Dr. José Ignacio Latorre
Center for Quantum Technologies, Singapore

April 20th, 2022

Report on the thesis

“Structured Unitary Matrices and Quantum Entanglement”

by Wojciech Bruzda

The present thesis presents research in the area of Quantum Information, specially focus in the understanding of the mathematical structure underlying multipartite entanglement.

The thesis is structured in several parts. The first presents a solid mathematical background on Hadamard matrices. It digs into the properties of isolated complex Hadamard matrices and its relation to quantum states. This work may open new lines of applications to Mutually Unbiased basis or Unitary Error Basis. The chapter is deeply mathematical, and the results are spelled as theorems or well-founded conjectures.

A relevant result for the thesis is described in the part of the thesis devoted to Absolutely Maximally Entangled States (AME) for a system made out of four subsystems of dimension six. The field of AME is a fascinating one as it explores the quantum states that carry maximal quantum correlations in all their partitions. Prior to this thesis, an standing conjecture in the literature was the possible existence of AME(4,6), that is a maximally entangled state in all partitions for a system of four subsystem of dimension 6, or quhexes. This problem ican be understood as a quantization of the Problem of 36 Officers by Euler. The main result of the thesis is to prove this conjecture in the positive: such a states exists and is explicitly delivered. I’ve myself coded the analytic solution provided in the text and verified the result, not because of any mistrust but out of curiosity to analyze the structure of the state. I’m particularly interested in the open question number 4 in the thesis, namely in finding optimal ways to represent a quantum circuit to deliver this system.

A final chapter of the thesis is devoted to Bell inequalities, relating it to the excess of a matrix. This is useful to obtain the classical limit of families of Bell Inequaltities. Natural relation to local hidden variables emerge.

The thesis is a solid contribution to quantum information. It is well written, well structured and clear. I consider it as necessary research, with progress to stand the pass of time.

I'd like to add some final minor comments about bibliography. The first one is related to first paper introducing QME states. This was in Ref. 104. A second one is related to the idea of building quantum circuits that deliver AME states. This has been done in "Quantum circuits for maximally entangled states" by A. Cervera-Lierta, J. I. Latorre and D. Goyeneche.

Prof. José Ignacio Latorre
Director of the Center for Quantum Technologies, Singapore

Centre for
Quantum
Technologies

National University
of Singapore

National University of Singapore, Blk S15, #03-18, Science Drive 2, Singapore 117543. Website: www.quantumlah.org