Warszawa, September 5, 2021r.

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Review of the Ph.D. Thesis of Yuriy Volkotrub
titled: *Covariance matrix of nucleon-nucleon potential parameters in few-nucleon studies.*

Doctoral thesis of Yuriy Volkotrub is devoted to studies of statistical uncertainties of 3N observables arising from propagation of uncertainties of nucleon-nucleon potential parameters. For this purpose covariance matrix technique has been applied. Moreover correlations among 2N and 3N observables have been investigated. Unfortunately I could not find information concerning publications on which the thesis is based. In the list of references, at the end of the thesis, I managed to find four papers coauthored by Yuriy Volkotrub. These include three papers in Physical Review C and one paper in Journal of Physics G. The number of coauthors of these papers ranges from five to twenty. In one of these papers Yuriy Volkotrub is the first author, which suggest that his contribution was dominating.

The thesis consists of an overview, six chapters, summary and three appendices. In the overview author presents the motivation for the study undertaken in the thesis.
The first chapter is devoted to the introduction to the problem of constructing and parametrizing nuclear Hamiltonian based on experimental information, coming mainly from NN scattering events. Author considers and investigates two types of parametrizations. The first one is based on one-pion-exchange-gaussian potential (OPE-Gaussian), and the second one originates from chiral effective field theory (up to N4LO terms). It is worth noting that author put his research in a broader context of adjusting NN potential parameters, in particular he discussed problems of setting 3-nucleon force (3NF) parameters.
In the second chapter Yuriy Volkotrub described properties of OPE-Gaussian potential and its parametrization. Next he discussed potentials based on chiral effective field theory, with semilocal regularization in momentum space (SMS potential).
Chapter 3 presents the procedure which allows to determine correlations of the potential parameters and to relate them to available 3N observables. Author decided to sample 50 sets of potential parameters and subsequently evaluate both 2N and 3N observables and their energy and scattering angle dependencies. For quantification of statistical uncertainties author has chosen a particular estimator. The truncation errors in the case of NN interaction based on chiral effective field theory has been extracted using both EKM method and Bayesian approach. Finally author has discussed the method to evaluate correlations between 2N and 3N observables by defining standard sample correlation coefficient.
Chapter 4 ends the introductory part of the thesis. Author presents there theoretical formalism which allows for extracting observables from a given set of potential parameters. These include Lippmann-Schwinger framework for 2N scattering and Faddeev equations for 3N scattering. Moreover author gives a brief overview of both 2N and 3N observables used in the thesis. It has to be emphasized that both Coulomb interaction and 3N forces have been neglected in the analysis.

Chapters 5 and 6 contain the main results of the thesis. In chapter 5 author has analyzed predictions obtained within both SMS interaction at different orders and OPE-Gaussian potential for neutron-deuteron scattering. The goal was to evaluate theoretical uncertainties for scattering observables extracted at various neutron laboratory energies up to 200 MeV. For the observables evaluated in the case of elastic scattering author has formulated the main conclusion that magnitude of statistical uncertainties for SMS interaction up to N4LO terms and for OPE-Gaussian potential are of the same magnitude. Moreover statistical uncertainties are smaller as compared to both the truncation

errors and uncertainties of predictions due to variations of the cutoff parameter. Author has studies also selected kinematical configurations for deuteron breakup reactions and concluded that statistical errors are still small in this case.
Subsequently in chapter 6 author has considered correlations between 2N and 3N observables and potential parameters. He has found that the correlation coefficients for both 2N and 3N observables reveal a nontrivial dependence on scattering angle. Moreover the polarization is weakly correlated with other 2N observables, whereas differential cross section is strongly correlated with other observables except of polarization at low energies (up to 30MeV). Correlations between other 2N observables are moderate or weak. For 3N observables correlation coefficients exhibit complex behavior as a function of scattering angle, which author attributed to nonlinear dependence of observables on potential parameters.

Summarizing, author has performed a thorough studies of two NN forces and their parameters. The first one based on OPE-Gaussian and the second one constructed in the spirit of chiral effective field theory up to N4LO terms. In both cases Coulomb interaction and 3NF terms were neglected. Author has applied covariance matrix of NN potential parameters in order to study statistical uncertainties of 3N observables and correlations between 2N and 3N observables. In particular he has shown that for certain pairs of 2N observables the correlation is almost linear. It was also shown that correlations may turn out to be strong for certain scattering angles. In the case of chiral potentials statistical uncertainties turned out to be smaller than trunctation errors.

In my opinion the thesis represents a solid and thorough research which will be useful for construction of models of NN interaction. Although there are no startling results in the thesis, it shows the usefulness of a technique based on the usage of covariance matrix, which may shed light on the dependencies between force parameters.
The thesis is clearly written and I did not notice any serious mistakes or omissions. I have only the following minor remarks:

- section 3.1.3 devoted to Bayesian model is described very briefly and it is therefore difficult to grasp the meaning of symbols in eqs. 3.7 and 3.8.
- In Fig. 5.1 there is no solid red line described in the caption, instead there is red line with circles. There is also no cyan line with open squares – instead there is a cyan line with crosses.

In general symbols in almost all figures in the thesis are slightly too small and sometimes it is difficult to distinguish their shapes or to distinguish the open symbols from the filled ones.
Despite these minor shortcomings, in my opinion, the thesis fulfills all requirements that have to be met by PhD theses and therefore I herewith request to admit Yuriy Volkotrub to further stages of PhD procedure.