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Report on the Ph.D. thesis
Linear and nonlinear perturbations of Einstein equations with matter
by **mgr Mieszko Rutkowski.**

As is well known the exact solutions to the classical Einstein field equations can be constructed with additional assumptions of symmetry, according to the simple rule: the higher symmetry - the better. This restricts the usage of the purely analytical nonperturbative approach to a limited class of rather idealized geometries. As the lack of symmetry usually invalidates exact treatment, one has to adopt either the numerical methods or some analytic approximation scheme, or both. Especially interesting in this regard is the perturbative approach, in which a real hard problem described by a complicated system of differential equations is replaced by an infinite set of relatively simple differential equations, that are (at least for a few lowest orders) analytically tractable. Of course, the construction of the solutions of the thus obtained chain of differential equations of ascending complexity may be (and usually is) a rather tedious process, requiring great calculational skills (or usage of the computer algebra) and is typically restricted to a few lowest orders. Moreover, generally speaking, the perturbative approach is inseparably related to the (divergent) asymptotic series. All this shows that although the main idea of the perturbative calculations is simple and clear, special care should be taken both in the calculations and in the interpretation of the thus obtained results. This, however, pays off, as the obtained results usually provide important information on the behavior of the system beyond the linear regime and may lead to new insights into the black hole physics.

The Ph.D. thesis of Mr. Mieszko Rutkowski entitled **Linear and nonlinear perturbations of Einstein equations with matter**, contains a detailed report of the calculations presented in his three papers. It should be emphasized that he is the sole author of one paper published in Physical Review D and a co-author of two papers published respectively in Physical Review D (with A. Rostworowski) and in Journal of High Energy Physics (with A. Jensen and A. Rostworowski). The thesis is composed of the introduction to the group of problems that are to be discussed, three chapters containing original results, final remarks, and four appendices. The reference list contains 56 items. Additionally, for the reader's

convenience, copies of the articles on which the work is based have also been attached.

Roughly speaking, the structure of the thesis reflects three (mutually related) research projects in which Mr. Rutkowski was involved. They are (in the order adopted in the thesis):

1. Construction of the master equation for the linear perturbations of the D-dimensional Einstein-Maxwell-scalar system.

2. Construction of the master equations for the nonlinear perturbations of the Reissner-Nordström spacetime.

3. An attempt to decide if the rotating gravastar may serve as a matter source for the Kerr spacetime.

Below I will briefly characterize the content of each chapter, with a special emphasis put on the original results. The objective of Chapter 2 is to generalize the well-known results of Kodama and Ishibashi to the Einstein-Maxwell-scalar systems in D-dimensions. It is shown that based on the assumptions regarding the number and type of perturbations in the scalar, vector, and tensor sector, assumptions regarding the type of the differential equations (and the form of the potential matrix) that 'master scalars' should satisfy, and finally the algebraic structure of the three gauge-invariant variables, it is possible to construct (rather complicated) formulae describing both the gauge invariants and the potentials. These equations and master scalars have been studied in several special cases. Moreover, when specialized to the Reissner-Nordström spacetime they reduce to the Kodama-Ishibashi master equations.

In Chapter 3 (which, according to the list of references, is based on Ref. [2]), the nonlinear perturbations of the Reissner-Nordström black holes are systematically studied. Specifically, it is shown that accepting a few assumptions regarding the number and type of the master equations at each perturbative level, the very nature of the source terms, and the structure of the perturbations, it is possible to study perturbations of the Einstein-Maxwell system in a systematic manner.

In Chapter 4 Mr. Rutkowski attempts to answer an important question if the (ultra-compact) gravastar may serve as a source term of the Kerr solution. And although the perturbative approach adopted by the author does not lead to a clear yes-no answer, the results presented in Ref. [3] may shed some light on this notoriously hard problem. The main idea is to use the Israel junction conditions for the external metric (the Kerr solution) and the internal metric (a gravastar) at some three-dimensional hypersurface. As the result of rather intricate calculations carried out with the aid of the computer algebra system, the conclusion may be drawn that the third-order approximation to a gravastar solution cannot be matched to the Kerr solution. In this demonstration an important role is played by the Kretschmann scalar ($R_{abcd}R^{abcd}$). Although the outcome of the calculations is somewhat pessimistic, this chapter contains a few very interesting results on their own, as, for example, the line element of the rotating gravastar calculated to the third order.

In the one-paragraph Chapter 5, Mr. Rutkowski gives some general information on the main subject of the thesis, emphasizing the role of the supervisor and co-author prof. Andrzej Rostworowski. Finally, at the end of the thesis, one can find four appendices containing material that has been relegated form the main text.

The thesis is well-written and has a crystal clear structure. Each chapter has its introduction and summary, and the material is introduced in an orderly fashion. Each chapter contains short information on possible extensions, generalizations, and applications of the presented results as well as their limitations. Moreover, anticipating some comments and (friendly) critique, Mr. Rutkowski explains why some seemingly obvious steps have not been taken or why they have been taken in some particular direction. And although it is a quite reasonable strategy from a point of view of the Ph.D. candidate, it simultaneously makes the life of a referee harder.

I have no critical comments and specific question may be asked during the public defense. However, a few equations are badly formatted. There are two rather mysterious reference items, namely [54] and [55], and Kerr's surname should start with a capital K. Vladimir Fock is the author of his famous book *The Theory of Space, Time and Gravitation*, not the editor.

In my opinion, results presented by Mr. Rutkowski in his thesis are solid, interesting, and important, substantially extending our understanding of the linear and nonlinear perturbations of the black holes. Indeed, generalization of the Kodama-Ishibashi analysis to the case of the Einstein-Maxwell-scalar systems in D-dimensions, as well as the study of the second order perturbations of the Reissner-Nordström black hole is certainly quite an achievment. Similarly, his attempt to match rotating gravastar solution to the Kerr metric, although inconclusive, contains some interesting ideas.

In conclusion, the Ph.D. thesis of Mr. Mieszko Rutkowski contains valuable and highly original results and satisfies all customary criteria as well as the formal requirements laid down by law. I strongly recommend that Mr. Mieszko Rutkowski should be granted the permission to defend his thesis.

Podsumowując, uważam że przedstawiona mi do recenzji praca doktorska dotyczy bardzo interesującej i aktualnej tematyki. Możliwe uogólnienia i nowe kierunki badań bezpośrednio związane z przedstawionymi przez autora rezultatami również przedstawiają się obiecująco. Samą zaś pracę oceniam bardzo wysoko i uważam, że zasługuje ona na wyróżnienie.

Uważam, że recenzowana praca spełnia zarówno ustawowe jak i zwyczajowe wymogi stawiane rozprawom doktorskim i wnoszę o jej przyjęcie i dopuszczenie mgr. Mieszka Rutkowskiego do dalszych etapów przewodu doktorskiego.

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