P_8 = \frac{1}{4(N_c^2 - 1)} \,. \tag{8.12}$$

For $m = 3$ on top of the above structures we need projectors of $8 \otimes 8 \otimes 8 \to 1$. There are two of them due to antisymmetric f^{abc} and symmetric d^{abc} structure functions for $N_c > 2$ and they read

$$P_f = \frac{1}{2N_c} \quad \text{and} \quad P_d = \frac{N_c}{2(N_c^2 - 4)}. \tag{8.13}$$

It is now trivial to write down few integrals

- $m = 1 \to \int dU[U_i^a U_b^{\dagger j}] = \frac{1}{N_c}\delta_b^a \delta_i^j$

- $m = 2 \rightarrow \int dU [U_i^a U_b^{\dagger j}]^2 = [\frac{1}{N_c} \delta_b^a \delta_i^j]^2 + \frac{1}{4(N_c^2-1)} [\lambda^i]_{b_1}^{a_1} [\lambda^i]_{b_2}^{a_2} [\lambda^i]_{i_1}^{j_1} [\lambda^k]_{i_2}^{j_2}$

- $m = 3 \rightarrow \int dU [U_i^a U_b^{\dagger j}]^3 = [\frac{1}{N_c} \delta_b^a \delta_i^j]^3$
$+ \frac{1}{4N_c(N_c^2-1)} \left([\lambda^i]_{b_1}^{a_1} [\lambda^i]_{b_2}^{a_2} [\lambda^j]_{j_1}^{i_1} [\lambda^j]_{j_2}^{i_2} \delta_{b_3}^{a_3} \delta_{i_3}^{j_3} + (3 \leftrightarrow 2) + (3 \leftrightarrow 1) \right)$
$+ \frac{N_c}{8(N_c^2-4)(N_c^2-1)} \; d_{ijk} \;\; d_{abc} [\lambda^i]_{b_1}^{a_1} [\lambda^j]_{b_2}^{a_2} [\lambda^k]_{b_3}^{a_3} [\lambda^a]_{j_1}^{i_1} [\lambda^b]_{j_2}^{i_2} [\lambda^c]_{j_3}^{i_3}$
$+ \frac{1}{8(N_c^2-1)N_c} \quad f_{ijk} \; f_{abc} [\lambda^i]_{b_1}^{a_1} [\lambda^j]_{b_2}^{a_2} [\lambda^k]_{b_3}^{a_3} [\lambda^a]_{j_1}^{i_1} [\lambda^b]_{j_2}^{i_2} [\lambda^c]_{j_3}^{i_3}$

This trick allows to write down the integrals in the relatively compact form, comparing to standard results involving the sum of $(m!)^2$ terms each being the product of $2m$ Kronecker delta's.

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