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To Whom It May Concern:

I am writing to provide a review of the thesis:

"*Numerical ranges and geometry in quantum information: Entanglement, uncertainty relations, phase transitions, and state interconversion*"

by Mr. Konrad Szymański of Jagiellonian University, Krakow.

The thesis is in the area of quantum information theory, which is a rapidly expanding an important area of research, driven in part by efforts to develop Quantum Technologies. Such technological goals offer dramatic advantages in computation, control over nano-scale devices, quantum-enhanced sensing and advances in communication. The present thesis contributes to this field from the mathematical perspective and draws on tools and concepts in geometry and algebra to deepen our understanding of a range of fundamental concepts in quantum information theory. These topics are both timely and important, and such analysis is valuable with a range of possible applications.

The thesis stems from 5 published works of the author (with collaborators), that have been published in world-class journals (Phys. Rev. A, Journal of Physics A: Mathematical and Theoretical, Linear Algebra and its Applications, Europhysics Letters).

One major focus of the thesis is on "numerical ranges" in quantum mechanics, which is a fundamental structural aspect of quantum theory that encodes such aspects as quantum uncertainty relations, aspects of phase transitions and aspects of entanglement. It provides an abstract and geometric means to delineate how quantum physics differs from classical physics.

A second focus of the thesis is on the ability to transform quantum states amongst themselves subject to fundamental constraints, such as a symmetry constraint. Such tools arise in the study of entanglement theory, thermodynamics of quantum systems, quantum computing, and other quantum information resource theories.

Chapter 1 provides a clear and concise overview of the entire thesis.

Chapter 2 introduces the fundamental concepts of quantum theory in a self-contained and accessible form. The provides the conceptual background for the rest of the thesis and good care is taken to present the relevant concepts in a clear form with consistent notation.

Chapter 3 discusses more specialised tools in convex geometry and algebraic geometry. This is a dense chapter, but is presented well with useful illustrations.

Chapter 4 begins by setting out the core feature under investigation — the numerical range of a quantum system — and establishes several properties and results that were new to me. A particularly elegant classification of the numerical range for a triple of observables is obtained, and illustrated with clear figures. This analysis is then applied to the topic of uncertainty relations, and provides novel techniques that are demonstrated in the case of the angular momentum of a quantum system. Following this an application of the earlier theory is made to the topic of quantum phase transitions and a means to bound the energy gap of a many-body system presented. This is done by linking the problem to the geometry of a particular numerical range. The author then demonstrates this with the XY spin chain system. Finally, an application to the problem of entanglement detection is provided.

Chapter 5 considers the interplay between symmetries and the processing of quantum information under quantum channels. It first re-caps on the fundamental results of this field before developing novel concepts that link symmetry-constrained transformations and the topic of majorization theory (which arose originally in statistics). The relation between these two is handled very carefully to circumvent obstacles such as the group theory setting requires infinite dimensions, but the majorization is finite dimensional. Clear and novel insights are presented that make previous results in the literature extremely simple. The author also establishes novel results on the number of states accessible from a given quantum state while respecting underlying energy conservation.

Chapter 6 provides a conclusion and discussion of the results in the thesis and outlines future research lines to build on this work. Appendices are provided with additional figures and code used. The bibliography is appropriate and covers the core works in these areas.

Overall the results of the thesis are significant and valuable. The most important results are from Chapter 5 on the numerical range, and it was impressive to see that these ideas could be applied to three very distinct problems (uncertainty relations, phase transitions and entanglement).
The thesis is very well written, although sometimes topics are described in words where I would have liked to have seen more mathematical detail. The work conveys mastery of the topic and a researcher who has engaged deeply with the material.
There are a range of typos throughout, but no more than one would expect with a thesis of this size and difficulty. The main aspects that I feel need work are some of the proofs and description of the concepts. In various points the details are very compressed and so difficult to easily verify claims. I feel this was a stylistic choice on the part of the author, however I have flagged a range of places where more detail should be provided. In particular this makes Chapter 4 a challenging body of material to fully digest and check. One way to maintain the overall narrative style while also being careful with technical details would be to provide explicit analysis in the appendices and then refer to this in the main text where appropriate.

In the PhD defence I would assess the candidate's fundamental understanding of the concepts exploited in the thesis, and in particular assess how they see these results fitting into the broader picture of research being conducted worldwide. For me the application to entanglement theory was the most opaque and so I will ask the candidate to expand more clearly on their precise claims in the defence. I will also ask for the candidate's views on how this work could be built on, and its significance in relation to prior results.

In summary, based on the above I consider that the candidate has met all the criteria as stipulated in the act on academic degrees (Dz. U. 2016, item 882). The work presented is both original, displays all the skills and outcomes expected from a PhD of this kind, makes a valuable contribution to the on-going research in this field, and meets the international standards of a PhD in theoretical physics. I judge that it has met all requirements needed to proceed to the PhD defence and I look forward to attending the defence once a date has been decided.

Yours sincerely

David Jennings