Report on the PhD thesis:

"Non-product geometries for particle physics and cosmology" written by Arkadiusz Bochniak,

under the supervision of prof. dr hab. Andrzej Sitarz
at the Department of Field Theory of the Institute of Theoretical Physics; Faculty of Physics, Astronomy and Applied Computer Science of the Jagiellonian University in Kraków.

The thesis is devoted to the study of geometry and dynamics behind the physical theories such as Einstein's theory of gravitation and gauge theory, and of their generalizations. On the mathematical side they are based on (Lorentzian) General Relativity and theory of connections and their curvatures on fiber bundles. A historical motivation for possible generalizations comes from another most successful theory of Quantum Mechanics, and on the mathematical side an appropriate area, complementary to string theory, is that of noncommutative geometry. More specifically, the appropriate notion is that of spectral triples, which consist of an algebra of operators on a Hilbert space together with a Dirac type opertor that satisfy certain analytic conditions. These mathematical structures have already been shown to be of importance for fundamental interactions, namely quantum field theory (standard model) of the strong and electroweak interactions, and to couple to general relativity (i.e. the gravitational interactions). The idea of unification of quantum and gravity theories has been attendant for about a century by now. Such a "quantum gravity" is being searched for not solely because of esthetical elegance, but more importantly to extend the validity of the individual theories besides their limits, as required by cosmology and by such 'phenomena' as extremely compact matter, black holes or big-bang. These really very difficult tasks require a profound examination and development of highly advanced concepts and methods.

The thesis of 143 pages long without doubt brings contributions in these directions, and incorporates three already published papers and two preprints (the input of the coauthors assumed equal):
1. A. Bochniak and A. Sitarz, *Spectral geometry for the standard model without fermion doubling*, Phys. Rev D 101, 075038 (2020)
2. A. Bochniak, A. Sitarz and P. Zalecki, *Spectral action and the electroweak θ-terms for the Standard Model without fermion doubling*, J. High Energ. Phys. 12, 142 (2021)
3. A. Bochniak and A. Sitarz, *Stability of Friedmann-Lemaître-Robertson-Walker solutions in doubled geometries*, Phys. Rev. D 103, 044041 (2021)
4. A. Bochniak and A. Sitarz, *Spectral interaction between universes*, arXiv: 2201.03839
5. A. Bochniak, *Towards modified bimetric theories within non-product spectral geometry*, arXiv: 2202.03765

The first section of the Thesis is a concise but efficient and updated introduction to Fredholm modules and spectral triples, their grading and reality, the so called first-order condition and Hodge property, reconstrution theorem of the geometrical data, twisted spectral triples, the spectral action principle, Wodzicki residue, gauge theories and almost-commutative geometries, spectral Standard Model, its fermion doubling problem, and Lorentzian formulation.

The main part, Section 2 "New directions", reports on the new and original material, based on the above mentioned references [1-5]. It extends the so far presented noncommutative (spectral) geometry of models in particle physics and gravitation theory based on the so called almost commutative framework, which is a product of the usual spectral triple on a spin manifold with a finite spectral triple describing the internal noncommuting degrees of freedom. Namely, therein non-product geometries are analysed from the perspective of their possible applications in particle physics and cosmology. It consists of two parts: Subsection 2.1 is dedicated to the Standard Model of particle physics, while subsection 2.2 to certain cosmological models.

In Sub-subsection 2.1.1 a triple from [1] is presented, that consists of the same algebra as in the case of the almost-commutative Standard Model, but acting on the (smaller) Hilbert space of rank 4 matrices (built from the Weyl spinors with a fixed chirality), and the Dirac-type operator differing from the usual (Minkowskian) one by a certain finite endomorphism. The fermionic action of this model with not product-type geometry by construction solves the fermion doubling problems, preserves color symmetry, and links the issue of real structure with the CP symmetry (my comment would be to express more clearly this issue in 2nd and 3rd sentence on p.38)

In Sub-subsection 2.1.2 the bosonic part is discussed basing on [2] which reproduces the right kinetic and interaction terms of the effective Lagrangian. The gauge transformations built as *fluctuations* of the Krein-shifted full Lorentzian Dirac operator, correspond to the unitary group $U(1) \times SU(2) \times U(3)$ (modulo a finite factor) as it should. Then, a suitable treatement of the bosonic action and Wick rotation, leads to interesting topological terms and to the Higgs potential with numerical value of one of the parameters different from the one obtained in the almost-commutative framework.

Sub-subsection 2.2.1, based on [3], generalizes some previous result with non-product geometry to (two-sheeted) doubled geometry model involving arbitrary Friedmann-Lemaître-Robertson-Walker spacetimes, and compares it with bimetric gravity theories. The derived equations of motion coupled with stress-energy tensor, are difficult to solve. It is shown that the classical solutions of the Einstein equations for each sheet separately are stable under infinitesimal perturbations.

Sub-subsection 2.2.2, based on [4], compares the doubled geometry model with the bimetric model. Though these two models possess similar features, if both their metrics are given as infinitesimal perturbation of the Euclidean ones, the expansions of their spectral actions in the perturbation parameter are different. The doubled geometry model, with a more clear geometric motivation, is proposed to describe e.g. the interaction of two four-dimensional branes living in a higher dimensional space.

Sub-subsection 2.2.2, based on [5], studies two-sheeted model with a more general class of metrics, and computes its spectral action, which contains more complicated logarithmic terms.

As already mentioned above, the thesis incorporates three original and significant publications in the international journals which have already undergone stringent refereeing processes, and two preprints on the arxive, which in my opinion are also of high quality. They are preceeded by brief introductions, which, like the main introductory section 1 are precise and well written; and the overall structure is clear and precise. This documents a systematic, original, independent and mathematically precise work which generalizes the product type almost commutative spectral description of both the Standard Model of particle physics and also of cosmological theories that go beyond the General Relativity.

Few minor comments to Subsection 1.1.1 which I can suggest would be to explain somewhat more in which sense 'The common understanding of geometric objects assets of points ... appears not to be fully satisfactory' (2nd sentence) and an impression that the adjective "easily" in the 6th sentence sounds perhaps too much optimistic. Few minor language corrections are "of" instead of "on" (p. 11_4), "which <u>is</u> a number" (p. 13_2), and "general" instead of "generic" (p. 17_6).

Summarizing, the thesis fulfils all requirements to award a doctoral degree. In my opinion it deserves to be awarded a distinction.

Prof. Dr Ludwik Dąbrowski
Mathematical Physics, SISSA
Trieste, Italy