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Report on the thesis
Symmetry and Classification of Multipartite Entangled States
by Adam Burchardt

Introduction:

The PhD thesis, entitled "Symmetry and Classification of Multipartite Entangled States" by Adam Burchardt, has been carried out at the Faculty of physics, astronomy, and applied informatics at the Jagiellonian University in Kraków. The supervisor of the thesis is Prof. Karol Zyczkowski.

The PhD thesis deals with the study of multipartite entanglement. It is written in English and consists of 160 pages (excluding the attached publications and preprints). Mr. Burchardt structured the thesis into eight chapters and five appendices. The appendices contain those publications and preprints on which the thesis is based on. Two of these papers were written without the supervisor of the PhD thesis. Mr. Burchardt co-authored three additional papers (one publication and two preprints).

In Chapter 1 a very brief introduction to some aspects of quantum information theory and the structure and aims of the thesis is given. Chapters 2,3,5,6,7, contain the research performed during the thesis and chapter 4 recalls some properties of absolutely maximally entangled and k-uniform states. In the last chapter, the results presented in the thesis are summarized and a brief overview over some open problems is given.

Assessment:

The main aim of the works presented in the thesis is to achieve a better understanding of multipartite entanglement, which is one of the most relevant and challenging problems within quantum information theory. To this end, several approaches are employed and developed. In the following I give a survey and assessment of each chapter.

Chapter 1 introduces some basic concepts in quantum information theory. In my opinion the introduction to a thesis should be much more elaborated. The author should explain there the main results and challenges in the field, and explain how the research presented in the thesis fits into that. In case the introduction would have been more extended, the author could have also included the content of Chapter 4 there. As mentioned already before, in Chapter 4 the author briefly recalls the relation between absolutely maximally entangled states (AME) and classical combinatorial designs. As far as I can see this chapter does not contain any new results, but rather serves as an introduction to the subsequent chapter. I believe that it might be better to include that in Chapter 1.

In the following, I summarize the content of the remaining chapters and comment on them below.

In Chapter 2 a link between the robustness of entanglement with respect to particle loss and knot theory is established. The notion of m-resistant states, which are states that remain entangled if one traces over m subsystems, but become separable when tracing over more than m subsystems is presented. Methods for constructing these states are shown and various examples are given.

In Chapter 3 (and the corresponding publication) states which are invariant under a subgroup of the permutation group are considered. Methods to construct these states are presented and it is shown that circuits generating states with a smaller symmetry group than the whole permutation group can be realized with a smaller computation complexity. As an illustration a five-qubit states is prepared on the IBM computer. Moreover, bipartite entanglement measures are computed for certain symmetric multipartite states.

In Chapter 5 one of the most important results of this thesis is presented, namely the AME state of four six-level systems. Despite a big effort in the community, the existence of this state was unclear until Mr. Burchardt together with collaborators presented their example. This instance of an AME state also leads to a solution for the quantum mechanical counterpart of the 36 offices problem of Euler and implies that not all AME states are stabilizer states, which is a very interesting consequence of this work.

In Chapter 6 the local equivalence (via local unitary operators as well as invertible operators) of AME states is investigated. In particular, it is shown that AME states of a given Hilbert space are not necessarily related to each other by local operators.

The main aim of Chapter 7 is to derive simple necessary and sufficient conditions for the SLOCC—equivalence of qubit states. It is shown that the roots of a SL-invariant polynomial can be employed to decide whether two states are SLOCC—equivalent or not.

The results presented in the thesis are highly interesting, especially Chapter 5,6, and 7. I believe that these results and the methods developed there will trigger future research on absolutely maximally entangled states, states with particular symmetry and SLOCC and local unitary equivalence.

However, I believe that the results could have been presented in a better way. The unusual approach of the author to focus in the various chapters on some aspects of the results obtained in the corresponding papers, makes it (in my opinion) more difficult to read. The only exception to this strategy seems to be Chapter 7, which contains all the relevant information. As the various chapters are very related to each other, I believe that an extended introduction to multipartite entanglement in general and AME states in particular, accompanying a clear summary of the results and the reprints of the papers would have increased the readability of the thesis significantly. I believe that the author's motivation for his way of presenting the results was that he wanted to state clearly what his main contributions were.

Summary:

The work presented in this thesis, is based on three publications and two preprints, which are very interesting and are all of high quality. Overall this work deals with multipartite entanglement, which is a highly relevant field of quantum information theory and well beyond. The work presented here is broad and innovative, and likely to lead to further research along the lines presented.

Recommendation:

In summary, it is very clear to me that this thesis presents work of the necessary standard to fulfil the requirements for the award of the PhD degree. I thus recommend, without hesitation, that the thesis is accepted for the oral defense.

Sincerely,

Barbara Kraus