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## Referee report for the thesis of Adam Burchardt

Multipartite entanglement plays an outstanding role in various applications of quantum information processing, such as quantum metrology or measurement-based quantum computation. Moreover, multiparticle correlations are relevant for foundational studies on quantum mechanics, e.g. in the form of non-locality proofs. The present thesis entitled “Symmetry and Classification of Multiparticle Entangled States” by Adam Burchardt deals with several topics in the theory of multipartite entanglement and presents groundbreaking results on some relevant and long-standing problems.

The thesis starts with an introduction, where an overview on the topics is given. This is rather short and concise, but demonstrates that the author has an overview over the topic.

In Chapter 2 the notion of  $m$ -resistant states is discussed. These are states, whose entanglement is robust under loss of  $m$  particles, but not more. Different families of such states are given and discussed.

Chapter 3 discusses multiparticle states with certain symmetries. Families of states with such symmetries are introduced and their physical properties are discussed. This is all interesting and relevant, but here I have a minor comment on the presentation: When constructing states based on the hypergraph formalism (p. 35), it would have been valuable to note that other types of “hypergraph states” exist [see, e.g. Rossi et al., New J. Phys. 15, 113022 (2013)]. This is mentioned in the associated publication, but not in the thesis.

The following chapters 4,5, and 6 are all concerning so-called absolutely maximally entangled (AME) states and contain the main results of this thesis. AME states have

recently moved into the center of attention in entanglement theory, as they are related to tasks like quantum error correction and secret sharing, moreover, some of the central open questions in entanglement theory concern AME states.

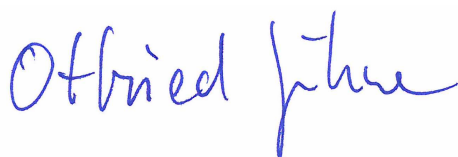
Chapter 4 starts the discussion by introducing the notions of AME states,  $k$ -uniform states and orthogonal arrays. This lays the ground for Chapter 5, which contains the highlight of this thesis: The explicit example of an AME state of four six-dimensional systems. The question whether this state exists or not may seem peculiar, but it was indeed one of the central open problems in the field of quantum information theory (see also P. Horodecki et al., Five open problems in quantum information, arXiv:2002.03233).

I know that many people (including me) spent a lot of effort in solving this, and the fact that the author solves this in his PhD thesis proves that he is one of the original thinkers in entanglement theory. Chapter 6 discusses then the situation, where several AME states for a given dimension and number of particles exist. It is shown by examples and general statements that two AME states do not have to be equivalent under stochastic local operations and classical communication (SLOCC). This is also an interesting result, which was discussed before in the community, but not solved.

Finally, Chapter 7 present generic conditions to decide whether two  $N$  qubit states are SLOCC equivalent or not. This is done by considering special entanglement measures.

In summary, this an outstanding thesis. The work has resulted in five publications, where the author acts in four of them as a leading author. It is also remarkable that two of the publications were written without the supervisor. In this way, the author demonstrates independent and original thinking. Clearly, this thesis meets all international standards and I am pleased to recommend to proceed with further steps of the Ph.D. procedure.

In addition, since in this thesis an outstanding open problem in entanglement theory is solved, I strongly support to award this thesis a distinction.



Prof. Dr. Otfried Gühne